calculated the event rates for $\nu+N\rightarrow\mu^-+$ hadrons and $\nu+Z\rightarrow\mu^-+W^++Z$ with $W^+\rightarrow\mu^++\nu$ in a Monte Carlo calculation which assumes the following: (1) the scaling functions described above, (2) the effects of a W propagator, (3) calculated W-production cross sections,[9] and the calculated decay distribution for $W^+\rightarrow\mu^++\nu$. Figure 1(c) shows the expected fraction of detected W events in our apparatus as a function of E_ν and M_W.

In our sample of 112 events, there are 18 kaon-neutrino and 94 pion-neutrino interactions. From the energy spectrum of these observed 112 neutrino interactions,[10] we have estimated the expected number of $W\rightarrow\mu^++\nu$ events for our apparatus. This is given for various-mass W bosons in Table I. Our 90% confidence bound as a function of the branching fraction is shown in Fig. 2. For comparison, the previous CERN neutrino limit is also shown.

In summary, we see no evidence for W bosons in this experiment. A lower bound has been set on the mass (i.e., $M_W>4.4$ GeV/c^2 for $B=0.5$) which depends on the branching fraction into leptons. This limit represents a significant improvement over past neutrino results.

We would like especially to acknowledge helpful discussions with J. Smith about W-production cross sections and use of his computer programs. We also appreciate the help and support of the NAL staff, especially Les Oleksiuk and members of the neutrino laboratory.

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*Work supported in part by the U. S. Atomic Energy Commission.

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Meson-Exchange Corrections to the Cross Section for $n+H^2\rightarrow H^3+\gamma$

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(Received 7 May 1973)

It is shown that agreement between theory and experiment regarding the cross section for radiative neutron-deuteron capture can be achieved by considering the effects of meson-exchange current on the magnetization density of the three-body system. The calculated cross section is 0.52 ± 0.05 mb.

The mesonic currents that mediate the nuclear force distort the free-state electromagnetic properties of nucleons when they are bound in a nuclear system. Taking into account the existence of meson-exchange currents, several investigators have recently eliminated some longstanding discrepancies between theory and experiment that could not otherwise be accounted for. Examples of this are the cross section for radiative n-p capture at thermal neutron energies,[1] the magnetic moments of He^3 and H^3,[2] and the β decay of H^3.[3,4]

Another instance where a discrepancy between theory and experiment exists is the cross section for radiative neutron-deuteron capture. The experimental value is $\sigma = 0.6 \pm 0.05$ mb[5] for incident neutrons of velocity $v = 2.2\times 10^6$ cm/sec, whereas theoretical calculations with reasonable three-body wave functions yield only a small fraction of this value.[6] At thermal neutron energies the reaction $n + d \rightarrow t + \gamma$ proceeds via a magnetic dipole transition, and hence the amplitude for this reaction is proportional to the matrix element of the $M1$ operator. No reasonable change in the components of the wave function for the three-body bound and scattered system can close the gap between theory and experiment.[6] On the other hand, it has been demonstrated phenomenologically[6] that a change in the one-body magnetization density of the three-body system,

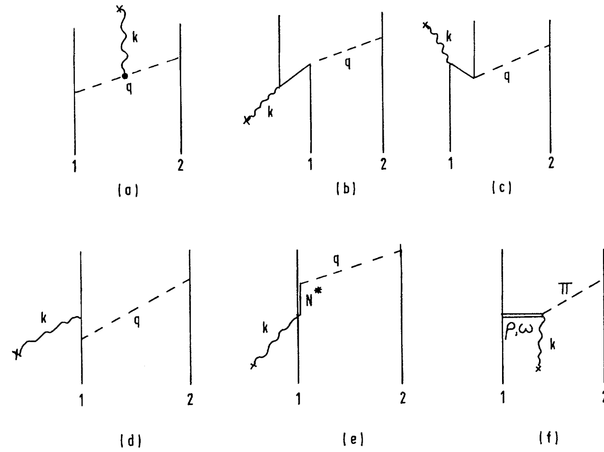

FIG. 1. Time-ordered graphs of meson-exchange currents.

$$\vec{\mathfrak{M}} = \frac{e\hbar}{2Mc}\sum_{i=1}^{3}\left[\tfrac{1}{2}(1+\tau_z{}^i)\mu_p\vec{\sigma}^i + \tfrac{1}{2}(1-\tau_z{}^i)\mu_n\vec{\sigma}^i\right] + \text{an orbital term}, \tag{1}$$

by a two-body term

$$\Delta\vec{\mathfrak{M}} = \frac{e\hbar}{2Mc}\sum_{i<j}^{3}(\tau^i - \tau^j)_z(\vec{\sigma}^i - \vec{\sigma}^j)W(r_{ij}), \tag{2}$$

with M the nucleon mass and $W(r_{ij})$ a Yukawa term whose range and strength is adjusted to give a correction of $0.37\mu_N$ to the isovector magnetic moment of H^3, will also increase the theoretical cross section to 0.5 mb, in good agreement with the experimental value. This favorable result points the way to the correct resolution of the discrepancy between theory and experiment.

In the present paper we investigate corrections to the n-H^2 radiative-capture cross section starting from first principles, i.e., by considering the interaction of the electromagnetic field with equivalent two-body currents, that correspond to processes shown by the elementary two-body graphs in Fig. 1. This is in addition to the usual photon-one-nucleon interaction which is handled by means of Eq. (1).

Following Chemtob and Rho[7] we divide the pion production amplitudes into a Born term, Figs. 1(a)-1(d), and a non-Born term, Figs. 1(e) and 1(f). In addition we consider graphs similar to Figs. 1(b)-1(d) where the exchanged particle is a ρ or a ω meson. These graphs are evaluated by the conventional S-matrix approach augmented by soft-pion and soft-current considerations.[7] Only leading terms in powers of $1/M$ have been kept. The general forms of the isovector and isoscalar components of the magnetization density arising from two-body currents and contributing to S-state-to-S-state transitions are, respectively,

$$\begin{aligned}\vec{\mathfrak{M}}_{ij}{}^V &= \tfrac{1}{2}(e\hbar/2Mc)[(\vec{\tau}^i\times\vec{\tau}^j)_z(\vec{\sigma}^i\times\vec{\sigma}^j)g_{\rm I} + (\vec{\tau}^i-\vec{\tau}^j)_z(\vec{\sigma}^i-\vec{\sigma}^j)h_{\rm I} + (\vec{\tau}^i+\vec{\tau}^j)_z(\vec{\sigma}^i+\vec{\sigma}^j)j_{\rm I}],\\ \vec{\mathfrak{M}}_{ij}{}^S &= \tfrac{1}{2}(e\hbar/2Mc)[\tau_z{}^i\tau_z{}^j(\vec{\sigma}^i+\vec{\sigma}^j)k_{\rm I} + (\vec{\sigma}^i+\vec{\sigma}^j)l_{\rm I} + \vec{\tau}^i\cdot\vec{\tau}^j(\vec{\sigma}^i+\vec{\sigma}^j)m_{\rm I}].\end{aligned} \tag{3}$$

Equivalent expressions contributing to S-state-to-D-state transitions are

$$\begin{aligned}\vec{\mathfrak{M}}_{ij}{}^V &= \tfrac{1}{2}(e\hbar/2Mc)[(\vec{\tau}^i\times\vec{\tau}^j)_z\vec{\rm T}_{ij}{}^{(\times)}g_{\rm II} + (\vec{\tau}^i-\vec{\tau}^j)_z\vec{\rm T}_{ij}{}^{(-)}h_{\rm II} + (\vec{\tau}^i+\vec{\tau}^j)_z\vec{\rm T}_{ij}{}^{(+)}j_{\rm II}],\\ \vec{\mathfrak{M}}_{ij}{}^S &= \tfrac{1}{2}(e\hbar/2Mc)[\tau_z{}^i\tau_z{}^j\vec{\rm T}_{ij}{}^{(+)}k_{\rm II} + \vec{\rm T}_{ij}{}^{(+)}l_{\rm II} + \vec{\tau}^i\cdot\vec{\tau}^j\vec{\rm T}_{ij}{}^{(+)}m_{\rm II}],\\ \vec{\rm T}_{ij}{}^{(\times)} &\equiv (\vec{\sigma}^i\times\vec{\sigma}^j)\cdot\hat{r}_{ij}\hat{r}_{ij} - \tfrac{1}{3}(\vec{\sigma}^i\times\vec{\sigma}^j)\end{aligned} \tag{4}$$

and similarly for $T_{ij}{}^{(\pm)}$ with the $\times$ replaced by $\pm$. The functions $g_{\rm I,II}$, $h_{\rm I,II}$, $j_{\rm I,II}$, $k_{\rm I,II}$, $l_{\rm I,II}$, $m_{\rm I,II}$ contain the radial dependence and are listed by Chemtob and Rho.[7] We shall only list here the contributions from the Born graphs for S-to-S transitions:

$$g_{\mathrm{I}} = (2M/3m_\pi) f_{\pi NN}^2 (2x_\pi - 1) e^{-x_\pi}/x_\pi + (4/3\pi) f_{\pi NN}^2 [K_0(x_\pi) - k_1(x_\pi)/x_\pi],$$
$$\tfrac{1}{2} m_{\mathrm{I}} = h_{\mathrm{I}} = j_{\mathrm{I}} = -(1/3\pi) f_{\pi NN}^2 [K_0(x_\pi) - K_1(x_\pi)/x_\pi], \quad (5)$$

with $f_{\pi NN}^2 = 0.08$. $K_0(x_\pi)$ and $K_1(x_\pi)$ are Bessel functions of the second kind.

The wave functions for the three-body system have been obtained from the solution of Faddeev equations using a separable approximation (unitary-pole approximation)[8] to the Reid interaction.[9] The unitary-pole approximation to local potentials has been discussed at length in several papers. It yields a triton binding energy of ~7.2 MeV and reproduces fairly successfully the exact two-body off-shell T-matrix elements. Furthermore, the quality of the wave function obtained thus has been checked[10] and, in the case of the bound state at least, has been found to be very good at all but very short internucleon distances where the short-range correlations in the nucleon-nucleon force are not fully reproduced. Yet this defect will introduce no larger error than, in the evaluation of, say, the expectation value of singular operators such as $e^{-\mu r}/r^2$, overestimates of 10 and 50% for $\mu = 0.7$ and 3.86, respectively, corresponding to one-π and one-ρ exchange. The error is much smaller in the case of the operators we deal with in S-state–to–S-state matrix elements.

The feature of the three-body wave function that is of dominant importance in the present calculation is the mixed-symmetry S'-state admixture. The contribution of Eq. (1) to the n-H^2 capture cross section goes entirely via the S' state. Additionally the D-state admixture features importantly in the meson-exchange corrections to the cross section. Theoretically then, the $M1$ n-H^2 capture can yield valuable information regarding the structure of the three-body wave function provided the meson-exchange effects are fully understood.

The present bound-state wave-function contains 1.6% S' state and an approximate D state of the form[4]

$$\Psi_D = -N\vec{\tau}^1 \cdot \vec{\tau}^2 S_{12} V_T \Psi_S, \quad (6)$$

where Ψ_S is the S state, S_{12} is the usual tensor operator, and $V_T = (1 - e^{-1.5x^2})(1 + 3/x + 3/x^2)e^{-x}/x$. N is a normalization constant such that $\langle \Psi_D | \Psi_D \rangle = 0.08$. We find $N = 1.84 \times 10^{-2}$. We have calculated the meson-exchange correction to the isovector magnetic moment using this wave function and have found $\nabla\mu^V = 0.355\,\mu_N$.

We define a total magnetization density

$$\vec{\mathfrak{M}}_T = \vec{\mathfrak{M}} + \sum_{i<j}^{3} (\vec{\mathfrak{M}}_{ij}^{\,V} + \vec{\mathfrak{M}}_{ij}^{\,S}),$$

and a magnetic interaction Hamiltonian $\mathcal{H} = \vec{\mathrm{M}} \cdot \hat{\epsilon}_m$, where $\hat{\epsilon}_\pm = \mp (\frac{1}{2})^{1/2} (\hat{e}_x \pm i\hat{e}_y)$ is the polarization of the magnetic field. Our results for the meson-exchange correction to the cross section using a 100% $S+S'$ state in the initial and final states are presented in Table I. In the same table we present results with a 92% $S+S'$ state and an 8% D state of the form (6), for the bound system. We have neglected contributions from the deuteron D state in the initial system, leading to a D state in the n-H^2 system which is not available presently.

Our results confirm the phenomenological calculations and the qualitative discussion in Ref. 6 regarding the importance of scattering effects in the initial state of the interacting n-H^2 system, and the dominant role played by the initial doublet spin state in the capture process when meson-exchange currents are considered. As indicated in Table I, the cross section for capture from

TABLE I. Contributions to the cross section for n-d capture (in mb). The theoretical uncertainty is ±0.05 mb.

	From nucleon spins		From nucleon spins and meson-exchange currents		
	Without scattering in the initial state	Exact solution	S and S' states, exact solution	S, S', and D states, exact solution	Experimental
Capture from doublets	0.37	0.12	0.25	0.36	
Capture from quartets	0.74	0.17	0.155	0.16	
Total	1.11	0.29	0.405	0.52	0.65±0.05

doublet spin states ($J=\frac{1}{2}$) was found to be 0.37 mb, and that for capture from quartet spin states ($J=\frac{3}{2}$) 0.74 mb, when meson-exchange currents and scattering effects were excluded. These numbers changed to 0.12 mb and 0.17 mb, respectively, when scattering in the initial state was considered, and to 0.25 mb and 0.155 mb, respectively, when meson-exchange currents were introduced, for a total $\sigma = 0.405$ mb. A further increase to $\sigma = 0.52$ mb was obtained when the contribution from S-to-D state was included.

Table II shows the relative contributions to the amplitude for capture from doublets, from the various graphs in Fig. 1. We note that the largest contribution comes from the N^* graph, Fig. 1(e), through an S-to-D transition. This is in full agreement with the findings in Refs. 1–4 regarding the role of this graph in trinucleon magnetic moments and the β decay of H^3. Additionally, it is seen that the N^* graph is the only one that is of consequence in S-to-D transitions. Consequently, its contribution is most sensitive to the D-state admixture in the trinucleon system and can, therefore, act as a probe of the nuclear structure of this system. It is unfortunate that at this time the largest theoretical uncertainty in our results is associated with this contribution, as will be discussed below.

Furthermore, we find that the renormalization correction,[7] necessary to take into account the admixture of pions into the two-nucleon state, only partially cancels the contribution from the recoil graph, Fig. 1(d). Finally, it is seen from Table I that the meson-exchange correction to the amplitude for capture from the quartet spin state is insignificant, and for this reason, a breakdown of the individual contributions from the various graphs in Fig. 1 to the amplitude is not displayed.

As a result of the approximate nature of the bound-system D state and the fact that the overall radial dependence of the S-D matrix elements [from Figs. 1(e) and 1(f)] is quite singular near the origin, the S-to-D contribution is prone to overestimation. We have attempted to remedy this situation by employing a cutoff function whose effect is to simulate a hard repulsive core in the N-N interaction. It was precisely this effect that was partially lost in solving the three-body system by employing a separable approximation to the Reid potential. The results displayed in Tables I and II are for an effective repulsive core set at 0.5 fm. The S-to-D contribution was increased by 0.045 mb with the core set at 0.34 fm. Assuming these to be reasonable limits for the repulsive core, and taking into account a 10% uncertainty in the radial integrals for S-to-S transitions, we have an overall uncertainty of approximately 0.05 mb in the total cross section (see Table I).

The results of the present calculations offer convincing evidence that considerations of mesonic currents bring theoretical calculations in agreement with experimental measurements of the cross section for n-H^2 capture. In addition, of great importance is the fact that these results, along with those of meson-exchange corrections to the β decay of H^3 and to the magnetic moments and the magnetic form factors[11] of H^3 and He^3, are indispensable for determining the S-, S'-, and D-state admixtures in the trinucleon bound system. In view of the role of this system as a sensitive probe of the short-range part of the nucleon-nucleon force,[12] a definitive determination of this admixture is necessary.

The author would like to acknowledge grateful-

TABLE II. Percent contribution to the amplitude for capture from the spin doublet n-H^2 state. In parentheses are shown the transitions through which the graphs contribute to the total amplitude.

Graphs in Fig. 1		Percent contribution
(a) π-current	(S-S)	3.0
(b), (c) Pair excitation	(S-S)	34.5
(d) Recoil + normalization	(S-S)	9.0
(b), (c) Heavy-meson exchange	(S-S)	10.0
(e), (f) Non-Born graphs	(S-S)	~0.0
(a)–(d) π-exchange	(S-D)	<1.0
(e), (f) Non-Born graphs	(S-D)	42.5 [a]
(b)–(d) Heavy-meson exchange	(S-D)	~0.0

[a] 40% from (e) and 2.5% from (f).

ly the hospitality of Professor R. J. Blin-Stoyle and the Department of Physical and Mathematical Sciences, University of Sussex, where most of this work was done.

*On leave of absence from Fairfield University, Fairfield, Conn. Work partially supported by the National Science Foundation.

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