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[8]In Ref. 1 the overall normalization was determined indirectly by extrapolating to measurements of Reaction (1) at 24 and 30 GeV. For the relative normalization between different energies we had assumed a constant total cross section. In view of the recent evidence for a rising total cross section a small s-dependent correction factor should be applied to the data of Ref. 1. The factor is simply $[\sigma_T/(38.5\text{ mb})]^2$ and amounts to a $\sim 5\%$ increase at the highest energy data of Ref. 1. This small correction reduced by $\sim 30\%$ the value of the parameter B in $A(1+B/\sqrt{s})$ which was used to fit the data in Ref. 1.

Determination of Triple Regge Couplings from a Study of the Reaction $p+p\to p+X$ between 50 and 400 GeV*

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We present an analysis, in the framework of the triple Regge model, of our recent experimental results on the reaction $p+p\to p+X$ between 50 and 400 GeV.

In a recent experiment[1] at the National Accelerator Laboratory we have studied the single-particle distribution for the inclusive reaction

$$p+p\to p+X \quad (1+2\to 3+X). \tag{1}$$

In the last two years several authors[2] have attempted to describe this reaction in terms of triple Regge couplings. Near the kinematic limit for particle 3, where $s\gg M^2\gg 1$ GeV^2 and t is small (s, M^2, and t are, respectively, the squares of the center-of-mass energy, the mass of X, and the four-momentum transfer between particles 2 and 3), the combination of Regge phenomenology with the generalized optical theorem leads to a prediction for the invariant cross section for particle 3 which is[3]

$$\frac{sd^2\sigma}{dt\,dM^2}=\frac{s_0}{s}\sum_{ijk}G_{ijk}(t)\left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)}\left(\frac{M^2}{s_0}\right)^{\alpha_k(0)}. \tag{2}$$

In the triple Regge (TR) formula (2), $G_{ijk}(t)$ is the product of a Reggeon-Reggeon-Reggeon coupling, $g_{ijk}(t)$, and three particle-particle-Reggeon couplings, i.e., $G_{ijk}(t)=g_{ijk}(t)\beta_{i23}(t)\beta_{j23}(t)\times\beta_{k11}(0)$. In what follows we shall refer to $G_{ijk}(t)$ as the TR couplings. The functions $\alpha_i(t)=\alpha_{0i}+\alpha_i't$ are Regge trajectories and, as usual, we take the scale factor $s_0=1$ GeV^2. We shall also make use of the Feynman variable $x\simeq 1-M^2/s$.

The TR formula (2) is an asymptotic expression and in the absence of a detailed theory the range of s, M^2, and t over which it is valid can only be guessed from a phenomenological analysis. In order to do this, however, we must know how many terms to allow in Eq. (2) and also what intercepts α_{0i} and slopes α_i' to use for the various trajectories. In principle all combinations ijk which are not forbidden by quantum-number conservation can contribute to Eq. (2), the only restriction being that diagonal terms with $i=j$ must be positive. Of course, exchange degeneracy simplifies the situation but even allowing only the Pomeranchuk (P) and leading meson (R) trajectories we obtain six terms (PPP, PPR, RRP, RRR, RPR, and RPP). The uncertainties on what $s \gg M^2 \gg 1$ really means, on what trajectory parameters to use, and on how many terms to allow in Eq. (2) make the determination of the TR couplings a highly speculative game. In what follows we shall try to play this game using the NAL data of Ref. 1. We shall assume for simplicity that the interference terms RPP and RPR ($i \neq j$) vanish. We shall also assume that the TR formula can be expected to hold for $s, M^2 \gtrsim 8$ GeV^2, $s/M^2 \gtrsim 5$, and $|t| \lesssim 0.5$ GeV^2. The choice of the conditions $s, M^2 \gtrsim 8$ GeV^2 and $|t| \lesssim 0.5$ GeV^2 is motivated by the success of Regge phenomenology in two-body reactions and by the desire to avoid complications from the resonance region.[4] The choice of $s/M^2 > 5$ is motivated[5] by the requirement that the absolute value of

$$\cos\theta_t = \frac{2st^2 + t^2 - t(3m^2 + M^2)}{\{t(t-4m^2)[t-(M-m)^2][t-(M+m)^2]\}^{1/2}} \tag{3}$$

should be >2 (m is the proton mass). For $s \gg M^2 \gg m^2 \gg |t|$ expression (3) reduces to $|\cos\theta_t| \simeq (s/M^2)|t|^{1/2}m^{-1}$ and for $|t| = 0.16$ which is the lowest value in our data the condition $|\cos\theta_t| > 2$ implies $s/M^2 > 5$. As pointed out in Ref. 5 this condition may not be sufficient to ensure the validity of a TR expansion.

The data on Reaction (1) to be fitted by the TR formula (2) came from a single experiment[1] covering the range $100 \leq s \leq 750$ GeV^2, $0.14 \leq t \leq 0.38$ GeV^2, and $5 \leq s/M^2 \leq 12.5$. The wide energy range covered should allow a clean separation of the energy-dependent terms PPR and RRR from the energy-independent terms PPP and RRP. We shall present the results of five different fits and discuss them as we proceed. The data are divided into four t intervals: $0.14 < |t| < 0.18$, $0.18 < |t| < 0.22$, $0.22 < |t| < 0.28$, and $0.28 < |t| < 0.38$ GeV^2, and the TR couplings are free parameters for each t interval. In all cases the errors are about $\pm 3\%$.

Fit I: We assume four terms PPP, PPR, RRP, RRR with conventional trajectories $\alpha_P = 1 + 0.25t$, $\alpha_R = 0.5 + t$. The resulting fit is shown in Fig. 1 and the TR couplings are given in Table I. We conclude that the fit is poor, the main problem being that it does not reproduce the dip in the data around $x = 0.88$. This failure is more pronounced for low $|t|$. For $x > 0.87$ the fit is reasonable. Notice in Table I that $G_{PPP}(t)$ shows a maximum around $|t| = 0.22$ GeV^2.

Fit II: In order to improve the fit we reject all points with $x < 0.84$. As mentioned earlier the validity of the TR formula is questionable for $s/M^2 \simeq 5$. The requirement $x \geq 0.84$ implies $s/M^2 \geq 6.2$. As for Fit I, we fit the data with four terms and with conventional trajectories. The resulting couplings $G_{ijk}(t)$ are given in Table I. We notice that whereas $G_{RRR}(t)$ and $G_{PPR}(t)$ are essentially the same as for Fit I, the relative magnitude of the couplings $G_{PPP}(t)$ and $G_{RRP}(t)$ has changed. This is understandable in view of the fact that over the limited x range of our data the contributions of the PPP and RRP terms are of the same order. Both terms are energy independent and although the x dependence is different it

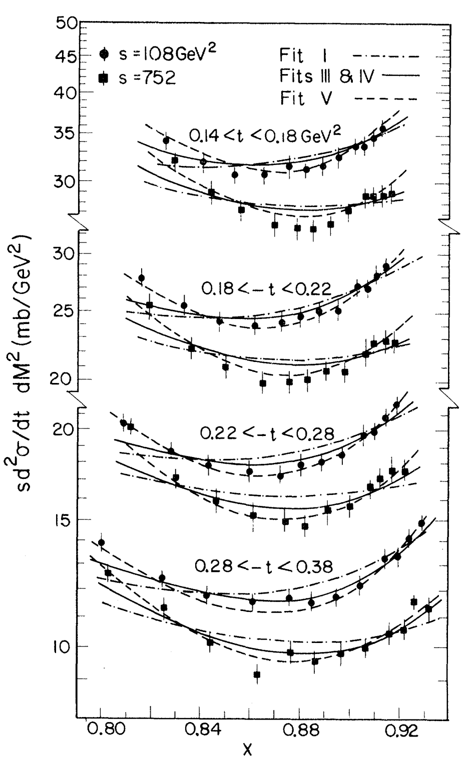

FIG. 1. The results of our TR fits to the experimental data of Ref. 1 at energies $s = 108$, 213, 285, 503, and 752 GeV^2. For clarity we show only the data and curves at the two extreme energies. For all fits the curves for the other three energies lie between the two extremes.

TABLE I. The TR couplings $G_{ijk}(t)$ resulting from fits of Eq. (2) to the data of Ref. 1. Each fit at fixed t has 56 degrees of freedom except Fit II which has 46. For all fits we find $G_{RRR} \simeq 0$. For Fits I and II we use conventional trajectories $\alpha_P = 1 + 0.25t$, $\alpha_R = 0.5 + t$. Fit II is restricted to data with $x > 0.84$. In Fit III we use $\alpha_R = 0.2 + t$ for the RRP term keeping $\alpha_R = 0.5 + t$ for the PPR term. Fit IV is like Fit I with additional fixed $\pi\pi$P term.

FIT	$-t$ GeV^2	G_{PPP} mb/GeV^2	G_{PPR} mb/GeV^2	G_{RRP} mb/GeV^2	χ^2
I	0.16	0.34	3.7	45	135
	0.20	0.48	3.2	38	162
	0.25	0.54	2.3	32	194
	0.33	0.43	1.8	26	204
II	0.16	0.61	3.8	40	55
	0.20	0.85	3.2	31	40
	0.25	0.80	2.3	26	44
	0.33	0.57	1.7	22	61
III	0.16	1.3	3.8	108	82
	0.20	1.2	3.3	91	94
	0.25	1.0	2.3	78	99
	0.33	0.7	1.8	67	84
IV	0.16	0.92	3.7	26	96
	0.20	0.84	3.6	24	118
	0.25	0.75	2.3	23	144
	0.33	0.52	1.8	21	158

is not possible to decouple the two contributions. In the case of the energy-dependent (like $s^{-1/2}$ for fixed x) terms PPR and RRR, on the other hand, the fits show a clear preference[6] for the PPR term, i.e., the s dependence of the data is adequately described by the single term PPR. Notice again that $G_{PPP}(t)$ has a maximum around $|t| \simeq 0.22$ GeV^2. This fit is better than Fit I but the above observations lead us to try different parametrizations in an attempt to improve the fit over the entire x range of the experiment and to study further the interplay between the PPP and RRP terms.

The difficulty of the four-term TR formula (with conventional trajectories) in describing the dip in the x distribution has been noticed by several authors. This is essentially because the RRP term does not drop fast enough with increasing x. Chan, Miettinen, and Roberts[7] in fitting data on $\pi^- + p \to p + X$ find it necessary to lower the intercept of the leading meson trajectory to 0.2. This is interpreted as being due to the contributions from lower-lying trajectories which may be important in the low x region.

Fit III: We follow the prescription of Ref. 7 and fit the entire x range with four terms PPP, PPR, RRP, RRR. For the RRP term we take $\alpha_R = 0.2 + t$ while for PPR and RRR we keep $\alpha_R = 0.5 + t$. For $\alpha_P(t)$ we take $1 + 0.25t$. The resulting fit is shown in Fig. 1 and the couplings are given in Table I. The fit is considerably better than Fit I. Notice that G_{PPP} does not turn over at $|t| = 0.22$ GeV^2. As for Fits I and II, $G_{PPR}(t)$ does not change.

Instead of accounting for lower-lying trajectories by an effective $\alpha_R(t)$ we can introduce them explicitly in the TR formula. This has been advocated by Bishari[8] and more recently by Yem,[8] who point out that a $\pi\pi$P term should be taken into account especially at low $|t|$.

Fit IV: We take the normalized expression for the $\pi\pi$P contribution from Bishari[8] and try a four-term fit (PPP, PPR, RRP, RRR, $\pi\pi$P fixed) with conventional trajectories. The resulting fit is very similar to Fit III and the couplings are listed in Table I. As in Fit III the triple-Pomeron coupling $G_{PPP}(t)$ does not turn over at low t.

A third, and much more *ad hoc*, way to produce a steeper drop with increasing x and therefore a more pronounced dip in the x distribution has been advocated by Capella, Hogaasen, and Peterson.[9] They have obtained a very good fit to Reaction (1) at 24 GeV/c by replacing the RRP term by an exponential $e^{d(1-x)}$ where d may depend on t. The need for such a term has also been pointed out by Berger[4] and has been advocated on theoretical grounds by Salin and Thomas[10] from a study of the behavior of a six-point amplitude in a dual model. Presumably this exponential reflects phase-space effects[4,9,10] and possibly lower-lying trajectories.[10]

Fit V: We replace the RRP term by an exponential and try a fit of the form PPP + PPR + $ce^{d(1-x)}$. The resulting fit is shown in Fig. 1 and the values of the couplings and the parameters c and d are given in Table II. This is clearly a much better fit than I to IV but has been achieved by violating the rules of the game. The only theoretical justification for an exponential as presented by Salin and Thomas[10] goes beyond the usual assumptions of a theory with factorizing Regge poles. As expected, the values for $G_{PPR}(t)$ from this fit are the same as for I to IV while $G_{PPP}(t)$ shows

TABLE II. The parameters resulting from Fit V (see text) obtained by replacing the RRP term in the TR formula (2) by a term of the form $c \exp[d(1-x)]$. Each fit at fixed t has 56 degrees of freedom.

$-t$ (GeV^2)	G_{PPP} ($\mathrm{mb/GeV}^2$)	G_{PPR} ($\mathrm{mb/GeV}^2$)	c ($\mathrm{mb/GeV}^2$)	d	χ^2
0.16	1.9	3.8	2.0	13	26
0.20	1.6	3.3	1.4	13	24
0.25	1.3	2.2	1.2	12	22
0.33	0.7	1.7	1.2	10	35

the same behavior as for III and IV.

In conclusion, our data on $p+p \rightarrow p+X$ allow a clean separation of the energy-dependent TR terms. We find that $G_{\mathrm{RRR}} \simeq 0$ and obtain values for G_{PPR} as a function of t which we believe to be reliable. For the triple-Pomeron coupling $G_{\mathrm{PPP}}(t)$ our data imply two possibilities typified by Fits II and III. Using conventional parameters for the Pomeron and leading meson trajectories and neglecting interference terms and lower-lying trajectories, we obtain a reasonable fit to the data for $x > 0.84$. In this fit $G_{\mathrm{PPP}}(t)$ goes through a maximum at about $|t| = 0.22\ \mathrm{GeV}^2$. When we include terms such as $\pi\pi\mathrm{P}$ either directly or by lowering the intercept of $\alpha_{\mathrm{R}}(t)$ or by introducing an *ad hoc* exponential dependence on x we obtain better fits and $G_{\mathrm{PPP}}(t)$ does not have a maximum at least down to $|t| = 0.16\ \mathrm{GeV}^2$. In order to distinguish between the two possibilities data with $x > 0.92$ would be needed.

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