[9]M. Nelkin, Phys. Fluids 15, 1685 (1972).

[10]B. J. Alder and T. E. Wainwright, J. Phys. Soc. Jap., Suppl. 26, 267 (1969).

[11]B. J. Alder and T. E. Wainwright, Phys. Rev. A 1, 18 (1970).

[12]The Oseen correction is neglected in the Langevin equation because it has no significant contribution in the present analysis.

[13]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Addison-Wesley, Reading, Mass. 1959), p. 96.

[14]Judging from Eqs. (3i) and (3ii), one must have $t \gtrsim 40\tau$ in order that $V(t)/V_0$ lie within 10% of its asymptote. Neither the Alder-Wainwright data nor the present measurements satisfy this criterion.

[15]Y. W. Kim and J. E. Matta, Bull. Amer. Phys. Soc. 18, 139 (1973).

[16]The runs in argon are expected to lie slightly above the points for air (not exceeding 5.7% of the air data) as a result of the difference in density. This difference is too small to notice in the figure.

Parametrically Driven Ion Cyclotron Waves and Intense Ion Heating*

T. K. Chu, S. Bernabei, and R. W. Motley

Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540

(Received 9 April 1973)

Finite-amplitude plasma waves excited by plates outside a plasma column are shown to decay into other plasma waves and electrostatic ion cyclotron waves, accompanied by ion heating.

A number of experiments[1] on parametric instabilities have been reported recently, including measurements of energy deposition into the tails of the particle velocity distributions[2,3] and electron heating.[4] In this Letter we report measurements of strong heating of the bulk of the ions. The increase in ion temperature coincides with the onset of ion cyclotron waves driven parametrically unstable by nearly perpendicularly propagating electron plasma waves resulting from rf electric fields applied to a plasma column. The experimental results include an identification of the pump wave as a Trivelpiece-Gould mode, and measurements of the dispersion relation of the parametric ion cyclotron wave, the threshold conditions, pump-field depletion accompanying instability onset, and ion temperatures up to a factor of 100 higher than the initial temperature.

The importance of this experiment rests on the following points. First, the pump-wave frequency ω_0 is close to the lower-hybrid frequency ω_{LH} which in fusion-reactor plasmas ($\omega_{LH}/2\pi \sim 2$ GHz $\sim \omega_{pi}/2\pi$, where ω_{pi} is the ion plasma frequency) represents the practical upper limit of available high-power sources.[5] Second, in accordance with the Manley-Rowe relation, the present parametric process, when compared with others has a high frequency ratio ($\sim \Omega_i/\omega_{pi}$) of the low frequency component of the decay wave to the pump, allowing a relatively high level of power to be stored in the ion wave. Third, the frequency of the ion cyclotron wave can be very close to the ion cyclotron frequency Ω_i; thus heating of the bulk of the ions should be expected.

Linear calculations of the relevant parametric processes,[6,7] using the dipole approximation, show that two branches ($k_\parallel \ll k_\perp$ and $k_\parallel \sim k_\perp$) of ion cyclotron and ion sound waves may be parametrically destabilized. Their frequencies for the lowest thresholds (in the usual notation) are, for $k^2 c_s^2/\Omega_i^2 \ll 1$,

$$\omega^2/\Omega_i^2 = 1 + k^2 c_s^2/\Omega_i^2, \quad k_\parallel \ll k_\perp, \tag{1a}$$

$$\omega^2 = k_\parallel^2 c_s^2, \quad k_\parallel \sim k_\perp; \tag{1b}$$

and for $k^2 c_s^2/\Omega_i^2 \gg 1$,

$$\omega^2 = k^2 c_s^2, \quad k_\parallel \ll k_\perp, \tag{2a}$$

$$\omega^2 = \Omega_i^2 (k_\parallel^2/k_\perp^2), \quad k_\parallel \sim k_\perp. \tag{2b}$$

The parametric process described by Eq. (2a) has been previously reported.[8] The corresponding threshold condition[7] for these instabilities is

$$(\vec{k}\cdot\vec{\epsilon})^2 = 2\left(\frac{kv_e}{\omega_{pe}}\right)^2 \frac{\omega_e}{\omega_0}\frac{\nu_i}{\Omega_i}\frac{T_e}{T_i},$$

where $|\vec{k}|$ is the instability wave number, $\vec{\epsilon}$ is the particle displacement due to the rf field $\vec{E}$ at frequency ω_0, v_e is the electron thermal velocity, ν_e is the collisional damping of the high-frequency component of the parametric instability, and

$$\frac{\nu_i}{\Omega_i} = 2\sqrt{\pi}\frac{k^2 v_i}{\Omega_i^2}\frac{\omega}{k_\parallel v_i}\exp\left[-\left(\frac{\omega - \Omega_i}{k_\parallel v_i}\right)^2\right]$$

denotes ion cyclotron damping. The dominant coupling occurs through the electron drift (at the pump frequency) either parallel to the magnetic field or perpendicular to both the magnetic and the electric fields, depending on whether $(\omega_0/\omega_{pe})\times\Omega_e/\omega_{pe}>1$ or <1.[7]

The experiments were performed in the Princeton University H-1 device in a density range $5\times10^8<n_0<2\times10^{11}$ cm^{-3} $(10^{-1}>\omega_0/\omega_{pe}>10^{-2})$, magnetic field $B\leq 15$ kG $(30>\Omega_e/\omega_{pe}>5)$, in Cs and K plasmas with $T_i \gtrsim T_e = T_{\text{end plate}} = 0.2$ eV. The plasma column was 120 cm long and 4 cm in diameter. The background pressure was less than 10^{-6} Torr, so that collisions with atoms are negligible. The two half-cylindrical-shell plates[9] for wave coupling (to an $m=0$ or 1 mode) form a 35-cm-long cylinder 4.5 cm in diameter concentric with the plasma column. T_e was measured by monitoring plasma radiation at the upper-hybrid frequency,[10] T_i by a biased probe collecting (transverse) ion current,[11] and the plasma density by a microwave interferometer and by Langmuir probes.

The pump wave has been identified as the Trivelpiece-Gould mode,[12] based on measurements by probes of the wave radial profile and of ω_0 versus $k_\parallel$, which agree with values calculated from the dispersion relation $\omega_0/\omega_{pe}\simeq k_\parallel a/p_m$, where the electron plasma frequency ω_{pe} is determined from the measured density, a is the plasma radius, and p_m is the value of the first zero of the Bessel function (for the mth pump mode). The wave number $k_\parallel$ in the frequency range up to ω_{pe} was measured by two methods: (1) wave interferometry, in the range $1>\omega_0/\omega_{pe}>0.1$, and (2) time-of-flight measurements of the launched wave and the mode structure parallel to the field by axially moving probes, for $\omega_0/\omega_{pe}<0.1$. $k_\parallel$ has been measured both with and without the onset of the parametric instability.

A decay spectrum of the $m=1$ pump wave into another electron plasma wave and an ion cyclotron wave is shown in Fig. 1(a). Generally, higher harmonics appear at higher power levels. Figure 1(b) shows the dispersion characteristics of the waves described by Eqs. (1) and (2). In Fig. 2, the curve for the ion cyclotron wave $(\omega>\Omega_i)$ is calculated according to Eq. (1a), using the measured azimuthal wave number $m=6$ and

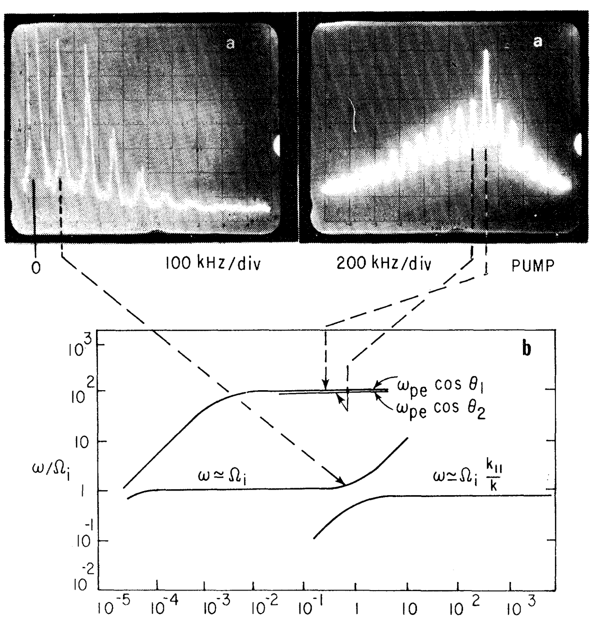

FIG. 1. (a) Parametric decay spectrum. The left-hand picture shows the low-frequency component, the right-hand one shows the pump and the high-frequency component. (Cs plasma, $n=10^{10}$ cm^{-3}, $B=8.6$ kG, $\omega_0/2\pi=15$ MHz.) (b) Dispersion relations of the parametrically coupled waves [Eqs. (1) and (2)]. In the cylindrical geometry of the present experiment, the waves have transverse resonance with the plasma column.

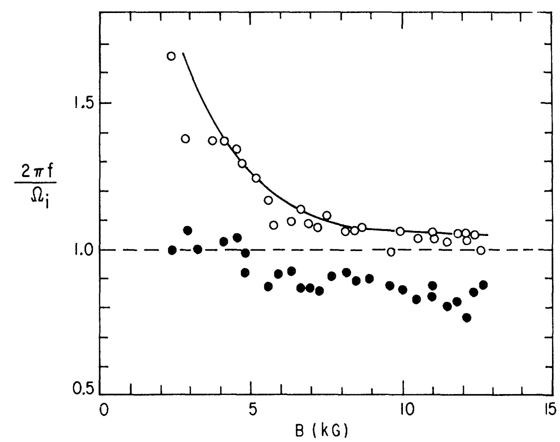

FIG. 2. Measured frequency versus magnetic field. Curve for the upper branch, calculated from Eq. (1a). The frequency of the lower branch is consistent with that of the obliquely propagating ion cyclotron wave. For the upper branch, measured $k_\parallel$ values are, for the pump, 0.05 ± 0.03 cm^{-1}; sideband, 0.48 ± 0.10 cm^{-1}; ion cyclotron wave, 0.39 ± 0.08 cm^{-1}; measured $k_\perp$ values: pump, $m=1$; cyclotron wave, $m=6$. [m of the first sideband cannot be measured because of its proximity to the large-amplitude pump. Assuming it to be $m=-7$, $k_\parallel$ determined from the equation $(k_\parallel a/p_m)_{\text{sideband}}\simeq(k_\parallel a/p_m)_{\text{pump}}$ is 0.15 ± 0.09 cm^{-1} for the first radial mode. The discrepancy might be due to the finite length of the plasma column as versus infinitely long column in theory (private communication, G. Van Hoven) and higher experimental radial-mode number.]

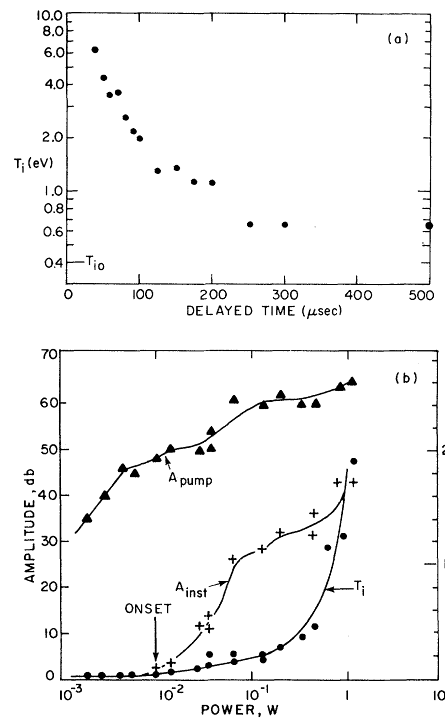

FIG. 3. (a) Measured ion temperature in the afterglow of rf pulse versus delay time. ($B = 8.6$ kG, rf power $= 1$ W, Cs plasma.) (b) Amplitude of the pump and high-frequency component of the instability (as determined from the floating-potential measurements by probes), and ion temperature as a function of rf power. Estimate of statistical error of T_i measurement, ~20%.

assuming $k_\perp \simeq m/r$, r being the radial location of peak instability amplitude. We note that a rigorous comparison requires calculations of normal-mode solutions in cylindrical geometry for both the pump and the decay waves satisfying the appropriate boundary conditions.

The frequency dependence of the wave as a function of magnetic field has also been verified in He, A, Ne, and K plasmas. (For noble-gas plasmas, the rf coupling plates are also used for plasma production.) In addition, the density-independent character of the ion cyclotron wave has been verified in the density range of 10^9 to 10^{11} cm^{-3} for a fixed pump frequency. The instability threshold power was measured by monitoring incident and reflected power and by measurement of rf voltage, current, and their phase difference. The threshold field was obtained from this power measurement and the experimentally measured damping rate for the pump wave. At $n = 2\times 10^{10}$ cm^{-3}, $B = 8.6$ kG, $\Omega_i/2\pi = 100$ kHz, $T_e = 0.2$ eV, $T_i = 0.35$ eV, and $\omega_0/2\pi = 15$ MHz, the measured threshold is 4 V/cm, and the calculated is 2.8 V/cm. The weak dependence of the threshold field on B ($E_{\text{th}} \propto \sqrt{B}$) is in qualitative agreement with the measured dependence. We have not measured the detailed mode structure of the wave below the cyclotron frequency. The threshold power for this branch is always higher than that for the branch propagating above the cyclotron frequency. This and the measured frequency suggest that it may be the obliquely propagating second ion cyclotron wave with $k_\parallel \sim k_\perp$.

Since the applied rf fields produce a time-varying space potential in the plasma, all probe measurements of (transverse) ion temperature were performed after the pulse of rf power, as a function of time in the afterglow. Figure 3(a) shows the measured ion temperature versus delay time for an rf pulse of width 500 μsec. (This width is sufficient to allow saturation of the ion temperature.) The pulse is longer than the ion cooling time, ~100 μsec, and shorter than the time for the plasma density to change. Generally, probe characteristics taken after the termination of the rf pulse (delay time > 40 μsec) shows that the bulk of the ions are heated, consistent with (transverse) ion heating by a wave field near the ion cyclotron frequency. In Fig. 3(b) are shown the ion temperature, pump-wave amplitude, and instability amplitude determined by probes, as a function of rf power. We note that pump depletion is shown by the slower rise of pump-wave amplitude in the unstable regime when compared to that in the stable regime

At still higher power levels (~5 W) severe plasma losses occurred (the power stored in the steady-state plasma is between 1 and 10 mW). The measured ion cooling time, 100 μsec, is a factor of 4 shorter than that due to the ions striking the end plates. This difference, however, is consistent with the preferential loss of high-energy ions ($r_L > 1$ cm) by scrapeoff on the rf plates. The measured electron temperature rise is about 20% of the ion temperature increase and the electron cooling time is ~20 μsec, indicating that ions and electrons share approximately equally the heating power.

Several checks have beem made to validate the ion temperature measurement. (1) T_i was observed to decrease with the addition of argon gas to the chamber. (2) For a fixed rf power level, T_i increases linearly with increasing pulse width, i.e., increasing energy content (up to a pulse duration such that heating equilibrium is reached). (3) An energy analyzer, placed at the end of the plasma column where the magnetic-field lines diverge, also measures similar enhanced ion temperatures. (4) At high rf levels, high-energy (large-orbit) ions could be collected outside the plasma column limiter. (5) For a given magnetic field the maximum possible ion temperature was found to be such that the average ion cyclotron radius is $\frac{1}{3}$ to $\frac{1}{2}$ of the plasma column radius, indicating that very high-energy ions cannot be confined.

Since the measured ion heating time is shorter than the collisional heating time, and the ion temperature is higher than the electron temperature, a collisional heating mechanism can be excluded. By using short pulses [90 μsec at moderate power levels (~0.5 W)], ion temperatures of ~5 eV were measured, while the measured change of plasma potential was less than 0.2 V. This observation shows that ion heating (up to this power level) cannot be attributed to the change of plasma equilibrium. Based on these measurements and the observation of concomitant pump-field depletion and instability onset accompanying the ion heating, we conclude that the observed ion heating is qualitatively consistent with that by ion cyclotron waves, driven parametrically unstable by the incident electron plasma waves.

In conclusion, we emphasize that the present parametric process possesses a number of highly desirable features for plasma heating. First, the process is not subject to resonance detuning, because a continuous spectrum of eigenmodes exists for both the pump wave and the high-frequency plasma wave. Also, the low-frequency decay wave is independent of the plasma density. Finally, the scaling laws of the characteristic frequencies based on the temperature, magnetic field, and ion mass suggest that this parametric process may be useful as supplementary heating in toroidal fusion reactors.[13]

This experiment was stimulated by discussions with Dr. W. Hooke. We also acknowledge with pleasure discussions with Dr. I. Fidone, Dr. D. Jassby, and Dr. Y. C. Lee and the technical assistance of Mr. J. Frangipani.

---

*Work supported by the U. S. Atomic Energy Commission under Contract No. AT (11-1)-3073, and by the Advanced Research Projects Agency under Contract No. F 30602-73-C-0163.

[1]For a summary, see T. K. Chu, in Proceedings of the First Topical Conference on RF Heating, Lubbock, Texas, July 1972 (to be published); M. Porkolab, in Proceedings of the Symposium on Plasma Heating and Injection, Varenna, Italy, 1972 (Academic, New York, to be published).

[2]H. Dreicer, C. Ingraham, and D. Henderson, Phys. Rev. Lett. 26, 1616 (1971).

[3]M. Porkolab, V. Arunasalam, and R. A. Ellis, Jr., Phys. Rev. Lett. 29, 1438 (1972).

[4]T. K. Chu and H. W. Hendel, Phys. Rev. Lett. 29, 634 (1972).

[5]W. M. Hooke, in Proceedings of the Symposium on Plasma Heating and Injection, Varenna, Italy, 1972 (Academic, New York, to be published).

[6]V. P. Silin, Zh. Tekh. Fiz. 37, 991 (1967) [Sov. Phys. Tech. Phys. 12, 717 (1967)]; N. E. Andreev and A. Yu. Kirii, Zh. Tekh. Fiz. 41, 1080 (1970) [Sov. Phys. Tech. Phys. 16, 854 (1971)].

[7]T. K. Chu and Y. C. Lee, to be published.

[8]J. M. Kindel, H. Okuda, and J. M. Dawson, Phys. Rev. Lett. 29, 995 (1972); W. M. Hooke and S. Bernabei, Phys. Rev. Lett. 29, 1218 (1972).

[9]W. M. Hooke and S. Bernabei, Phys. Rev. Lett. 28, 407 (1972).

[10]A. Y. Wong and A. F. Kuckes, Phys. Rev. Lett. 13, 536 (1964).

[11]I. Katsumata and M. Okazaki, Jap. J. Appl. Phys. 6, 123 (1967); R. W. Motley and T. Kawabe, Phys. Fluids 14, 1019 (1971).

[12]A. W. Trivelpiece and R. W. Gould, J. Appl. Phys. 30, 1784 (1959).

[13]An additional length-scale ratio, the rf wavelength λ to that of plasma size or plasma inhomogeneity scale length R, might also become important. In the present regime, $\lambda/R \gg 1$; for reactor-regime plasmas, $\lambda/R \lesssim 1$.

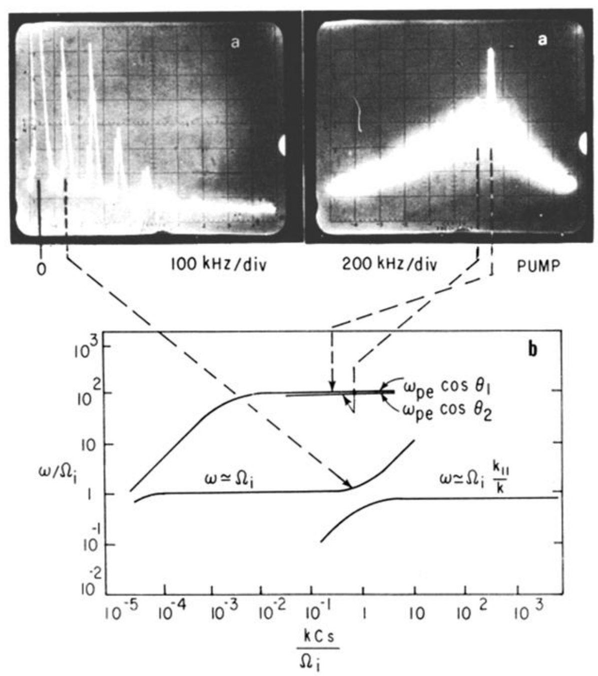

FIG. 1. (a) Parametric decay spectrum. The left-hand picture shows the low-frequency component, the right-hand one shows the pump and the high-frequency component. (Cs plasma, $n = 10^{10}$ cm^{-3}, $B = 8.6$ kG, $\omega_0/2\pi = 15$ MHz.) (b) Dispersion relations of the parametrically coupled waves [Eqs. (1) and (2)]. In the cylindrical geometry of the present experiment, the waves have transverse resonance with the plasma column.