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π Condensation in Nuclear Matter

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(Received 17 April 1973)

It is shown that in nuclear matter at $Z=0$ (neutron star) at a density $n_1 < n_{\text{nucl}}$ a π^0 condensate appears. Nearly at the same density an electrically neutral π^+, π^- condensate arises. The π^- condensate assumed by other workers apparently does not arise even at very high densities.

The π condensation in nuclear matter was first considered by the present author.[1,2] Later the problem was considered once more by others[3,4] for the case of a neutron star ($Z \ll N$). Sawyer[3] and Scalapino[4] came to the conclusion that at some nuclear densities, a π^- condensate arises (the charge is compensated by the same amount of protons). The same result is assumed by Kogut and Manassah.[5] A more realistic consideration given below does not confirm these conclusions.

Let us write the condition for the instability of nuclear matter with respect to the reaction $n \to p + \pi^-$. We have

$$\mu_p - \mu_n + \omega(k_0) = 0,$$

where μ_p, μ_n are the proton and neutron chemical potentials, $\omega(k_0)$ is the minimal energy of π^- in nuclear matter, and k_0 is the corresponding wave vector. For small proton densities, $\mu_n - \mu_p \cong \epsilon_F^{(n)}$. Thus the instability arises only when the π^- energy is less than the Fermi energy of the neutrons. A detailed consideration of the pion energy-momentum relation taking into account the pion-pion interaction shows that an instability does not arise up to very high nucleon densities at least. Even if the instability arises, increase of the π^- density in any case would be limited by the pion-pion interaction. Let us start with the case $N=Z$ investigated by Migdal.[6] There are two branches of the meson spectrum: the "meson" branch, which tends to the free meson energy as the nuclear density n tends to zero, and the "spin-sound" branch which coincides with the spin-sound excitations[7] in nuclear matter, when the meson-nucleon interaction is switched off. At $n \cong 0.5 n_0$ (n_0 is the usual nuclear density) the spin-sound branch becomes unstable [$\omega^2(k)$ is negative for some $k \cong k_0$]. This instability leads to formation of an electrically neutral meson condensate $\varphi_1 = \varphi_2 = \varphi_3$, with

$$\varphi_{\pi^\pm} = 2^{-1/2}(\varphi_1 \pm i\varphi_2), \quad \varphi_{\pi^0} = \varphi_3.$$

In the case $Z \ll N$ these results hold for π^0 mesons (Fig. 1) but the spectrum of π^+ and π^- entirely changes.

The polarization operator $\Pi(\omega, k)$ for ω and k of interest ($\omega \lesssim 1$, $k \lesssim m$, $\hbar = c = m_\pi = 1$, m is the nucleonic mass) is given by two types of graphs:

$$\Pi(\omega, k) = \omega^2(k) - 1 - k^2 = D_1 + D_2, \tag{1}$$

where D_x is the term represented by diagram x in Fig. 2. The term D_1 corresponds to the absorption of a pion by a nucleon with the formation of a hole in the Fermi distribution. The shaded vertex means that the nucleon-nucleon interaction is taken into account. This vertex can be ex-

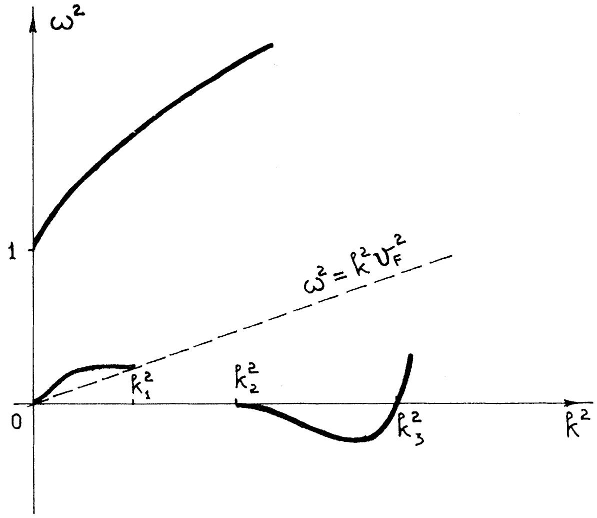

FIG. 1. Branches of π^0 spectrum for $n = n_0$. There is a region of instability with $\omega^2 < 0$.

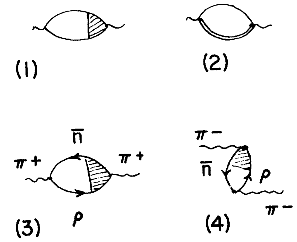

FIG. 2. Diagrammatic terms for Eqs. (1) and (2).

pressed through the constants characterizing the internucleon spin-spin interaction in nuclei.[7] D_2 corresponds to the transition of the pion and nucleon into the Δ_{33} resonance. All other graphs involve large four-momenta in intermediate states and differ slightly from the corresponding graphs in vacuum. They are taken into account in the observable mass of the pion used in Eq. (1). The left vertex of the D_1 equals $\Gamma_\alpha = (g/2m)\vec{\sigma}\cdot\vec{k}\tau_\alpha$, where $g^2/4\pi = 14$ and α is the pion isotopic index. The vertices of D_2 are determined by the amplitude of (π, N) resonance scattering.

In the case of $Z = 0$, the term D_1 for π^+, π^- mesons can be written in the form

$$\Pi_+^{\mathcal{P}} = D_3, \quad \Pi_-^{\mathcal{P}} = D_4. \tag{2}$$

The graphs for π^+ and π^- mesons differ only by the sign of ω. A similar relation takes place for the graphs containing the Δ_{33} resonance.

The calculation of the first of Eqs. (2) gives, in the case of nonrelativistic nucleons [omitting the internucleon interaction at this stage, and using for the vertex $\Gamma = (\sqrt{2}g/2m)\vec{\sigma}\cdot\vec{k} \equiv \sqrt{2}f\vec{\sigma}\cdot\vec{k}$, $f \cong 1$)],

$$\Pi_-^{\mathcal{P}}(\omega, k) = 4f^2k^2\int \frac{n^{(n)}(p)}{-\omega - E^{(p)}(\vec{k}-\vec{p}) + E^{(n)}(p)}\frac{d^3p}{(2\pi)^3} = -\frac{m^3}{2\pi^2 k}f^2\left\{-\tfrac{1}{2}(a^2 - b^2)\ln\frac{a+b}{a-b} + ab\right\}, \tag{3}$$

where $a = \omega + k^2/2m$, $b = kv_F$, $n^{(n)}(p)$ is the Fermi distribution function, $\Pi_+^{\mathcal{P}}(\omega, k) = \Pi_-^{\mathcal{P}}(-\omega, k)$. For $|a| \gg b$ we have

$$\Pi_-^{\mathcal{P}} = -\frac{2nf^2k^2}{\omega + k^2/2m}, \quad \Pi_+^{\mathcal{P}} = \frac{2nf^2k^2}{\omega - k^2/2m}. \tag{3$'$}$$

In addition to these graphs the S-wave scattering which is negligibly small at $N = Z$ should also be taken into account:

$$\Pi_-^{S} = -4\pi n\tilde{F}_-^{S}, \quad \Pi_+^{S} = -4\pi n\tilde{F}_+^{S}, \tag{4}$$

where $\tilde{F}_\pm^{S}$ are the amplitudes of the π^- and π^+ mesons undergoing S-wave scattering on a neutron. These amplitudes may be expressed through the amplitudes $F_\pm^{S}$ on the mass shell:

$$\tilde{F}_\pm^{S} = \tfrac{1}{2}(F_+^{S} + F_-^{S}) \mp \tfrac{1}{2}\omega(F_-^{S} - F_+^{S}) \cong \pm 0.1\omega.$$

For the second type of graphs (1) (resonance scattering) far from the resonance we have the following (neglecting the nucleon and Δ_{33} kinetic energy difference):

$$\Pi_-^{R}(\omega, k) = -4\pi nF_-^{R} = -0.75n\left(\frac{3}{\omega_R - \omega} + \frac{1}{\omega_R + \omega}\right)k^2,$$
$$\Pi_+^{R}(\omega, k) = \Pi_-^{R}(-\omega, k), \quad \Pi_0^{R}(\omega, k) = \tfrac{1}{2}[\Pi_+^{R}(\omega, k) + \Pi_-^{R}(\omega, k)]. \tag{5}$$

The vertices in the resonance graph are taken in the form $\Gamma = Ck$ (P-wave scattering); the resonance energy is $\omega_R = 2.4$. The constant C has been chosen so that at $\omega = (1+k^2)^{1/2}$ the observed amplitude of the resonance scattering could have been obtained. The second term in the brackets of the formula for Π_-^R takes into account the u channel, that makes a small contribution as $\omega \to \omega_R$, but is essential at $\omega \ll \omega_R$.

As a result we get the following expressions for the determination of π^-- and π^0-meson spectra:

$$\omega^2 = 1 + k^2 + 1.3n\omega - 0.75nk^2\left(\frac{3}{\omega_R - \omega} + \frac{1}{\omega_R + \omega}\right) - \frac{2nf^2k^2}{\omega + k^2/2m}, \tag{6}$$

$$\omega^2 = 1 + k^2 - 3nk^2\frac{\omega_R}{\omega_R^{\,2} - \omega^2} - \frac{mf^2p_{\rm F}k^2}{\pi^2}\Phi(\omega, k). \tag{7}$$

For simplicity in the last term in (6) the expression (3) is replaced by (3′).

$$\Phi(\omega, k) = \frac{\pi^2}{mf^2p_{\rm F}k^2}\left(\frac{\Pi_+^{\varphi} + \Pi_-^{\varphi}}{2}\right).$$

The account of the nucleon interaction (the dashed vertex in the term D_1), as is shown in the theory of the finite Fermi systems,[7] may be reduced to multiplication of the last term in Eq. (7) by the factor

$$[1 + g^{nn}(k)\Phi(\omega, k)]^{-1},$$

where $g^{nn}(k)$ is the quantity characterizing the neutron spin-spin interaction. For $Z = N$, $K \ll 2p_{\rm F}$, $g^{nn}(k) = 0.5p_{\rm F}$ and decreases with the increase of k. The account of the nucleon interaction in Eq. (6) is reduced to multiplication of the last term on the right-hand side by the factor

$$\left[1 + g^{np}(k)\frac{\pi^2}{mf^2p_{\rm F}k^2}\Pi_-^{\varphi}\right]^{-1},$$

where $g^{np}(k)$ for $N = Z$ and $k \ll 2p_{\rm F}$ is equal to $g^{np} = -0.3p_{\rm F}$. For $Z \ll N$ the functions $g^{nn}(k)$ and $g^{np}(k)$ are not known, but they must not have a noticeable difference from analogous values for $N = Z$. The curves in Figs. 1, 3, and 4 are calculated for $g^{nn} = 0.5p_{\rm F}$ and $g^{np} = -0.3p_{\rm F}$. The results do not depend essentially on the magnitudes of these values. The spectrum of π^+ is obtained from the spectrum of π^- by replacing ω by $-\omega$. The result of the analysis (6) and (7) is given in Figs. 1, 3, and 4. For simplicity we have omitted in Figs. 1, 3, and 4 the branch of (6), (7) which corresponds to the energies ω close to the resonance energy, and is unimportant for our consideration. The dots indicate false solutions of (6). The selection of physical solutions is defined by the following rule. The second quantization of mesons moving in the medium and having a polarization operator $\Pi(\omega, k)$ dependent on the frequency is reduced to quantization of the field with a Lagrange function with time derivatives of ψ of arbitrary

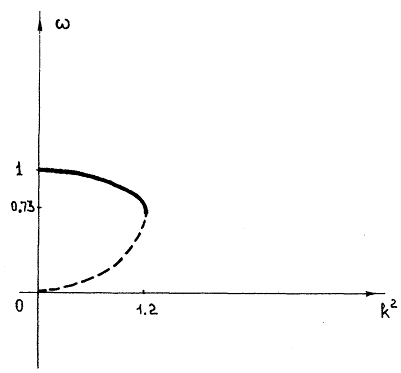

FIG. 3. The energy-momentum relation for π^- in the case of a neutron star with $n = 0.4 > n_c$.

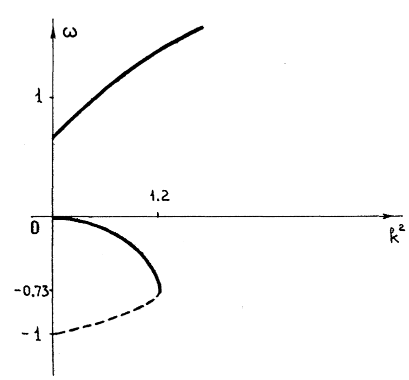

FIG. 4. The energy-momentum relation for π^+ in a neutron star with $n = 0.4 > n_c$.

order. Making use of the usual formalism and supposing that

$$\psi=\sum_k\{C_k^{(+)}a_k\exp[i(\omega_k^{(+)}t-\vec{k}\cdot\vec{r})]+C_k^{(-)}b_k^\dagger\exp[-i(\omega_k^{(-)}t-\vec{k}\cdot\vec{r})]\},$$

it is easy to obtain

$$C_k^{(+)}=[2\omega_k^{(+)}-\partial\Pi^+/\partial\omega]^{-1/2},$$

$$C_k^{(-)}=[2\omega_k^{(-)}-\partial\Pi^-/\partial\omega]^{-1/2}.$$

With this the Hamiltonian will acquire the form

$$H=\sum_k\{a_k^\dagger a_k\omega_k^{(+)}+b_k^\dagger b_k\omega_k^{(-)}\}.$$

Therefore, only solutions to (6) satisfying the following condition are of physical sense:

$$2\omega_k-(\partial\Pi/\partial\omega)_{\omega=\omega_k}>0.$$

At the points where $2\omega_k=(\partial\Pi/\partial\omega)_{\omega=\omega_k}$, an instability arises with respect to $\pi^+\pi^-$ pair creation and the meson interaction should be taken into account for the restoring of stability. With the neutron density increasing there arises an instability for π^0-meson creation. From (7) putting $\omega=0$ and $k\cong 2p_F$ we have $(p_F)_C\cong 2.2$, $n_C\cong 0.35$. At $n>n_C$ a π^0-meson condensate arises analogous to that in the medium with $N=Z$. Approximately at the same density an instability with respect to the meson-pair creation arises and an electrically neutral condensate of $\pi^+\pi^-$ mesons appears. The presence of the π^0 condensate leads effectively to a mass increase of π^+, π^- mesons. For instance, for the $\frac{1}{4}\lambda(\varphi_\alpha\varphi_\alpha)^2$ interaction type the positive term $\frac{3}{2}\lambda\varphi_0^2\psi^\dagger\psi$ is added to the $\pi^\pm$ meson Lagrangian, where φ_0 is the π^0-meson condensate field. For the same reason, within the total density region under consideration the minimal π^--meson energy exceeds the neutron Fermi energy and π^- condensate considered by Sawyer[3] and Scalapino[4] does not arise. It should be noted that if such a condensate had arisen, the meson interaction would have limited the increase in π^- meson density ν. For example, with a $\frac{1}{4}\lambda\varphi^4$ interaction, for the π^--meson frequency to be defined one should solve the problem of the anharmonic oscillator (1), and the meson condensate energy would be $\sim\lambda^{1/3}\nu^{4/3}$ that is, approximately the same value as that for relativistic electrons, and ν is, thus, of the same order as the electron density in a neutron star ($\nu/n\sim 0.01$).

Additional remarks.—(1) It seems at first sight that the exchange part of the pole graph is counted two times in $\Pi^{\mathcal{P}}$ and in Π^R. To avoid this we had extracted the corresponding term from the observable $(\frac{3}{2},\frac{3}{2})$ amplitude. In the expression $\Pi^R=-4\pi nF^R$, F^R is only the resonance part of F_{33} amplitude. At $\omega=\omega_R=(1+k_R^2)^{1/2}$ the exchange part introduces only 5%–7% but as $\omega\to 0$ it gives a very large contribution. To obtain a correct expression for Π at $\omega\to 0$ we should include the exchange part in $\Pi^{\mathcal{P}}$ but not in Π^R.

(2) When $2\omega=\partial\Pi/\partial\omega$ the velocity $d\omega/dk=\infty$ and $\omega_++\omega_-=0$ (see analogous situation in Ref. 1).

(3) It can be shown that graphs omitted in our calculations give a correction $\sim\mu/m$. So it is by no means a perturbation theory in terms of f^2 [see Eqs. (1) and (6)].

(4) The quantity $g^-(k)$ which determines the spin-spin interaction in the polarization operator differs from the corresponding quantity introduced in the theory of finite Fermi systems, because by definition, it does not contain the one-meson graph. As $k\to 0$ these quantities are equal and we can use the values $g^{nn}(0)$ and $g^{np}(0)$ given in Ref. 7.

(5) From our considerations it follows, evidently, that the phase transition of second order assumed in Refs 3–5 is impossible, but the first-order transition is not excluded. Really from Fig. 4 it follows that π^+ mesons have negative energies in a neutron star. This means that in a proton medium the π^- meson has negative energy. It is not excluded that a transition of first order is possible, i.e., at some density the neutron star may turn into a *proton star* in which the density of protons and π^- mesons is greater than the neutron density. This question will be discussed elsewhere.

A more detailed consideration will be given in a future paper.

The author expresses his thanks to A. A. Migdal, A. M. Polyakov, V. A. Khodel, and his collaborators O. Markin and I. Mishustin.

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