

turn, to coincide with a desired value of E . The entire operation requires ~ 260 sec on an IBM 370/168, for three values of E .

The resulting values of μ_α and $U_{i\alpha}$ are shown in Table I, together with the matrix elements for dipole excitation from the Hartree-Fock ground state,

$$D_\alpha = \int_{r_j \leq r_0} \Psi_\alpha^* \left(\sum_{j=1}^{18} z_j \right) \Psi_0 \prod_{j=1}^{18} d^3r_j, \quad (11)$$

and with the experimental values of the same parameters from Ref. 9. The differences of calculated and experimental values are not surprising in view of the inaccuracy of fitting in Ref. 9, and of the known importance of electron correlations—neglected here—for the excitation of Ar. The $U_{i\alpha}$ show the eigenchannels $\alpha = 1, 2, 3$ to consist mostly of d orbitals with LS coupling ($^3D, ^1P, ^3P$, respectively), while $\alpha = 4, 5$ correspond to s orbitals with 3P and 1P coupling; the D_α values are accordingly large for singlet-singlet transitions only.

Calculation of the R matrix by the alternative methods of Refs. 7 or 8, followed by its diagonal-

ization, should yield equivalent results.

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Hydrogenic Stark-Zeeman Spectra for Combined Static and Dynamic Fields

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The entire spectrum of hydrogen in combined static and periodic electric and magnetic fields is derived by using $O(4)$ algebra, and related to the corresponding simpler Lyman- α spectrum by a scaling relation. This time-dependent generalization of the Stark-Zeeman effects provides the necessary basis for the proper analysis of hydrogenic spectra for plasma diagnostic purposes.

In this Letter we provide a full description of the characteristics of the entire hydrogen spectrum under the influence of timewise periodic, but otherwise arbitrary, electric and magnetic fields. Within the framework of certain physical approximations, valid over a wide range of parameters, these results constitute a nonperturbative exact solution to the problem, based on Lie-algebraic methods, and thus remain valid for many physically interesting situations, including resonance domains,^{1,2} where perturbation

methods fail. One of the important areas where the knowledge of such spectra is required is plasma diagnostics by hydrogen spectroscopy, where the atoms are under the influence of combined time-dependent and quasistatic electric fields. Such situations arise both in laboratory plasmas^{3,4} and astrophysical plasmas (solar flares).⁵

The Blokhintsev⁶ theory describes the effect of simple dynamic electric fields; its inadequacy when both static (or quasistatic) and dynamic fields act simultaneously was pointed out by us

earlier.¹ In that paper we presented the main results for the Lyman- α spectrum for the case of a static and a *linearly polarized* dynamic field perpendicular to the former. However, the modification of the Balmer spectrum^{3,4} (which requires the description of higher- n levels) is of greater practical importance. Also, the more general physical situation would certainly require understanding the effects produced by static and dynamic electric fields (a) in arbitrary directions, (b) of arbitrary polarization, and (c) possibly in conjunction with static or dynamic magnetic fields. These, and related, problems provide the motivation for the present paper.

Our main results are as follows. There are only two types of basic spectra: For *type I*, which obtains for example when all the electric fields are coplanar and there is no magnetic field, (a) the n^2 -times-degenerate n multiplet of energy E_n splits into $2n - 1$ *equally spaced* quasilevels,

$$\epsilon_{n\mu} = E_n + \mu\kappa_n, \quad \mu = -(n-1), \dots, 0, \dots, (n-1), \quad (1)$$

plus a harmonic satellite structure $\epsilon_{n\mu} \pm r\omega$ at integral multiples of the basic frequency ω of the dynamical fields. (b) The basis spacings κ_n for various n 's are related to each other by a simple scaling relation, which allows one to determine the entire structure of an arbitrary n multiplet from the knowledge of the spacing for the much simpler Lyman- α problem. For the *type-II* spectrum, which prevails for example if the electric fields are not coplanar, (a) the remaining degeneracy is now completely removed, resulting in n^2 distinct quasilevels,

$$\begin{aligned} \epsilon_{n\mu^+\mu^-} &= E_n + \mu^+\kappa_n^+ + \mu^-\kappa_n^- \\ &= E_n + (\mu^+ + \mu^-)\sigma_n + (\mu^+ - \mu^-)\delta_n, \\ \mu^+, \mu^- &= -\frac{1}{2}(n-1), -\frac{1}{2}(n-3), \\ &\dots, \frac{1}{2}(n-1), \end{aligned} \quad (2)$$

plus the harmonic satellite structure at $r\omega$ around these levels. The second expression for ϵ_n may be viewed as a fine structure with spacings δ_n , superposed on the previously described type-I spectrum with spacings σ_n . (b) The basic spacings κ_n^{\pm} are related to the spacings κ_2^{\pm} of the corresponding Lyman- α problem which in turn are expressible in terms of the Lyman- α spacings κ_2 for an equivalent type-I problem. For both types I and II, (c) a strong enough magnetic field, in addition to the effects described above, further splits each level into a spin doublet.

We consider in the following (1) pure electric

fields, (2) pure magnetic fields, and (3) combined electric and magnetic fields, which are periodic: $\vec{E}(t+\tau) = \vec{E}(t)$, $\vec{B}(t+\tau) = \vec{B}(t)$. (This does not restrict the fields to be monochromatic, nor does it exclude static fields.)

(1) For pure *electric fields*, the field-dependent part of the Hamiltonian is $H_1 = -\vec{r} \cdot e\vec{E}(t)$, which in general has inter- n -multiplet as well as intra- n -multiplet matrix elements. The influence of the former on the dynamics of the system is, however, negligible as long as the splittings induced by the imposed field within each n multiplet (and the basic frequency of the field) remain small compared to the inter- n separations. Therefore we concentrate on the intramultiplet dynamics.

As a consequence of the O(4) symmetry^{7,8} of the hydrogen atom, the vector $\vec{K} = \vec{M}(me^4/2|E_n|)^{1/2}$, where \vec{M} is the Runge-Lenz vector, can be identified on the n multiplet with the position vector \vec{r} , in the form⁸ $\vec{r} = \frac{3}{2}na_0\vec{K}$, where a_0 is the Bohr radius. Thus, on the n multiplet

$$H_1 = -\vec{r} \cdot e\vec{E} = \vec{K} \cdot \vec{\mathcal{E}}_n, \quad \vec{\mathcal{E}}_n = \frac{3}{2}na_0e\vec{E}. \quad (3)$$

Also, \vec{K} and angular momentum \vec{L} satisfy the standard commutation relations of O(4), $[K_i, K_j] = i\epsilon_{ijk}L_k$, $[L_i, K_j] = i\epsilon_{ijk}K_k$, $[L_i, L_j] = i\epsilon_{ijk}L_k$.

(1a) If the electric field vector $\vec{E}(t)$ at all times remains confined to some given plane (designated the x - y plane), the Hamiltonian H_1 can be re-expressed in terms of an angular momentum operator, by constructing⁸ $\vec{J} = (K_x, L_y, K_z)$, which satisfies $[J_i, J_j] = i\epsilon_{ijk}J_k$:

$$H_1 = \vec{J} \cdot \vec{\mathcal{E}}_n(t). \quad (4)$$

The resulting Schrödinger equation for the evolution operator $U_n(t)$ for the n multiplet constitutes a set of n^2 linear equations with periodic coefficients, thus imparting a Floquet structure⁹ to the solution:

$$U_n(t) = \sum_{q=1}^{n^2} \exp(-ik_q t) P_q(t), \quad (5)$$

where $P_q(t)$ are periodic matrices and $\{k_q\}$ are constants, to be determined. The n^2 eigenvalues of $U_n(\tau)$ can be shown to form the set $\{\exp(ik_q\tau)\}$. To determine the $\{k_q\}$, we note that the solution for $U_n(t)$ can also be written in the Magnus form,^{10,11}

$$U_n(t) = \exp[-i\Omega_n(t)], \quad (6)$$

where $\Omega_n(t)$ is expressible in terms of quadratures of repeated commutators of H_1 at different times. Since the components of \vec{J} form a closed Lie algebra, it follows that the evolution opera-

tor necessarily has the structure¹¹

$$U_n(t) = \exp[-i\vec{J} \cdot \vec{\beta}_n(t)],$$

$$U_n(\tau) = \exp[-i\vec{J} \cdot \vec{\beta}_n(\tau)] \equiv \exp[-iJ_\beta \beta_n(\tau)], \tag{7}$$

where J_β is the component of \vec{J} in the direction of the vector $\vec{\beta}_n(\tau)$, and $\beta_n(t)$ is a unique functional¹¹ $\vec{\beta}$ of $\vec{\mathcal{E}}_n(t)$,

$$\vec{\beta}_n(t) = \vec{\beta}_n[\vec{E}] = \vec{\beta}[\vec{\mathcal{E}}_n]. \tag{8}$$

The entire spectrum can immediately be inferred from the structure of U_n ; detailed knowledge of $\vec{\beta}_n$ is not needed for this purpose. $U_n(\tau)$ is clearly diagonal in the n^2 (angular momentum) basis states $|nj\mu\rangle$: $J^2 \rightarrow j(j+1)$, $J_\beta \rightarrow \mu$. The eigenspectrum of $U_n(t)$ is then given simply as $\exp[-i\mu \times \beta_n(\tau)]$; we can thus identify the set of n^2 Floquet constants as $\{k_q\} = \{\mu \kappa_n\}$, $\kappa_n \tau \equiv \beta_n(\tau)$, $\mu = -(n-1), \dots, 0, \dots, (n-1)$; there are only $2j_{\max} + 1 = 2n - 1$ distinct values of k_q , all integral multiples of the basic unit κ_n . The spectrum is degenerate with respect to j , resulting in a multiplicity $n - |\mu|$. The time dependence of $U_n(t)$, Eq. (5), with this identification of $\{k_q\}$ immediately leads to the "energy spectrum" of type I, where the quasilevels arise from the Floquet exponentials, and the harmonic structure arises from the periodic parts.

The dynamics associated with the type-I spectrum is determined by the fact that $U_n(t)$, Eq. (7), generates a simple time-dependent rotation of the initial states by the angle $\beta_n(t)$ around the direction of $\vec{\beta}_n(t)$. The n^2 states of the n multiplet decouple into n independent completely nondegenerate j multiplets, and

$$U_n = \sum_{j=0}^{n-1} \oplus U_n^{(j)}.$$

In view of Eqs. (8) and (3), $\vec{\beta}_n(t)$ and thus the basic spacings κ_n for the different n multiplets obey the scaling relation

$$\vec{\beta}_n[\vec{E}] = \vec{\beta}_{n'}[(n/n')\vec{E}],$$

$$\kappa_n[\vec{E}] = \kappa_{n'}[(n/n')\vec{E}]. \tag{9}$$

In particular, with the choice $n' = 2$, we can infer the full spectrum as well as the dynamics (i.e., the harmonic structure) of an arbitrary n multiplet from the solution for the Lyman- α problem¹ by a simple scaling by $n/2$ of the amplitudes of the physical electric fields.

(1b) When $\vec{E}(t)$ does not remain confined to a plane, the Hamiltonian $K \cdot \vec{\mathcal{E}}_n$ cannot be re-expressed in the simpler $\vec{J} \cdot \vec{\mathcal{E}}_n$ form, and because of the commutation relation of \vec{K} the resulting

Magnus solution for $U_n(t)$ contains all six generators of the O(4) algebra. A more convenient characterization of $U_n(t)$ can however be obtained in terms of the commuting angular momenta $\vec{G}_\pm = \frac{1}{2}(\vec{L} \pm \vec{K})$,

$$i(\partial/\partial t)U_n^\pm = \pm(\vec{G}_\pm \cdot \vec{\mathcal{E}}_n)U_n^\pm, \tag{10}$$

$$U_n = U_n^+ U_n^- = \exp(-\vec{G}_+ \cdot \vec{\beta}_n^+) \exp(-\vec{G}_- \cdot \vec{\beta}_n^-),$$

$$\vec{\beta}_n^\pm = \vec{\beta}[\pm \vec{\mathcal{E}}_n].$$

The spectra of $U_n(\tau)$ are individually of the angular momentum form, $\{\exp(-i\mu^\pm \kappa_n^\pm \tau)\}$, $\tau \kappa_n^\pm = \beta_n^\pm(\tau)$. On the n multiplet, $G_\pm^2 = \frac{1}{4}(n^2 - 1)$, and thus we are led to the type-II energy spectrum, Eq. (2), with a large spacing $\sigma_n = \frac{1}{2}(\kappa_n^+ + \kappa_n^-)$ and a finer spacing $\delta_n = \frac{1}{2}(\kappa_n^+ - \kappa_n^-)$. Since a mere change of sign of the functional argument can result in a significant change¹¹ in the structure of $\vec{\beta}$, $\beta_n^+(\tau) \neq \beta_n^-(\tau)$ in general. Consequently, the degeneracy of the n multiplet is now completely removed. This results from the breaking of the symmetry of the Hamiltonian; now $[H, J^2] \neq 0$. This also mixes the j multiplets which were independent for a planar field: $U(t)$ is no longer reducible. Whenever $\beta_n^+(\tau) = \beta_n^-(\tau)$, a type-II spectrum reduces to a type-I spectrum, with the states of same $\mu^+ + \mu^-$ becoming degenerate.

(2) For pure *magnetic fields* (time-dependent Zeeman effect, including arbitrary static fields), the Hamiltonian in the dipole approximation, ignoring spin which is accounted for separately, is $H_2 = \vec{\mathcal{O}} \cdot \vec{L}$, $\vec{\mathcal{O}} = (e/2mc)\vec{B}$. The ensuing evolution operator $U(t) = \exp(-i\vec{L} \cdot \vec{\beta}[\vec{\mathcal{O}}])$ evidently leads to the type-I spectrum, irrespective of whether $\vec{B}(t)$ remains coplanar or not, in contrast to the case of the pure electric fields. The identity of the basic Zeeman splittings for all levels is also in contrast to the scaling, Eq. (9), of the Stark splittings.

(3) For combined *electric and magnetic fields*, the Hamiltonian and the evolution operator take the forms

$$H = \vec{K} \cdot \vec{\mathcal{E}}_n + \vec{L} \cdot \vec{\mathcal{O}}$$

$$= \vec{G}_+ \cdot (\vec{\mathcal{O}} + \vec{\mathcal{E}}_n) + \vec{G}_- \cdot (\vec{\mathcal{O}} - \vec{\mathcal{E}}_n), \tag{11}$$

$$U_n(t) = \exp(-i\vec{G}_+ \cdot \vec{\beta}_n^+) \exp(-i\vec{G}_- \cdot \vec{\beta}_n^-),$$

$$\beta_n^\pm = \beta[\vec{\mathcal{O}} \pm \vec{\mathcal{E}}_n]. \tag{12}$$

(3a) The special case where $\vec{E}(t)$ remains planar (x - z plane) and $\vec{B}(t)$ is normal to this plane can be described by the simpler form $H = \vec{J} \cdot \vec{\mathcal{F}}_n$, $\vec{\mathcal{F}}_n = (\mathcal{E}_{nx} \mathcal{O}_y \mathcal{E}_{nz})$, and the spectrum remains of type I, with $\kappa_n \tau = \beta_n(\tau)$, $\vec{\beta}_n = \vec{\beta}[\vec{\mathcal{F}}_n]$.

(3b) The simple case of combined static Stark-Zeeman effect has the spectrum, immediately inferred¹³ from Eq. (11),

$$\epsilon_{n\mu^+\mu^-} = \mu^+|\mathfrak{B} + \mathcal{E}_n| + \mu^-|\mathfrak{B} - \mathcal{E}_n|. \quad (13)$$

For perpendicular fields, this reduces to $\epsilon_{n\mu} = (\mathfrak{B}^2 + \mathcal{E}_n^2)^{1/2}$, which is the degenerate type I. For parallel fields, it is type II with the larger (smaller) spacing equal to the larger (smaller) of the two effective fields.

(3c) In general, the combined time-dependent Stark-Zeeman effect produces a spectrum of type II since $\beta[\mathfrak{B} + \mathcal{E}_n] \neq \beta[\mathfrak{B} - \mathcal{E}_n]$, except for special situations, some of which have been described above. However, the entire spectrum and dynamics for any level can always be inferred from the corresponding information for Lyman- α by scaling \vec{E} by $n/2$ and leaving \vec{B} unchanged.

The conditions for the validity of our results are the following: (i) The field-induced splittings exceed the fine-structure splittings, i.e., $E \gg 91/n^4$ kV/cm, or $B \gg 124/n^3$ kG. The latter condition also ensures the doublet spin structure (complete Paschen-Back effect) described above under (c). (ii) The mixing of levels of different n is negligible, i.e., the quadratic Stark-effect term is small compared to the linear one ($E \ll 7 \times 10^6/n^5$ kV/cm), and $\omega \ll 10^{15}$ sec⁻¹, the natural level separations. (iii) The coupling with the radiation field can be ignored, i.e., $\omega > 10^9$ sec⁻¹, the inverse lifetime of the fastest decaying state within a multiplet.

The *line spectrum* resulting from type-I and -II level structures has the following features: (i) The type-I line spectrum for the transition $n \rightarrow n'$, apart from the harmonic structure, is basically static Stark-like with $(2n-1)(2n'-1)$ independent components at $\{\mu\kappa_n - \mu'\kappa_{n'}\}$ modulo ω . Since $\kappa_n/\kappa_{n'} \neq n/n'$ in general, the spacings between the shifted components are, however, not integral multiples of a basic unit spacing, and also the special degeneracies of the static Stark effect such as in H_β (where $n=2n'$) are removed. (ii) The type-II line spectrum for $n \rightarrow n'$, apart from the harmonic structure, contains $n^2n'^2$ independent components together with harmonics at multiples of ω , which on a gross scale appear as a type-I line spectrum at $\{\mu\sigma_n - \mu'\sigma_{n'}\}$ ($\mu = \mu^+ + \mu^-$), but with a finer structure governed by δ_n and $\delta_{n'}$. (iii) The spin doubling of the multiplet levels in the complete Paschen-Back domain has no effect on the line spectrum because of the absence of spin-flip transitions.

Regarding the experimental implications of the

present theory, these results would apply directly to the situations where the fields are macroscopically generated and are uniform over the observed sample. For turbulent plasmas, the high-frequency field originates from electron plasma oscillations, while the quasistatic field arises from low-frequency, typically strong ion-acoustic turbulence, as well as the usually smaller quasistatic Holtzmark fields. Different atoms see fields which differ in magnitude, direction, and phase, and whose statistical distribution depends on the specific nature of the turbulence. Thus, the observed spectral profiles are to be obtained by an averaging of our preceding results over the appropriate field parameters, leading to a *turbulence broadening* of the spectral lines, different in character from and often exceeding the broadening caused by other mechanisms (Doppler, Holtzmark, etc.). Then it is the precise shapes of these turbulence-broadened lines which carry the diagnostic information on the turbulent plasma fields.

Calculations pertaining to state structure, transition probabilities, and line intensities, as well as a diagnostic analysis of the observed Balmer spectrum from a turbulent plasma,³ will be the subject of forthcoming publications.

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Anomalous Kapitza Resistance to Solid Helium*

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The Kapitza thermal boundary resistance at temperatures near 1 K is anomalously small to solid ^3He or solid ^4He just as it is to liquid ^3He or liquid ^4He .

The measured Kapitza thermal boundary resistance R_B between two solids is generally in good agreement ($\approx 10\%$) with the acoustic mismatch model of thermal energy exchange at low temperature.¹⁻³ In this model the reflection and refraction of thermal phonons at the interface is calculated by use of classical acoustics. Even R_B to liquid He may be explained in terms of acoustic mismatch at temperatures below ≈ 0.1 K.² An explanation of R_B to liquid He at temperatures of ≈ 1 K, however, remains elusive. No theory introduced thus far can explain the very small magnitude of R_B ,⁴⁻⁶ the small pressure dependence,⁴ the fact that transverse phonons in the solid efficiently transfer energy to the liquid,⁷⁻⁹ and that the thermal impedance is qualitatively independent of whether the liquid is ^3He or normal or superfluid ^4He .^{10,11} The purpose of the present Letter is to correct a fallacy which is introduced^{12,13} in searching for an explanation of R_B at ≥ 1 K, namely that the anomaly occurs only when liquid He is present. R_B to solid He at ≈ 1 K is *not* in agreement with the acoustic mismatch model, it is also anomalously small. Hence the anomalous behavior is not to be associated solely with properties of the liquid.

The experimental arrangement for measuring R_B was similar to that of Anderson and Johnson.¹⁴ Two closely spaced 2.5-cm-diam plates of electropolished Cu were separated by a thin layer of He. The thermal impedance of the sandwich, $2R_B$, was measured under conditions of constant heat flux. Some of the data thus obtained are shown in Fig. 1 as $R_B T^3$ to emphasize the T^{-3} temperature

dependence predicted for R_B by the acoustic mismatch theory. Data below 0.3 K were obtained in a dilution refrigerator; data above 0.4 K were obtained in a different cell in a ^3He refrigerator. The dashed lines represent previous data obtained in a magnetic refrigerator using an electropolished Cu cell of completely different geometry.¹⁰ Near 1 K the present data agree with pre-

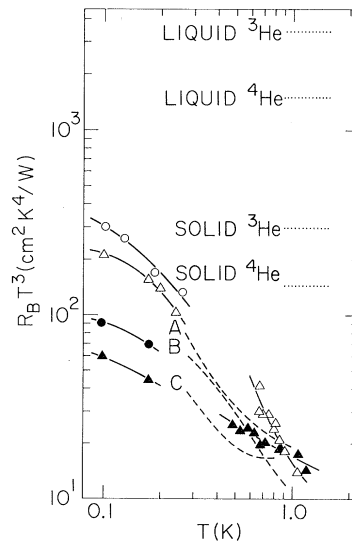


FIG. 1. The Kapitza resistance between Cu and He. Open circles, liquid ^3He ; closed circles, solid ^3He ; open triangles, liquid ^4He ; closed triangles, solid ^4He . The sample pressure for liquid He was ≈ 0 atm, for solid He ≈ 37 atm. Curves A, B, and C are for liquid ^3He , liquid ^4He , and solid ^3He , respectively, as obtained from Ref. 10. The dotted lines represent calculations using an acoustic mismatch theory.