we can then write the partition function of Eq. (6) as

$$Z(H,T)=Z_{Ld-1}(T)\int_{-\infty}^{\infty}dx\exp[-\beta mHNx-\tfrac{1}{2}NU_{Ld}(2^{1/2}x)], \tag{14}$$

where for $2^L \gg \xi$, $U_{Ld}(x)$ has minima[10] which depend only on T. For N large we can perform the final integral by using the saddle-point approximation. Expanding

$$U_{Ld}(2^{1/2}x)\approx U_{Ld}(2^{1/2}x_0)+U_{Ld}''(2^{1/2}x_0)(x-x_0)^2+\cdots, \tag{15}$$

we then have

$$\lim_{H\to 0}\chi(T)=\tfrac{1}{2}M^2\beta N/U_{Ld}''(2^{1/2}x_0), \tag{16}$$

where we have $x_0=0$ for $T>T_c$ and $x_0=M$, the reduced spontaneous magnetization per spin, for $T<T_c$.

Using the above formulas the spontaneous magnetization and the susceptibility were computed numerically to determine their temperature dependences above and below T_c. We find for $d>2\sigma$, $\gamma=\gamma'=1.000$, $\beta=0.500$ as expected. For $d=3$, $\eta=0.06$ as a comparison with the three-dimensional Ising model, we find $\gamma=\gamma'=1.256$, $\beta=0.3429$, with a numerical precision of ± 1 in the last place quoted for all the preceding numbers. Thus we find Eq. (3) to hold in these cases and all the other cases we have investigated numerically.

Using the techniques described above it is also possible to determine the spin-spin correlation function once the fluctuations become small. For $T>T_c$ we find,

$$\langle \nu_1\nu_j\rangle \approx \frac{\chi_0^{\,2}(1-2^{-\sigma/d})K}{2^{(d+\sigma)l/d}(1-2^{-(1+\sigma/d)})}, \tag{17}$$

where $\chi_0=\chi/(m^2\beta N)$ is the reduced susceptibility. Since $2^{l/d}\approx r$, we have a decay for r large, but still small compared to the system size, proportional to $r^{-(d+\sigma)}$ as expected,[7,8] for $0<\sigma<d$.

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Experimental Study of Oscillatory Values of g^* of a Two-Dimensional Electron Gas

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We have measured the surface electron density and the magnetic field dependence of the extremes of the oscillatory quasiparticle g factor in a two-dimensional electron gas.

An experimental approach with which one can measure the extreme values of the oscillations of the quasiparticle g value, g^*, of a two-dimensional electron gas (2DEG) is reported. We also report preliminary measurements which confirm this approach.

Electrons or holes can be confined to the surface of a semiconductor by the application of a sufficiently strong electric field normal to the surface. If the resulting potential well in the semiconductor is steep enough, then motion perpendicular to the surface will be quantized. At sufficiently low temperatures when the electron scattering time τ is large enough, such a system of electrons may behave as a 2DEG which has a density of states independent of energy. This two-dimensional nature of the surface electrons was shown by the experiments of Fowler *et al.*,[1] who studied Shubnikov–de Haas (SdH) oscillations of the electrons in inversion-layer conductivity in a Si metal-oxide-semiconductor (MOS) structure in high magnetic fields. They found a constant period of oscillation as a function of surface electron concentration n_s. The two-dimensionality of such field-induced electron gas was further confirmed by the experiments of Fang and Stiles.[2] These authors studied the effects of a tilted magnetic field on the electron system and found that the period of SdH oscillations depends only on the normal component of the applied magnetic field.

A 2DEG obtained by using an MOS structure is a rather unique system for studying many-body effects because n_s can be varied in a single sample over a wide range by simply changing the applied electric field. The influence of electron-electron interaction on the quasiparticle g factor, g^*, in such a system was first studied by Fang and Stiles.[2] They found that the g factor was not only considerably higher than the bulk value (1.998), but also depended on n_s. A similar behavior of the effective mass, m^*, although smaller in magnitude, was recently observed by Smith and Stiles.[3]

It was proposed by Janak[4] that the exchange interactions among the surface electrons were the cause of the observed g shift. He considered the self-energies of the quasiparticle states. Such states are filled up to the Fermi energy. When the density of electrons is increased, the Fermi energy increases and more quasiparticle states are occupied. Any change in the occupation results in a change in the energy of the quasiparticle states. The definition of the quasiparticle g factor is obtained by

$$g_k^*\beta H \equiv E_{k\uparrow} - E_{k\downarrow}$$
$$= g\beta H + \Sigma_\uparrow(k, E_{k\uparrow}) - \Sigma_\downarrow(k, E_{k\downarrow}),$$

where

$$E_{k\uparrow} = \epsilon_k + \tfrac{1}{2}g\beta H + \Sigma_\uparrow(k, E_{k\uparrow}),$$

ϵ_k is the noninteracting energy, and $\Sigma_\uparrow(k, E_{k\uparrow})$, is the self-energy of spin-up quasiparticle states. β is the Bohr magneton.

Janak calculated the self-energy in the random phase approximation by using the screened Coulomb interaction,[5] which led to both an enhanced quasiparticle g factor as well as the correct trend with respect to the electron concentration. However, the theoretical values of g^* are much higher than the values determined experimentally by Fang and Stiles.[2]

The treatment of Janak did not consider any dependence of the quasiparticle g factor on the magnitude of the applied magnetic field. Just as a change in the electron concentration affects the quasiparticle self-energy, so may a change in the applied magnetic field. We expect that the effects of the magnetic field may be more pronounced at high fields when the Landau levels are well resolved. The energy separation between the Landau levels is given by $\Delta E = \hbar\omega_c$, where $\omega_c = eH/m^*c$ is the cyclotron frequency and m^* is the effective mass of the electrons. To simplify the calculations we take the effective mass to be $0.21m_0$ and independent of n_s throughout. (It was found to vary only by 7% in the appropriate concentration range[3] from this value, which is not important here.) If we consider a magnetic field of 50 kOe, we find that the Landau levels are separated by about 2.5 meV, while the level broadening is expected to be of the order of 1 meV or less, depending on the electron scattering time τ. The quasiparticle self-energy depends mainly on the electrons in the highest occupied Landau level. Each Landau level has a number of states proportional to the magnetic field. As we increase n_s and the nth level is filled, the electrons first go into the spin-down states so that the self-energy $\Sigma_\downarrow$ increases while $\Sigma_\uparrow$ is zero (assuming that the density of states of spin down and spin up do not overlap). When essentially all the spin-down states are full electrons go into the spin-up states, thus keeping $\Sigma_\downarrow \simeq$ const and increasing $\Sigma_\uparrow$. When both the spin states are full, we expect $\Sigma_\uparrow \simeq \Sigma_\downarrow$, therefore $g^* = g = 2$. Thus g^* oscillates between 2 and some maximum value which depends on the electron density as was predicted and calculated by Ando *et al.*[6,7]

We have carried out experiments on the 2DEG in an attempt to observe the oscillatory behavior of the quasiparticle g factor as the electron concentration is varied. Circular field-effect transistors[8] with p-type Si substrate and (100) surfaces were used. The channel width was 10 μm and the length-to-width ratio was 50 μm. The insulating oxide thickness was 3000 Å. All the data were taken with the samples immersed in liquid helium at 1.3°K. There are two reasons for working at such low temperature. The electron scattering time τ becomes larger as the temperature is lowered so that one of the conditions, $\omega_c\tau > 1$, necessary to observe SdH oscillations is satisfied. The other condition, namely, $kT < \hbar\omega_c$, is more easily satisfied with a moderate magnetic field.

The channel conductance was measured by using the usual ac technique while the source drain modulation amplitude was kept < 250 μV, so that hot-carrier effects were negligible.[9] The magnetic field was produced by using a superconducting magnet capable of producing fields up to 83 kOe.

SdH oscillations in the inversion layer were studied at different values of the magnetic field applied normal to the sample surface. The quasiparticle g factor for a fixed electron concentration was determined by making the following assumptions. We consider the density of states of each energy level to be a broadened δ function. Next, we assume that the electron scattering time, τ, which gives rise to the level broadening, is the same for both inter- and intra-Landau levels and also that it is independent of the magnetic field.[10] Finally, we assume that the inversion-layer conductivity is directly proportional to the density of states at a given surface electron density. This assumption implies that the magnitude of the conductivity minima is determined by the amount of overlap of the density of states of the two adjacent levels. Since we have assumed that the level broadening is independent of the magnetic field, the overlap of the density of states of two neighboring levels depends only on their energy separation.[11] Thus, the intra-Landau-level conductivity minimum depends on $\Delta E_s = g_s^*\beta H$. Similarly, the inter-Landau-level conductivity minimum depends on $\Delta E_L = \hbar\omega_c - g_L^*\beta H$. For a given electronic concentration we consider the special case when the intra-Landau-level conductivity minimum corresponding to a field H_s is equal to the inter-Landau-level conductivity minimum for a field H_L. In this situation $\Delta E_s = \Delta E_L$ or,

$$g_s^*\beta H_s = \hbar\omega_c - g_L^*\beta H_L,$$

at constant n_s. By simplifying above we get

$$g_s^* = [2(m/m^*) - g_L^*](H_L/H_s).$$

We note that the inter-Landau-level conductivity minimum occurs when a Landau level is completely full. In this situation the quasiparticle self-energies for spin-up and spin-down states are equal and this leads to $g_L^* = g = 2$. It must be emphasized that this argument is only valid for the high-field case when the inter-Landau-level spin interactions are negligible due to the large energy separations between the levels.

In Fig. 1(a) we illustrate the values of g^* obtained from the experiments at 45, 50, and 65 kOe. The range of n_s covered in this experiment is from 5×10^{11} to 3.2×10^{12} cm^{-2}. It is believed that this experiment yields the envelope of the maximum values of g^* because the con-

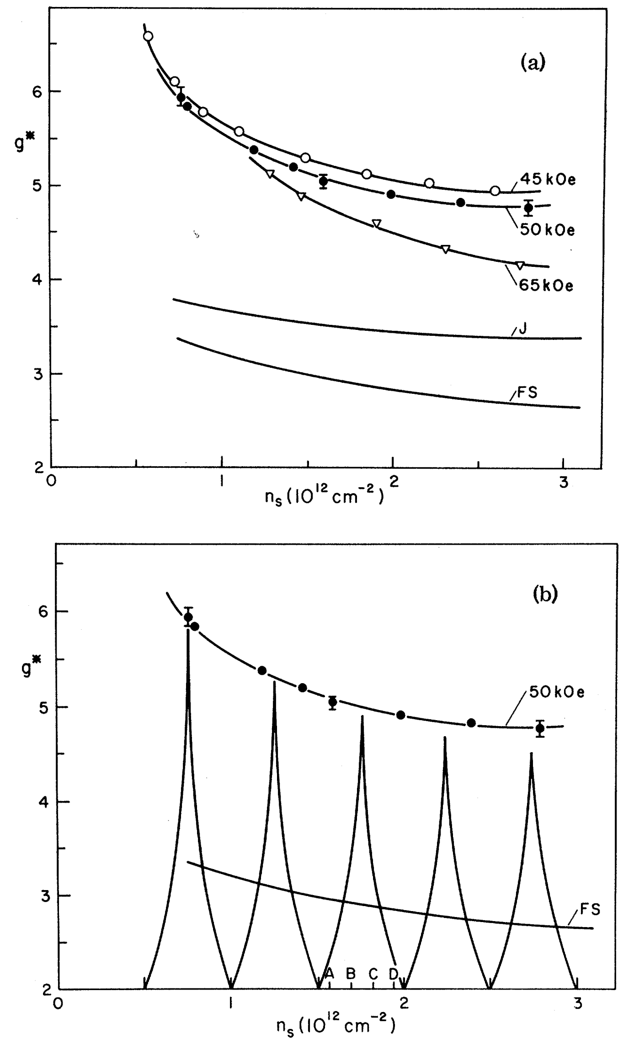

FIG. 1. (a) Concentration dependence of the quasiparticle g factor. Curves indicated by 45, 50, and 65 kOe are the values of g^* obtained by us at those fields. Curve J is the result of the calculation by Janak and curve FS is the experimental result of Fang and Stiles. (b) Quasiparticle g-factor oscillation as a function of electron concentration partially reproduced from Ref. 6. The envelope of g^* obtained by us at 50 kOe, as well as the result of Fang and Stiles, are also shown.

ductivity minima occur in the middle of the Landau levels when the spin-down states are full and the spin-up states are empty which results in the maximum value of $(\Sigma_\uparrow - \Sigma_\downarrow)$. It is interesting to note that the envelope of g^* is a function of magnetic field particularly at electron concentration greater than 1×10^{12} cm^{-2}. Although the theory of Ando, Matsumoto, and Uemura[6] does predict a similar trend in g^* with respect to the magnetic field, it exhibits a much weaker dependence than the one determined experimentally here. For comparison we have shown in Fig. 1(a) the result of the theoretical calculation by Janak (curve J) and also the experimental result of Fang and Stiles (curve FS).

In Fig. 1(b) we have reproduced the result of the calculation made in Ref. 6 for a magnetic field of 50 kOe. We have also shown the envelope of the maximum values of g^* obtained by us at 50 kOe. Such agreement between theory and experiment is somewhat fortuitous because the experimental values of g^* exhibit a stronger dependence on the magnetic field than the one predicted by theory.

We have also shown in Fig. 1(b) the experimental values of the quasiparticle g factor obtained by Fang and Stiles. These authors measured the g factor by studying the maxima and the minima of the transconductance. The relative separations of the spin and Landau levels was changed by tilting the sample in the magnetic field because the spin splitting depends on the total field while the Landau splitting is due only to the normal component of the magnetic field. The angle of tilt was chosen so that the spin splitting was equal to the Landau splitting minus the spin splitting. The smaller values of g^* obtained by these authors are due to the fact that the turning points in transconductance occur at electron concentrations at points A, B, C, and D[12] in Fig. 1(b). Thus, the g values measured would be an average of the four g values corresponding to these concentrations.

In this report we have proposed a method of measuring the quasiparticle g factor of a 2DEG by making use of the SdH oscillations. The experimental results show that the maxima attained by the g factor are a function of the magnetic field, particularly at electron concentrations greater than 1×10^{12} cm^{-2}. To determine this functional dependence of the g factor on the magnetic field, we hope to carry out similar experiments at higher magnetic fields. We also intend to measure the g factor for a tilted case. Some of our initial results for this geometry indicate that the g factor decreases as the tilt angle increases.

Quantization of the 2DEG can also be studied by measuring the capacitance of the inversion layer.[13] This method has the advantage that the capacitance measures the density of states directly while the conductance experiments yield the product of electron mobility and the density of states. We plan to measure g^* by the capacitance technique also.

A more refined theory of SdH oscillations in the 2DEG as a function of n_s and including electron-electron interaction may make it possible to obtain the intermediate values of the quasiparticle g factor. A combination of the method proposed in this report with the tilting of the magnetic field may make it possible to determine the full oscillations of g^*.

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[12]If the number of electrons in each Landau level is n_0, the turning points in transconductance occur approximately at $\frac{1}{8}n_0$ (point A), $\frac{3}{8}n_0$ (point B), $\frac{5}{8}n_0$ (point C), and $\frac{7}{8}n_0$ (point D).

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