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where $p(n)=6(3n+14)/(n+8)^2$.
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Drift Waves in Turbulent Plasmas

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The influence of a turbulent spectrum of high-frequency, short-wavelength electron plasma waves on the linear dispersion relation for low-frequency, long-wavelength drift waves has been analytically investigated.

The presence of high-frequency, short-wavelength microturbulence in a plasma can modify the dispersion relation for low-frequency waves in two different ways: (a) indirectly, by producing "anomalous" transport coefficients, and (b) directly, by mode-coupling effects. In this Letter we investigate an example of the latter interaction.

In a way this interaction is similar to parametric excitation or suppression of low-frequency waves by high-frequency fields.[1] Two important differences exist, however. First, in parametric processes one usually considers the effect of one coherent high-frequency wave on low-frequency dispersion relations; here, on the other hand, we consider the more general problem of the effect of a spectrum of a large number of waves with random phases. Second, in usual parametric calculations, one considers the excitation/suppression of wavelengths shorter than those of the applied field, whereas in the present calculation we investigate the modification of low-frequency waves with wavelengths longer than that of the background turbulence.

Vedenov *et al.*[2] have developed a general method for studying the interaction of low-frequency long waves with high-frequency, short-wavelength turbulence. Basically, the method is an "adiabatic" method in which one treats the high-frequency microturbulence as wave packets with a distribution in k space that satisfies the wave-kinetic equation. One then studies the motion of these wave packets in a medium varying "slowly" in space and time—the variation being due to the low-frequency, long-wavelength wave. The reverse influence of the high-frequency turbulence on low-frequency waves comes through the average electric-field pressure of ∇E^2 type which modifies the electron dynamics.[2] In this paper we use the technique of Vedenov *et al.*[2] to study the influence of given background electron-plasma-wave turbulence on the dispersion relation of low-frequency drift waves.

While this work was nearing completion we learned about the work of Krivorutsky *et al.*[3] who have also investigated a similar problem. Unfortunately, their paper does not seem to have received the amount of attention it deserves—perhaps because they use a rather cumbersome mathematical procedure. Our results, based on a simpler and physically more transparent model,[2] agree with those of Krivorutsky *et al.*[3] in the one case where they cover similar ground; in addition, our calculations contain many new features not discussed by them.

Consider a low-β inhomogeneous plasma (gradients along the x direction) immersed in a uniform magnetic field $\vec{B}_0=(0,0,B_0)$. Let us assume that there is stationary[4] high-frequency, short-wavelength electron-plasma-wave turbulence in this plasma and that its behavior is governed by the wave-kinetic equation,[2]

$$\frac{\partial N_k}{\partial t}+\vec{v}_g\cdot\nabla_r N_k-\frac{\partial\omega_k}{\partial\vec{r}}\cdot\nabla_k N_k=0, \qquad (1)$$

where $N_k=|E_k|^2/4\pi\omega_k$ is the plasmon distribution function, $\vec{v}_g=\partial\omega_k/\partial\vec{k}$ is the group velocity of plasma waves, and other symbols have their usual meanings. In Eq. (1), the space and time dependences are slow compared to space-time variation of the microturbulence. Thus, if this slow dependence is provided by a drift wave $(\Omega,\vec{q})$ in the medium, we must require $\Omega\ll\omega_k$, $q\ll k$ for the validity of Eq. (1). The presence of a small density perturbation $\tilde{n}_e$ due to a drift wave modifies the plasmon distribution N_k. This is evident from the plasma-wave dispersion relation (for an electron-cyclotron frequency $\omega_{ce}\gg\omega_{pe}$, the plasma frequency),

$$\omega_k^2=(k_z^2/k^2)(\omega_{pe}^2+k^2v_e^2), \qquad (2)$$

which shows that $\partial\omega_k/\partial\vec{r}\simeq\frac{1}{2}i\vec{q}\omega_k\tilde{n}_e/n_0$; substitution into a linearized version of Eq. (1) then yields

$$\tilde{N}_k=-\frac{\omega_{pe}}{2}\frac{\tilde{n}_e}{n_0}\frac{\vec{q}\cdot(\partial N_k^0/\partial\vec{k})}{\Omega-\vec{q}\cdot\vec{v}_g}. \qquad (3)$$

Equation (3) shows that fluctuations in electron density due to the drift wave directly induce corresponding fluctuations in the intensity of electron plasma waves.

The modified plasmon distribution reacts back on the drift wave through the averaged ponderomotive-force term $\langle(\vec{v}_k\cdot\nabla)\vec{v}_k\rangle$ in the low-frequency electron equation of motion.[2] The parallel component of the linearized electron equation of motion now takes the form

$$0\simeq-\frac{T_e}{mn_0}\frac{\partial\tilde{n}_e}{\partial z}+\frac{e}{m}\frac{\partial\tilde{\varphi}}{\partial z}-\frac{e^2}{2m^2}\frac{\partial}{\partial z}\left(\sum\frac{\tilde{N}_k}{\omega_k}\right), \qquad (4)$$

where in the last term we have used the definition

$$\tilde{N}_k=|\widetilde{E_k^2}|/4\pi\omega_k.$$

Electron inertia has been ignored because $\Omega\ll q_z v_e$ for drift waves. We have also neglected electron collisions (normal or "anomalous") to exclude the conventional drift-dissipative instability. Equations (3) and (4) show that

$$\tilde{n}_e/n_0=(e\tilde{\varphi}/T_e)[1-(A/n_0T_e)]^{-1}, \qquad (5)$$

where

$$A=\frac{W}{4}\omega_{pe}\int\frac{\vec{q}\cdot(\partial N_k/\partial\vec{k})}{\Omega-\vec{q}\cdot\vec{v}_g}d^3k\left[\int N_k^0\frac{k_z}{k}d^3k\right]^{-1}, \qquad (6)$$

and $W=c\int N_k^0\omega_k d^3k$ is the energy density of the plasma-wave turbulence, c being a normalization constant taking care of dimensions while going from $\sum_k$ to $\int dk$.

The plasma-wave turbulence leaves the ion dynamics unaltered. We can therefore follow the conventional treatment of drift waves for subsequent analysis. For $T_i=0$ and $\Omega\ll\omega_{ci}$, the ion-cyclotron frequency, one can show[5]

$$\tilde{n}_i/n_0\simeq(\omega_*/\Omega)(e\tilde{\varphi}/T_e), \qquad (7)$$

where $\omega_*=(-cq_yT_e/eB_0n_0)(dn_0/dx)$ is the drift frequency. The parallel motion of ions and the corrections due to finite ion Larmor radius are ignored. Using quasineutrality, Eqs. (5) and (7) yield the dispersion relation

$$\Omega=\omega_*[1-A/n_0T_e]. \qquad (8)$$

Note that A is a complicated function of Ω [Eq. (6)].

When ion temperature is finite ($T_e=T_i=T$) and strong temperature gradients exist ($d\ln T/d\ln n\gg1$), one has to retain parallel motion of ions and gets[6]

$$\frac{\tilde{n}_i}{n_0}\simeq\frac{e\tilde{\varphi}}{T}\frac{q_z^2v_s^2}{\Omega^2}\frac{\omega_T^*}{\Omega}, \qquad (9)$$

where $\omega_T^*=-(q_yc/eB_0)(dT/dx)$, and $v_s=(2T/M)^{1/2}$ is the speed of sound. Together with Eq. (15), this yields the modified dispersion relation for the drift-temperature-gradient instability, viz.,

$$\Omega^3\simeq(e\tilde{\varphi}/T)(q_z^2v_s^2\omega_T^*)(1-A/n_0T_e). \qquad (10)$$

We begin our discussion of Eqs. (8) and (10) by considering one-dimensional turbulence of electron plasma waves propagating along the magnetic field ($k_z\equiv k$) and having an equilibrium spectrum

$$N_k^0=\frac{N_0}{(2\pi)^{1/2}\Delta}\exp\left[-\frac{1}{2\Delta^2}(k-k_0)^2\right]. \qquad (11)$$

Equation (6) may now be expressed in terms of the plasma dispersion function[7]:

$$\frac{A}{n_0T_e}=\frac{W}{4n_0T_e}\frac{\omega_{pe}^2}{v_e^2\Delta^2}\left[1+\frac{k_0}{q_z}\frac{\Omega-\vec{q}\cdot\vec{v}_0}{\Delta v_0}Z\left(\frac{k_0}{q_z}\frac{\Omega-\vec{q}\cdot\vec{v}_0}{\Delta v_0}\right)\right], \tag{12}$$

where $\vec{v}_0=\vec{v}_g|_{k=k_0}$. Two limiting cases are of interest: (i) "Cold" limit. This corresponds to a sharply peaked spectrum of plasma waves in k space. Choosing $\Delta \ll k_0(\Omega-\vec{q}\cdot\vec{v}_0)/\vec{q}\cdot\vec{v}_0$ we get

$$A/n_0T_e \simeq \alpha/(\Omega-\vec{q}\cdot\vec{v}_0)^2 \tag{13a}$$

with

$$\alpha=-q_z^2v_e^2(W/4n_0T_e). \tag{13b}$$

(ii) "Warm" limit. This corresponds to a broad spectrum of plasma waves satisfying $\Delta \gg k_0(\Omega-\vec{q}\cdot\vec{v}_0)/q_zv_0$, and here

$$\frac{A}{n_0T_e}\simeq\left(\frac{W}{4n_0T_e}\right)\frac{\omega_{pe}^2}{\Delta^2v_e^2}\left[1+i\left(\frac{\pi}{2}\right)^{1/2}\frac{k_0}{q_z}\frac{\Omega-\vec{q}\cdot\vec{v}_0}{v_0\Delta}-\left(\frac{k_0}{q_z}\frac{\Omega-\vec{q}\cdot\vec{v}_0}{v_0\Delta}\right)^2\right]. \tag{14}$$

In the "cold" limit, Eq. (8) takes the form

$$(\Omega-\omega^*)(\Omega-\vec{q}\cdot\vec{v}_0)^2+\alpha\omega^*=0. \tag{15}$$

This cubic equation can be readily solved by Cardan's method. For large α, the solution is

$$\Omega\simeq(\alpha\omega_*)^{1/3}[-1,\exp(\pm\tfrac{1}{3}i\pi)],\quad |\alpha|\gg 4|\vec{q}\cdot\vec{v}_0-\omega_*|^3/27|\omega_*|; \tag{16}$$

an equation similar to (16) has also been derived by Krivorutsky *et al.*[3] For small α,

$$\Omega\simeq\omega_*,\ \vec{q}\cdot\vec{v}_0\pm i[\alpha\omega_*/(\omega_*-\vec{q}\cdot\vec{v}_0)]^{1/2},\quad |\alpha|\ll 4|\vec{q}\cdot\vec{v}_0-\omega_*|^3/27|\omega_*|. \tag{17}$$

Equations (16) and (17) show that even small magnitudes of plasma turbulence ($W/n_0T_e \sim \omega_*^2/q_z^2v_e^2 \ll 1$) can drastically alter the properties of drift oscillations in a plasma. In particular, strong instabilities (e.g., with a growth rate $\sim W^{1/3}$) can be driven by the turbulence. Our analysis gives a simple physical picture for these instabilities. Under conditions of instability, the phase relationship between $\tilde{n}_e$ and $\tilde{N}_k$ is such that plasma waves accumulate in the troughs of the drift wave. This drives more particles out of the trough through pondermotive forces and thus enhances the density perturbations.

The drift temperature-gradient instability is also strongly modified:

$$(\Omega^3-q_z^2v_s^2\omega_T^*)(\Omega-\vec{q}\cdot\vec{v}_0)^2+\alpha q_z^2v_s^2\omega_T^*=0. \tag{18}$$

For large and small α, this gives the roots

$$\Omega\simeq(\alpha q_z^2v_s^2\omega_T^*)^{1/5}[-1,\exp(\pm\tfrac{1}{5}i\pi),\exp(\pm\tfrac{3}{5}i\pi)] \tag{19}$$

and

$$\Omega=(q_z^2v_s^2\omega_T^*)^{1/3}\left(\frac{-1+i\sqrt{3}}{2}\right)+\frac{\alpha}{6(q_z^2v_s^2\omega_T^*)^{1/3}}\frac{1+4a+4a^2+i\sqrt{3}(1-a^2)}{1+2a+3a^2+2a^3+a^4},$$

where

$$a=(\vec{q}\cdot\vec{v}_0)(q_z^2v_s^2\omega_T^*)^{-1/3}. \tag{20}$$

Equation (19) shows a stronger dependence of the growth rate on the temperature gradient (compared to case of no background turbulence). Since α is negative [Eq. (13b)], Eq. (20), for $a^2<1$, shows a possible suppression of the drift-temperature instability.

In the "warm" limit, Eq. (8) takes the form

$$\Omega\left[1+i\left(\frac{\pi}{2}\right)^{1/2}\frac{W}{n_0T_e}\frac{\omega_*}{q_zv_e}\frac{\omega_{pe}^2}{\Delta^3v_e^3}\right]=\omega_*\left\{1-\frac{W}{4n_0T_e}\frac{\omega_{pe}^2}{\Delta^2v_e^2}\left[1-i\left(\frac{\pi}{2}\right)^{1/2}\frac{k_0}{\Delta}\right]\right\}. \tag{21}$$

This equation can be readily solved; for small W/n_0T_e, the solution is

$$\mathrm{Re}\Omega \simeq \omega_*\left[1-\frac{\omega_{pe}^2}{\Delta^2 v_e^2}\frac{W}{4n_0T_e}\right], \quad \mathrm{Im}\Omega = -\left(\frac{\pi}{2}\right)^{1/2}\frac{W}{4n_0T_e}\frac{\omega_{pe}^2}{\Delta^2 v_e^2}\frac{\omega_* k_0}{q_z}\left(\frac{\Omega_R-\vec{q}\cdot\vec{v}_0}{v_0\Delta}\right). \tag{22}$$

In this limit, resonance interaction between the modulations on the plasma waves (propagating at the group velocity) and the parallel phase velocity of drift waves assumes an important role. Mode-coupling effects thus lead to "emission" and "absorption" of drift waves by propagating plasma waves. Instability may result if $v_0 > \Omega_R/q_z$.

To investigate the effect of two-dimensional plasma turbulence (in y-z plane), we choose a spectrum $N_k^0 = N_0\delta(k_y-k_{0y})\delta(k_z-k_{0z})$ for the "cold" limit. Equations (8) and (10) again take the form of Eqs. (15) and (18) with a new definition of α:

$$\alpha = -\frac{W}{4n_0T_e}\omega_p\frac{k_0}{k_z}\left[\frac{q_z v_e^2}{\omega_{pe}}\frac{\vec{q}\cdot\vec{k}_0}{k_0}+\frac{\omega_{pe}}{k_0^3}(k_{0y}q_z-k_{0z}q_y)\left(q_y-3k_{0y}\frac{\vec{k}_0\cdot\vec{q}}{k_0^2}\right)\right]. \tag{23}$$

The basic new feature of two-dimensional turbulence is a modified group velocity. Equation (2) shows that in the general case, the group velocity of plasma waves is controlled by two effects: (a) dispersion due to thermal corrections and (b) dispersion due to angular dependence. Equation (23) shows that both contribute to the modified α. It is interesting to note that for $k_{0y}\to 0$, we do not recover the one-dimensional case discussed above; in fact α takes the form

$$\alpha \simeq (W/4n_0T_e)\omega_p^2(q_y^2/k_0^2). \tag{24}$$

It has not only a different sign from (13b) but also a much larger magnitude. Physically, the difference arises because a spectral choice $N_0\delta(k_y)\delta(k_z-k_{0z})$ is different from the corresponding one-dimensional case with no dependence on k_y. It seems to us that for application to realistic finite systems in the laboratory, Eq. (24) is better suited than the idealized one-dimensional limit (13b). The "warm" limit for the two-dimensional case is considerably more complicated but produces similar effects.[8]

In conclusion we have shown that moderate amounts of short-wavelength electron-plasma-wave turbulence can drastically alter the dispersion properties of drift waves. Compared to the work of Krivorutsky *et al.*[3] our calculation has the following new features: (a) discussion of the "warm" limit, Eq. (22); (b) discussion of drift temperature-gradient instability, Eqs. (18)–(20); (c) effect of two-dimensional anisotropic plasmon distributions, Eqs. (23) and (24); and finally (d) a simpler mathematical and physical description. In many ways (both mathematically and physically) the plasma-wave packets behave like a "beam" of quasiparticles giving both "hydrodynamic"- and "kinetic"-type "beam" instabilities. Taking this analogy further one can speculate that for "weak beams" this instability will nonlinearly saturate when the "beam" is trapped by the growing waves, i.e., when a good fraction of the plasma waves are trapped in the density fluctuations created by drift waves.

The particular choice of microturbulence and low-frequency waves in this Letter was governed by reasons of simplicity. Future work should look at more realistic problems like the effects of Buneman, ion-plasma, or lower hybrid turbulence on trapped-particle modes, drift temperature modes, etc. At the same time laboratory investigation of this interaction should be quite useful. In this connection, the threshold amplitudes of turbulence for instability, etc., can be readily obtained by equating the growth rates derived above with the typical damping rate of drift waves.

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Observation of Drift Instability Due to Particle Trapping in a Corrugated Geometry

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A drift instability due to electrons trapped in a series of shallow magnetic troughs has been observed and compared to theoretical estimates. The instability, identified as Kadomtsev's trapped-electron mode, is maximum at a density lower than estimated from the theory.

Plasma confinement in magnetic configurations periodic along the field lines divides particles into two classes: (i) circulating particles which are not reflected at the field maxima, and (ii) trapped particles localized in the field depressions. If these depressions are shallow enough, the rate at which the particles are scattered from one class into the other can be higher than the collision rate for large-angle collisions. As was first pointed out by Kadomtsev and Pogutse,[1] this is a dissipative mechanism capable of driving unstable the oscillations associated with the plasma radial pressure gradient. This instability is of special importance in the plasma of present tokomak experiments. For the temperature and density achieved, the particles trapped in the field depression should locally excite unstable oscillations which will not be affected by shear, unlike the long-wavelength drift waves studied so far. In this Letter we report an attempt to investigate the effect of the trapping-detrapping mechanism on drift oscillations.

In order to investigate this effect the 5-m-long homogeneous magnetic field in the ODE device, used previously to study collisionless drift waves,[2] was modified to produce a spatial dependence of the form $B = B_0[1 - \epsilon\cos(2\pi z/L)]$ ($\epsilon \sim 0.12$, $L = 1.20$ m, and $B_0 \leq 3750$ G) with three field periods as shown in Fig. 1. The plasma parameters remained as described in Ref. 2, with constant density along a flux tube ($n_0 \sim 5\times10^9 - 2\times10^{10}$ cm^{-3}, $T_e \sim 10$ eV, $T_i \sim 2$ eV, $\nabla n_0/n_0 \sim \nabla T/T \sim 1.3$ cm^{-1}, and $p_0 < 10^{-6}$ Torr). Under the conditions reported below, low-frequency instabilities arise, with maximum amplitude in the density gradient at $r \sim 0.8$ cm. These instabilities differ from the collisionless drift waves that have been already investigated in homogeneous fields at the same magnitude of B_0.

In both cases for which the wave vectors have been measured, the waves propagate azimuthally in the electron drift direction and are standing waves both radially and axially; the perturbed-density axial distribution is recorded by a probe moving parallel to the machine axis over 2.90 m. For $m = 1$ modes, the measurements show a new wave structure (Fig. 2) with maxima localized in the magnetic troughs. The data may suggest that the "trapped" drift mode is a superposition of independent modes with parallel wavelengths of order of the machine length and equal to the mirror length, around the same frequency. However, by correlating the moving probe to reference probes

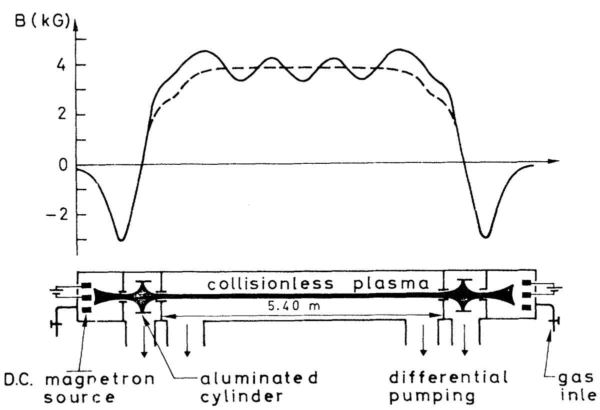

FIG. 1. Experimental setup.