Equation of State for Hadronic Matter Produced in High-Energy Collisions

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Conflicting inclusive data from the CERN intersecting storage rings, treated with Landau's original hydrodynamical differential equations for an arbitrary equation of state ($p=H\epsilon$) describing the created hadronic matter fluid, lead to two different physical pictures. A recent Pisa–Stony Brook experiment suggests $p=0.3\epsilon$; $p=\epsilon/3$ implies scale invariance, suggesting we are near an asymptotic region governed by scaling. However, earlier experiments imply that the dynamics may be quite different ($p\approx\frac{1}{2}\epsilon$), with viscous hadronic "fluid" being produced.

The hydrodynamical model for very high-energy collisions was proposed by Landau[1] twenty years ago to provide "significant improvement" on Fermi's statistical model. Increasing attention[2-4] has been given this model because laboratory experiments, particularly at the CERN intersecting-storage-ring (ISR) facility, are now being done at energies where many of the very basic physical assumptions might be expected to hold or be tested. The model depicts two ultrafast extended particles, Lorentz contracted into flattened disks, approaching each other in the c.m. system. At the moment of impact one hot, compressed thin disk exists with a large undetermined number of particles whose mean free paths are exceedingly short so that hydrodynamical differential equations describe the evolution of this hadron fluid. It is necessary to assume an equation of state for this matter fluid as it expands adiabatically with conservation of entropy. Landau assumed relations for black-body radiation would be applicable, i.e., the chemical potential is zero and the pressure p is one third of the energy density ϵ,

$$p=\epsilon/3. \tag{1}$$

Following the expansion hydrodynamically to a critical temperature T_c, such that $kT_c\approx m_\pi c^2$, when the mean free paths are expected to be large enough for fluid breakup, Landau arrived at a number of relationships which appear very relevant for interpreting recent ISR data. The prediction of a power-law dependence between multiplicity and lab energy ($E_{\rm lab}$) has been checked to hold over many decades in $E_{\rm lab}$.[3] The energy dependence of the "plateau" region [where $\eta=-\ln(\tan\frac{1}{2}\theta)$ is small in magnitude] and the distribution of produced inclusive pions as a function of η are both well predicted.[3,4] Also, the small transverse momentum and longitudinal momentum dependences are well described when Hagedorn's thermodynamic considerations are incorporated[2] into the breakup part of Landau's model. Correlation data from the National Accelerator Laboratory (NAL) seem also to be described by the model.[2]

However, these comparisons with data[2,3] are based *not* on the rough distribution Landau derived (*or* his original equations),

$$N^{-1}dN/d\eta=(2\pi L)^{-1/2}\exp(-\eta^2/2L), \tag{2}$$

but on a similar equation in which Landau's variable $\lambda=\eta=-\ln(\tan\frac{1}{2}\theta)$ is replaced by rapidity y with L kept unchanged, $L=\ln(\gamma)=\frac{1}{2}\ln(s/4M_p^2)$, where γ is the Lorentz transformation factor connecting c.m. and lab frames. This change appears to be unjustified, although the authors do refer to Milekhin[5] for this modification. Milekhin did indeed derive a distribution from Landau's equations with y as the variable, but L was simultaneously required to be different from that found by Landau.

To correct this inconsistent situation, a more precise, consistent numerical computer solution of Landau's differential equations has been obtained without assuming the correctness of the approximate Gaussian-type solutions. It was found possible to eliminate the black-body radiation assumption and replace Eq. (1) by a more general equation of state,

$$p = H\epsilon. \quad (3)$$

The usual thermodynamical considerations suggest that H is the square of the sound velocity in the relativistic hadron "fluid." The ultrarelativistic limit can be shown to be coincident with black-body radiation for which $H = c^2/3$ (c = speed of light = 1). Besides improving on the calculations appearing in the literature, we can study the validity of Landau's black-body radiation assumption and answer the question: "How relativistic is the hadron fluid produced in ISR experiments (within present data limitations)?" Recent work by Shuryak[6] investigates the effect of varying the equation of state in the 10–70-GeV/c momentum range, but using the Gaussian approximation.

Two quite different possibilities reveal themselves when the weakest assumption of Landau's theory is removed. Either $H \approx 0.3$ or 0.5, depending on which of two mutually different ISR results is more nearly correct. The physical interpretations are likewise mutually exclusive and forecast profoundly different physics at high energies: The smaller number, $H = 0.3$, implies that the ultrarelativistic limit $\frac{1}{3}$ for the hadron fluid produced at ISR has not quite yet been reached, and is being approached from smaller values as is appropriate for weakly interacting, closely compressed fluid particles described by a scale-invariant theory. On the other hand the physics needed for the $H = 0.5$ result requires that the fluid in the hydrodynamic model be relatively viscous as one expects if strong forces are required to move one of the particles through it. This means dissipative forces are not negligible, and the entropy-conservation assumption made by Landau for the initial expansion stage is not valid.

An important result is that the equation of state now influences the multiplicity distribution,

$$N = KE_{\text{c.m.}}{}^{(1-H)/(1+H)} \propto E_{\text{lab}}{}^{(1-H)/2(1+H)}, \quad (4)$$

which for $H = \frac{1}{3}$ gives the $E_{\text{lab}}{}^{1/4}$ dependence derived by Landau. Figure 1 shows some power-law fits to multiplicity data for relevant values, $H = 0.28$, 0.50, and $\frac{1}{3}$, along with a logarithmic

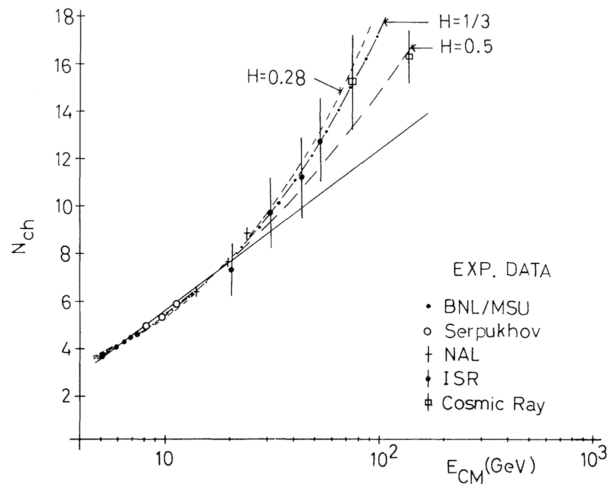

FIG. 1. Multiplicity as a function of c.m. energy. The lines represent best fits to the data for the following functional forms: solid line, $a + b\ln(s)$; dot-dashed, $a + k(E_{\text{c.m.}} - 2m_p)^{1/2}$; short dashes, $a + k(E_{\text{c.m.}} - 2m_p)^{0.55}$; and long dashes, $a + k(E_{\text{c.m.}} - 2m_p)^{1/3}$. The appropriate values for H in the equation of state label the curves.

dependence for comparison. The high-statistical-weight low-energy points in Fig. 1 exert sufficient leverage on the curves so that it seemed advisable not to constrain them to pass through the origin. The inclusion of these lower-energy conventional-accelerator data in serious definitive fitting is questionable because of uncertainty as to how low an energy the assumptions basic to the hydrodynamical model are valid. It seems that the trend of the higher-energy data is towards power-law behavior, and it is critical that experiments in the near future improve these data to allow stronger conclusions to be drawn. From the purely statistical viewpoint, the three power-law curves are equivalent if the highest-energy cosmic-ray point is dropped. (Higher-energy cosmic-ray points exist with error bars such that they include the three power-law curves.)

Shuryak[6] recently modified Eq. (1) in Landau's model by using a power approximation for the mass-density function $\rho(m) \sim m^{\kappa}$ with $\kappa \approx 3$, giving a reasonable fit to the mass spectrum produced in the incident-particle energy range 10–70 GeV. In our notation this corresponds to $H = (\kappa + 4)^{-1} = 0.14$ for which value the multiplicity data below $E_{\text{lab}} \approx 100$ GeV are well fitted. But in comparison with all the data shown in Fig. 1, $H = 0.14$ produces a curve which rises too rapidly above NAL energies. In general, Fig. 1 indicates that a wide *range* of power-law curves is consistent with the present multiplicity data. Shuryak's case of p

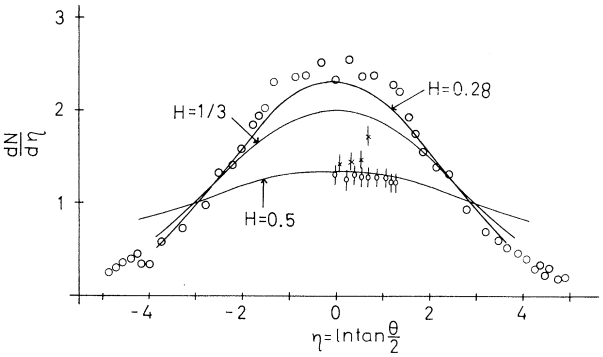

FIG. 2. Charged-pion distribution $dN/d\eta$ versus η for $E_{\text{c.m.}}=30.8$ GeV. The solid curves show how changing H effects the intensity distribution. The large circles show the Pisa–Stony Brook results as reported by Bellettini (Ref. 7). The data shown with small circles and crosses are representative of the earlier published data of Ref. 7. The normalizations differ, and the theory curves are calculated for N-versus-$E_{\text{c.m.}}$ curves which are normalized as in Ref. 3 to the $E_{\text{c.m.}}=52$-GeV point.

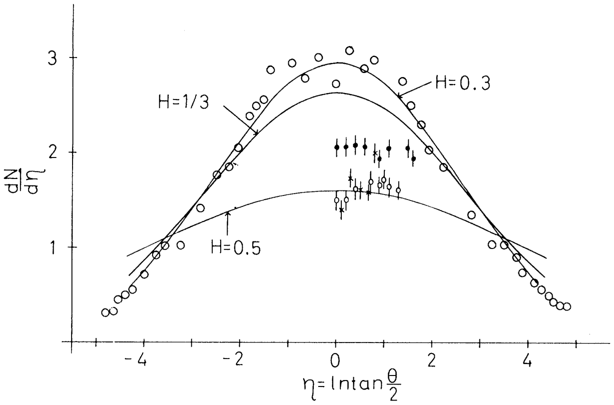

FIG. 3. Distribution $dN/d\eta$ versus η for $E_{\text{c.m.}}=53.4$ GeV. The same comments as in the legend for Fig. 2 apply, except that at this energy, the emulsion results of Ref. 9 are also shown. These data have a normalization requiring a smaller H value than that for Ref. 8, but still larger than that for Ref. 6. The data of Ref. 6 appear to be better fitted with $H=0.3$ than with 0.28, possibly indicating this higher energy is more asymptotic (assuming Refs. 8 and 9 have too low a normalization).

$=0.14\epsilon$ corresponds to the dynamical situation where higher resonances become more important. In fact, the limiting case $H=0$ shows the behavior given an exponential mass spectrum which arises when an infinite number of resonances are involved.

In the manner of Eq. (2), the theory prescribes the normalization of $dN/d\eta$ distributions in terms of N. Rather than use the more general N forms in Fig. 1, we show curves in Figs. 2 and 3 which are computed as in Ref. 3, where N of Eq. (4) is constrained to fit the ISR point at $E_{\text{c.m.}}=52$ GeV. The plots shown and conclusions drawn are basically unchanged by this, but differences will be thoroughly discussed elsewhere. In Figs. 2 and 3, the circles give the result for $dN/d\eta$ found by a recent Pisa-Stony Brook (PS) collaboration.[7] We note that the curve for $H\approx 0.29$ gives an excellent fit at both energies while those for $H=\frac{1}{3}$ and $\frac{1}{2}$ are both too broad. These data are rumored to have normalization problems and also the transverse motion of the c.m. frame cannot be removed, but a 100% increase over earlier data[8] is still surprising. These earlier data[8] (as seen from Figs. 2 and 3) in the "plateau" region have a normalization which is lower than the PS data, are flatter, and strongly support the $H=0.5$ conclusion. It is clear that this data difference is a crucial, important one. The emulsion result of Babecki *et al.*[9] is slightly higher than that of Ref. 8, and it can be well fitted by reducing H from 0.50 to 0.44.[10] Also, our computer-generated curves in Figs. 2 and 3 are *not* Gaussian as they tend to be flatter at the top. When analytical-form approximate solutions are sought, these inevitably turn out to be Gaussian because Landau's physically motivated intuitive steps [which led him to Eq. (2)] are followed or slightly deviated from. As one example with more deviation, Shuryak obtains a Gaussian in rapidity with $L=[8H/(3-3H^2)]\ln(\gamma)$. This is different from the L obtained by Milekhin[5] who deviated from (or improved on) Landau in a different fashion. The apparent freedom in the width parameter L thus depends on what approximations to the exact solution an author makes.

We have noted that the physical conditions for $p=\epsilon/3$ obtain in the ultrarelativistic limit when the particles of the matter fluid begin to "behave" like black-body radiation. Through the present analysis the PS data (though they are rumored to be somewhat suspect) suggest that a limit is being reached at ISR energies wherein the normal strong-interaction correlation length ceases to matter when a regime reminiscent of scale invariance (in many-body calculations[11]) appears. It is by no means clear that critical phenomena, the renormalization group, and expansions in dimensionality are relevant here; nevertheless, $H=\frac{1}{3}$ is the appropriate result in such a limit for a three-dimensional fluid. This interesting question seems best pursued elsewhere.

If the PS data are too high, Landau's assumptions regarding the hadronic fluid properties need modification. The situation then is that the hadron fluid being "produced" at ISR is more like tar than compressed water, and the entropy conservation assumption of Landau will have to be changed according to future experiments. Three older, independent ISR experiments[9,10] support this view, but these data are of limited utility because of the short η or γ region they span, and the importance of measurements over the widest possible range is clear.

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[9]J. Babecki *et al.*, Phys. Lett. 40B, 507 (1972).

[10]We note that at $\eta = 0$ the dependence of $dN/d\eta$ on the equation of state parameter is $(dN/d\eta)_{\eta=0} \propto \{H^{-3/2}(1-H)[4-H^{1/2}(1+H)]\}^{1/2}$.

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ERRATUM

DISORDER-INDUCED MAGNETIC PHASE TRANSITIONS. B. A. Huberman [Phys. Rev. Lett. 31, 1251 (1973)].

The second sentence of the second paragraph on page 1253 should read, "Since Γ is not known even in simple systems of this type, and for $J = 2$ it provides only a 'fine tuning' of the transition, we set it equal to 1." Also, the parameter values for Fig. 3 are $U_i = 1$ eV, $U = 1.7U_i$, $A = 0.07U_i$, and $J = 2$.