eral make a very sizable contribution to the (^{3}He, t) amplitude. The general effect would be that, in second-order DWBA or CRC calculations which ignore the nonorthogonality correction, the direct amplitude would need to have the wrong phase relative to the two-step terms in order to fit the data. This phase difficulty has already been noticed in a number of instances.[1,3,4]

Such effects of nonorthogonality are by no means restricted to charge-exchange reactions, but would be expected to manifest themselves in any reaction with a significant multistep contribution. For example, recent calculations for successive nucleon-transfer processes in heavy-ion reactions clearly show the importance of the nonorthogonality contributions.[11,18]

Better estimates of the nonorthogonality contributions to the cases we have discussed require full finite-range treatment of the overlap integrals. Detailed calculations are now underway by the present authors.

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Simple Potential Model for Nucleus-Nucleus Interactions*

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We construct a simple optical potential for nucleus-nucleus processes by a folding method. With these potentials, we have obtained high-quality fits to a variety of elastic-scattering data. We also successfully reproduce one-particle transfer data. However, two-particle transfer results may indicate the need for further refinements of the model. In addition, we calculate the bound states of an $\alpha + {}^{16}$O folded potential. There is good agreement with experimental energies and α widths for two α-cluster bands in ^{20}Ne.

A simple folding model, where the optical potential is given by a convolution of the target density with an effective nucleon-nucleon interaction, is quite successful in the description of nucleon-nucleus elastic scattering.[1] In view of the considerable amount of heavy-ion elastic scattering and transfer data which has recently become available, it seems particularly relevant to study the application of this simple and physically appealing technique to heavy-ion processes. A generalization of this model to nucleus-nucleus interactions yields an optical potential of the form

$$V_{\rm opt}(\vec{r}) = -\frac{2\pi\hbar^2\bar{f}}{M}\int d^3r'\,\rho_1(\vec{r}'-\vec{r})\rho_2(\vec{r}'). \qquad (1)$$

In Eq. (1), ρ_1 and ρ_2 are the projectile and target total densities, respectively, M is the nucleon

mass, and $\bar{f}$ is an adjustable complex strength constant (in femtometers). A different approach, given by a convolution of a nucleon-projectile optical potential with a target density, has been used successfully to describe the elastic and inelastic scattering of α particles.[2]

Equation (1) may be viewed as an approximation to the first (Hartree) term in a systematic density expansion of the nucleus-nucleus optical potential.[3,4] Exchange terms as well as Pauli and short-range correlations are neglected. Also, the effective nucleon-nucleon interaction is taken to have zero range.[5] This model is expected to be a reasonable approximation for those processes which are primarily sensitive to the long-range part of the potential, such as elastic scattering and few-nucleon transfer reactions. For such reactions, we are chiefly interested in testing the *shape* given by Eq. (1) and, therefore, we assume that corrections to Eq. (1) can largely be absorbed into $\bar{f}$. Thus, we treat $\bar{f}$ as a single complex adjustable parameter.

An estimate for $\bar{f}$ can be obtained from the free-space *NN* forward-scattering amplitude by performing an appropriate average over spin, isospin, and Fermi motion.[4] For α scattering above 100 MeV, this estimate agrees with the empirical value of $\bar{f}$ to within a factor of 2; for the lower-energy heavy-ion data, there is a considerably greater renormalization of $\bar{f}$ in some cases.

The results presented here use a value $\bar{f}$ determined from a least-squares fit to elastic-scattering data, with the real and imaginary well geometry fixed by Eq. (1). We prefer to retain this very simple approach, which makes optimal use of our knowledge of nuclear densities to minimize the number of free parameters in the optical potential. For example, the folded potential is free of continuous ambiguities.

We chose proton densities which fit elastic electron scattering[6]:

$$\rho(r)=\rho_0\left(1+\frac{wr^2}{R^2}\right)\times\begin{cases}\left[1+\exp\left(\dfrac{r-R}{a}\right)\right]^{-1} & \text{(2a)}\\ \exp\left(-\dfrac{wr^2}{\alpha R^2}\right) & \text{(2b)}\end{cases},$$

where $w=3\alpha(2+5\alpha)/2(2+3\alpha)$ for (2b). The neutrons were assumed to have the same radial distribution as the protons. For ^{41}Ca, ^{49}Sc, and ^{66}Ni, where no densities were available, the parameters were chosen from ^{40}Ca, ^{48}Ca, and ^{64}Ni, respectively, with the radius scaled up by $R=r_0A^{1/3}$.

Typical fits to elastic-scattering data[7] using the potential of Eq. (1) are shown in Fig. 1. For α scattering, only the data in the region of strong

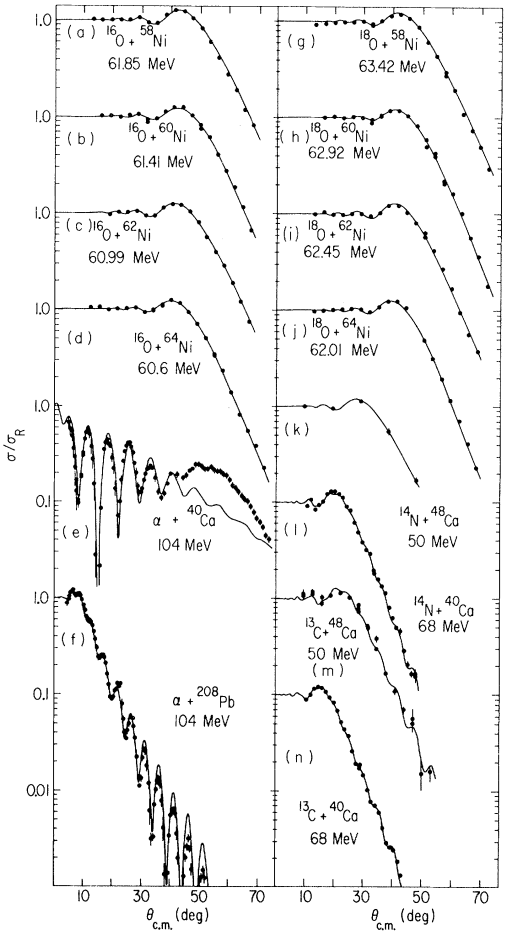

FIG. 1. Best fits to the elastic cross section σ divided by the Rutherford cross section σ_R. All data were taken at Brookhaven National Laboratory (Refs. 9–11) except for α-scattering results of Habs *et al.* (Ref. 12). When no error bars appear, they are smaller than the size of the data points. The incident lab energy is depicted. We denote the density parameters by $[R,a,w]$ if the density of Eq. (2a) was used, or by $[R,\alpha]$ if the density of Eq. (2b) was used. The projectile density parameters are, for α, [1.01, 0.33, 0.45]; ^{13}C, [2.32, 1.33]; ^{14}N, [2.48, 1.67]; ^{16}O, [2.75, 1.6]; ^{18}O [2.77, 2.0]. The target density parameters are, for ^{40}Ca, [3.68, 0.59, −0.1]; ^{48}Ca, [3.74, 0.53, 0]; ^{58}Ni, [4.35, 0.5, −0.185]; ^{60}Ni, [4.57, 0.53, −0.35]; ^{62}Ni, [4.51, 0.52, −0.284]; ^{64}Ni, [4.21, 0.58, 0]; ^{208}Pb, [6.4, 0.54, 0.32]. The best-fit values of $\bar{f}$ in femtometers are (a) $4.53+1.42i$, (b) $6.0+1.12i$, (c) $4.78+1.63i$, (d) $2.53+1.35i$, (e) $2.36+1.31i$, (f) $3.07+1.85i$, (g) $5.37+2.30i$, (h) $10.5+2.1i$, (i) $6.84+1.77i$, (j) $3.43+1.6i$, (k) $5.49+3.59i$, (l) $3.77+1.29i$, (m) $6.33+0.78i$, and (n) $4.57+2.32i$.

diffractive oscillations were used in the fit ($\theta < 41°$ for ^{40}Ca, $\theta < 50°$ for ^{208}Pb). The results are comparable to those of Ref. 2. In all cases, the Coulomb potential was taken to be that of a uniform charged sphere of radius $1.25(A_1^{1/3} + A_2^{1/3})$ fm.

It is not difficult for any reasonable model to fit the elastic-scattering data. A more stringent test of the model consists in examining its predictions for one- and two-particle transfer reactions. In Fig. 2, we display a sample of transfer results obtained using a no-recoil, finite-range distorted-wave Born-approximation code.[8] The optical potentials were taken directly from the elastic-scattering best fits, without further adjustments. The transfer form factors were obtained from previous work.[9-11]

The transfer cases we have chosen[9-11] are significant, since some of them differ markedly from the semiclassical bell-shaped angular distribution. For one-particle transfer, most of the fine details in the experimental angular distribution are well reproduced. This is an important test of our model.

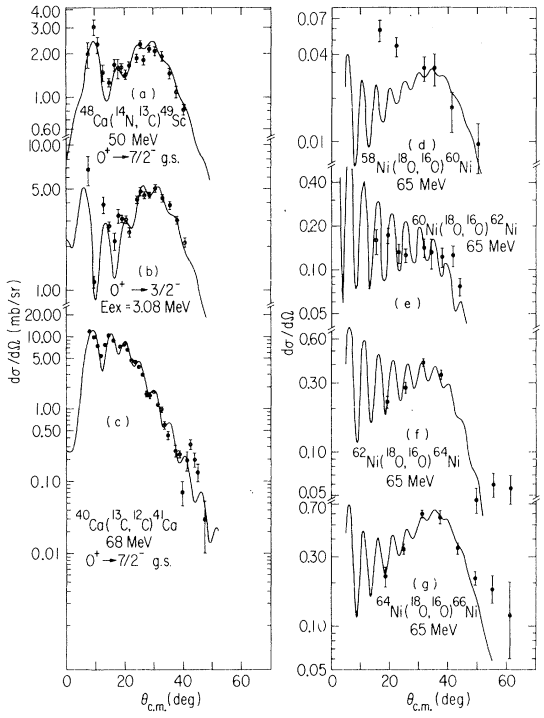

FIG. 2. Cross sections for one- and two-particle transfer reactions. No spectroscopic factors were included in the calculation, so each theoretical curve has been multiplied by a normalization factor N in order to make the comparison with data. The values of N are (a) 0.79 (b) 0.38, (c) 1.2, (d) 19., (e) 8.1, (f) 7.2, and (g) 5.1. Because of the lack of elastic data for ^{13}C + ^{49}Sc, ^{12}C + ^{41}Ca, and ^{16}O + ^{66}Ni, we have used the same strength $\bar{f}$ as for ^{13}C + ^{48}Ca, ^{13}C + ^{40}Ca, and ^{16}O + ^{64}Ni, respectively. In each case, $\bar{f}$ is taken directly from the elastic-scattering best fit.

In the (^{18}O, ^{16}O) transfer results shown in Fig. 2, there are strong oscillations predicted by the theory at small angles, whereas they have not yet been seen experimentally in this reaction. A proper treatment of recoil might alter this situation.[9] Another difficulty is that the theory does not predict enough forward peaking in the reaction[13] ^{58}Ni(^{18}O, ^{16}O)^{60}Ni. Also, a significantly larger normalization factor N is required for this case. This problem is handled in Ref. 9 by changing the geometry of the imaginary well, as dictated by the elastic-scattering data. One should also estimate the importance of second-order processes for these reactions.

We have also examined[14] the bound states of clusters such as ^{3}He, ^{3}H, and α, using the potential of Eq. (1). A classic example[15] is the ground-state band of ^{20}Ne, which is viewed as an α cluster on an ^{16}O core, based on α elastic-scattering and transfer results. For this case, we have taken $\mathrm{Im}\bar{f} = 0$ and adjusted $\mathrm{Re}\bar{f}$ to position the 4^+ level correctly. The other members of the ground band, except for the 8^+, are then in agreement with experiment, as shown in Fig. 3. Experimentally, the 8^+ deviates markedly from the $J(J+1)$ rule while the theoretical spectrum is very close to the pure rotational limit. Since the level spacing is sensitive to the geometry of the potential, these results provide evidence that the folded potential has approximately the correct shape. It may be possible to obtain equivalent results by appropriate variations of the radius and diffuseness of a Woods-Saxon potential; however, the parameters of Fortune *et al.*[16] yield an approximately degenerate spectrum. Also depicted in Fig. 3 are the $K^\pi = 0^-$ excited-band spectra. Here, an 8% renormalization of $\mathrm{Re}\bar{f}$ was necessary to yield the correct absolute energy. Finally, we give the α widths for these states in Fig. 3; they are in essential agreement with experiment. Studies are in progress[14] to explore the usefulness of the folded potential in the description of energies, widths, and electromagnetic transition rates of cluster states in other light nuclei.

In summary, we find that the simple potential of Eq. (1) gives a physically motivated and unified description of a variety of processes involving composite particles, including elastic scat-

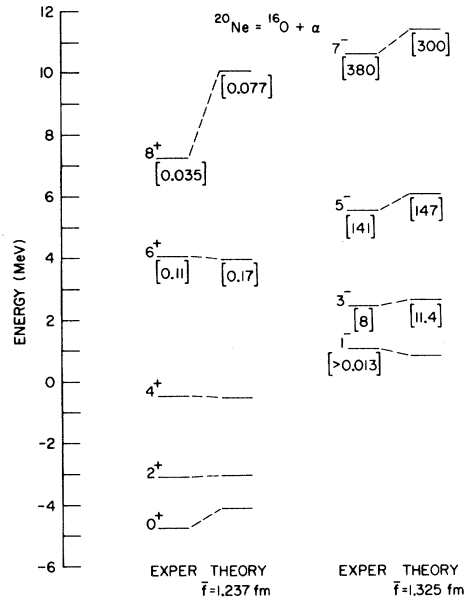

FIG. 3. Experimental and theoretical energy spectra and α widths for two "α cluster" bands of ^{20}Ne. The width for the 8^+ state was obtained after increasing $\bar{f}$ to obtain the experimental energy. The energy is measured with respect to the $\alpha + {}^{16}$O threshold in ^{20}Ne. The levels are labeled by the spin, parity, and α width in keV.

tering, transfer reactions, and cluster bound states.

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