tions. The equilibrium drift velocity of the carriers in the ring state was measured by injecting bare charge carriers into the drift region and then jumping the field up to a high value for a short time (~5 μsec) to produce a sharp pulse of such carriers. The field was then reduced to the value of interest, and the propagation properties of this pulse were determined. As can be seen in Fig. 3(a), a certain minimum field $\mathcal{E}_M$ is necessary to sustain a carrier in the ring-coupled state. For fields just below $\mathcal{E}_M$ the ring pulse is observed to decay quickly back into the bare carrier state, and at lower fields this process is so fast that one simply observes a well-defined pulse with the bare carrier velocity.

As the He^3 concentration is increased, $\mathcal{E}_M$ approaches $\mathcal{E}_c$, and as $\mathcal{E}_M$ crosses the transition region, the transition goes over into the continuous type. As far as can be determined, the qualitative behavior of the transition rates remains unaffected by the addition of He^3. These two observations indicate rather unambiguously that the continuous transition arises, not because of any specific feature of the ring creation process, but because the field $\mathcal{E}_M$ needed to maintain the ring-coupled state lies above the field $\sim\mathcal{E}_c$ necessary to produce it. In the range $\mathcal{E}_c < \mathcal{E} < \mathcal{E}_M$ individual carriers will then cycle between the bare and the ring-coupled states, with the fraction of time spent in the slower ring state increasing as $\mathcal{E}$ increases. Macroscopically, this will lead to a well-defined $\bar{v}$ which decreases continuously as $\mathcal{E}$ increases beyond $\mathcal{E}_c$. It is particularly interesting to note that when $\mathcal{E}$ reaches $\mathcal{E}_M$, the ring-coupled states will again be stable, so that the variation of $\bar{v}$ with $\mathcal{E}$ above $\mathcal{E}_M$ may be quite different from that in the range $\mathcal{E}_c < \mathcal{E} < \mathcal{E}_M$. Thus in addition to the decrease in $\bar{v}$ which begins at $\sim\mathcal{E}_c$, one would expect a second somewhat less drastic change in $\bar{v}(\mathcal{E})$ to occur at some higher field $\mathcal{E}_M$. This would seem to provide a very natural explanation for the existence of the second transition field $\mathcal{E}_{C2}$ observed by Kuchnir, Ketterson, and Roach.

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Plasma Modes Due to Impurity and Magnetically Trapped Ions*

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The effects of unstable modes, connected with the presence of impurity ions in a magnetically confined high-temperature plasma column and not dependent on wave-particle resonances that involve only a small portion of velocity space, are discussed for two significant regimes: a collisional one corresponding to relatively short mean free paths of the main ion population and a collisionless one in which a significant fraction of this population is magnetically trapped.

A number of recent high-temperature toroidal plasma experiments have brought to light the importance that a small amount of impurity ions can have, in terms of raising the average particle collision frequencies,[1] on the transport parameters of the involved plasmas. In this context recall that an early theoretical discussion was given[2] on the direct effects of impurities on the magnetic confinement of high-temperature plasmas. Special drift modes were shown to be produced by impurity ions and, in particular, to be suitable for transporting these ions from the walls of the container into the plasma column itself. If we label the impurity ion, the main ion, and the electron species by the subscripts I, i, and e, respectively, and the thermal velocity of the species j by $v_{\mathrm{th}j}$ we have, in realistic situations,

$$v_{\mathrm{th}I} \ll v_{\mathrm{th}i} \ll v_{\mathrm{th}e}.$$

Thus a new interval $(v_{\mathrm{th}I}, v_{\mathrm{th}i})$ for the phase velocities of modes, driven by a radial gradient of the longitudinal pressure (parallel to the magnetic field) of the main ion population, is introduced with the impurities. When more than one kind of impurity is present with sufficiently well-separated phase velocities, then analogous impurity modes with phase velocity contained between the thermal velocities of two kinds of impurities can be found. In this connection it has been noticed[3,4] that the effects of magnetically trapped particles, which are a considerable fraction of the total particle population in realistic toroidal experiments, can be simulated by those of two cold populations of electrons and ions in regard to the type of unstable modes[5] that can result from them. We also recall that the instabilities treated in Ref. 2 are relevant to longitudinal wavelengths shorter than the main-ion (i) collisional mean free paths and shorter than the periodicity length of the magnetic field. They are of kinetic type involving, in significant special cases, a Landau resonance with a relatively small portion of the ion distribution in velocity space.

Here we point out that in collisional regimes where the ion-ion (i) collision mean free path is considerably shorter than the periodicity length of the magnetic field, for a given confinement configuration, the corresponding modes become the fluid type,[6] resulting from well-known moment equations,[7] and are associated with the finite longitudinal thermal conductivity of the main-ion (i) population. In addition, when we consider regimes where the main-ion mean free path is longer than the magnetic field periodicity length, we show that new "macroscopic" modes, which have the same periodicity and do not depend on wave-particle resonanct processes, can be excited.

We refer in particular to an axisymmetric toroidal configuration in which ζ and θ indicate the toroidal and poloidal angles, respectively. The magnetic field is represented by $B \approx B_\zeta = B_0R_0/R$, where $R = R_0 + r\cos\theta$; r and R_0, indicate the minor and major radius for a given magnetic surface. We consider electrostatic perturbations with $\vec{E}_1 = -\nabla\tilde{\Phi}$ and $\tilde{\Phi} = \tilde{\varphi}_m(\theta, r)\exp(-i\omega t - im^0\theta + in^0\zeta)$, where m^0 and n^0 are integers. We limit attention to modes that are radially localized around the rational surface $r = r_0$ such that $q(r_0) = m^0/n^0$ for $q(r) \approx rB_\zeta/RB_\theta$, B_θ being the poloidal magnetic field component. We assume for simplicity that magnetic shear is negligible, avoid considering[8] the radial modulation that this induces on $\tilde{\varphi}_m(\theta, r)$, and refer to $\tilde{\varphi}_m(\theta, r_0) \equiv \tilde{\varphi}_m(\theta)$ from here on.

We assume that the species i is singly ionized, while Z_I indicates the impurity charge number. So, $Z_In_I + n_i = n_e$, where n_j is the particle density of species j. Here we consider only regimes in which the average effective collision frequency of trapped ions (i) is smaller than their average bounce frequency $\hat{\omega}_{bi}$. For this we recall that $\hat{\omega}_{bj} = (v_{\mathrm{th}j}/q_0R_0)(2r_0/R_0)^{1/2}$ and the average transit frequency of circulating particles is $\hat{\omega}_{Cj} = \nu_{\mathrm{th}j}/q_0R_0$, where $v_{\mathrm{th}j} = (2T_j/m_j)^{1/2}$, T_j and m_j indicating the temperature and particle mass for the species j. We assume at first that all temperature gradients are negligible with respect to the density gradients [i.e., $d\ln(T_j)/dr \ll d\ln(n_j)/dr$] and consider the large aspect-ratio limit, $R_0 \gg r_0$, in any case.

The interesting frequency range is

$$\hat{\omega}_{CI} < \omega \lesssim \hat{\omega}_{bi} \ll \hat{\omega}_{be}. \tag{1}$$

For toroidal modes of the type we have indicated earlier and with $\tilde{\varphi}_m(\theta)$ given by an *odd* function around $\theta = 0$, we recall[4] that the perturbed electron density is simply

$$\tilde{n}_{em} = (e/T_e)n_e\tilde{\varphi}_m. \tag{2}$$

For the impurity ions we can adopt the fluid approximation and have, from the particle conservation equation,

$$-i\omega\tilde{n}_{Im} + i\frac{m^0c}{r_0B}\tilde{\varphi}_m\frac{dn_I}{dr} + n_IB\frac{\partial}{\partial l}\left(\frac{1}{B}\tilde{u}_{Im\parallel}\right) = 0, \tag{3}$$

and, from momentum conservation along the magnetic field,

$$-i\omega m_In_I\tilde{u}_{Im\parallel} = -eZ_In_I\,\partial\tilde{\varphi}_m/\partial l \tag{4}$$

so that

$$\tilde{n}_{Im} = -n_I\frac{e}{T_i}\left(\frac{\omega_{*I}}{\omega} + Z_I\frac{T_i}{m_I}\frac{B}{\omega^2}\frac{\partial}{\partial l}\frac{1}{B}\frac{\partial}{\partial l}\right)\tilde{\varphi}_m, \tag{5}$$

where

$$\omega_{*I} = -cm^0T_i[d\ln(n_I)/dr]/er_0B$$

and $l \equiv q\int^{\theta} R\,d\theta$. Now we refer to the quadratic form in $\tilde{\varphi}_m$ of the quasineutrality condition,

$$\frac{e}{T_e}n_e\oint\frac{dl}{B}|\tilde{\varphi}_m|^2 = -Z_I n_I\frac{e}{T_i}\left[\frac{\omega_{*I}}{\omega}\oint\frac{dl}{B}|\tilde{\varphi}_m|^2 - \frac{Z_I T_i}{m_I\omega^2}\oint\frac{dl}{B}\left|\frac{\partial\tilde{\varphi}_m}{\partial l}\right|^2\right] + \oint\frac{dl}{B}\tilde{n}_{im}\tilde{\varphi}_m{}^*. \tag{6}$$

In order to give a simple presentation of the expression for the last term we recall that if we were dealing with a one-dimensional equilibrium configuration, or with modes having longitudinal wavelengths much shorter than the magnetic connection length $2\pi qR$, we would consider *traveling* modes with $\tilde{\Phi} = \tilde{\varphi}_k \exp(-i\omega t - im^0\theta + ik_{\parallel}l)$ and,[9] for $\omega < k_{\parallel}v_{\mathrm{th}i}$,

$$\oint\frac{dl}{B}\tilde{n}_{ik}\tilde{\varphi}_k{}^* = -\frac{e}{T_i}n_i\left[1 - i\sqrt{\pi}\left(1-\frac{\omega_{*i}}{\omega}\right)\frac{\omega}{|k_{\parallel}|v_{\mathrm{th}i}}\right]\oint\frac{dl}{B}|\tilde{\varphi}_k|^2. \tag{7}$$

In the case of a two-dimensional equilibrium configuration where we consider *standing* modes of the form indicated earlier and with $\tilde{\varphi}_m(\theta)$ odd, we obtain an expression for $\oint dl\,\tilde{n}_{im}\tilde{\varphi}_m{}^*$ similar to Eq. (7), in the limit $\omega \ll \hat{\omega}_{bi}$, but with the first-order term in $\omega/\hat{\omega}_{bi}$ missing.[4] Specifically,

$$\oint\frac{dl}{B}\tilde{n}_{im}\tilde{\varphi}_m{}^* = -\frac{e}{T_i}n_i\left[1 + \left(\frac{r_0}{R_0}\right)^{1/2}\left(1-\frac{\omega_{*i}}{\omega}\right)\left(J_1\frac{\omega^2}{\hat{\omega}_{bi}{}^2} + iJ_2\frac{\omega^3}{\hat{\omega}_{bi}{}^3}\right)\right]\oint\frac{dl}{B}|\tilde{\varphi}_m|^2, \tag{8}$$

where J_1 and J_2 are finite ratios of positive integrals, quadratic in $\tilde{\varphi}_m$, $\omega_{*i} = -m^0 cT_i[d\ln(n_i)/dr]/r_0 eB$, and the factor $(r_0/R_0)^{1/2}$ indicates that the contribution of the terms in J_1 and J_2 is proportional to the fraction of magnetically trapped ions.

If we assume that ω is real, we notice that the imaginary term in Eq. (8) results from a wave-particle resonance $\omega = \omega_b(\epsilon, \mu)$, where ω_b is the bounce frequency[4,8] of a particle with kinetic energy ϵ and magnetic moment μ. This term is smaller by a factor of the order of $\omega/\hat{\omega}_{bi}$ than the term in J_1, which does not correspond to the interaction of the considered mode with a relatively small portion of the ion distribution in velocity space. On the other hand, we see that in Eq. (7) the imaginary term corresponding to a Landau resonance of the type $\omega/k_{\parallel} = v_{\parallel}$, $v_{\parallel}$ being the particle longitudinal velocity, is the lowest-order correction to $\tilde{n}_i = -e\tilde{\varphi}_m n_i/T_i$. This corresponds to the fact that in the case of Eq. (7) the resonating particles may have any value of $v_{\perp}$, the transverse velocity, and the relative amount of phase space involved is larger than for the corresponding resonance of the type $\omega = \omega_b(\epsilon, \mu)$.

We consider the asymptotic limit where $\omega\omega_{*i} \sim \hat{\omega}_{bi}{}^2(R_0/r_0)^{1/2}$ and $T_i \sim T_e$ and obtain, from Eq. (6),

$$\left(\frac{T_i}{T_e}+\frac{n_i}{n_e}\right) - Z_I{}^2\frac{n_I}{n_e}\frac{T_i}{m_I\omega^2 q_0{}^2R_0{}^2}J_3 + Z_I\frac{n_I}{n_e}\frac{\omega_{*I}}{\omega} - \frac{n_i}{n_e}\left(\frac{r_0}{R_0}\right)^{1/2}\omega_{*i}\left(\frac{\omega^2}{\hat{\omega}_{bi}{}^2}J_1 + i\frac{\omega^2}{\hat{\omega}_{bi}{}^3}J_2\right) = 0, \tag{9}$$

where J_3 is also a finite ratio of real and positive integrals that are quadratic in $\tilde{\varphi}_m$.

Now we can refer to Eq. (9) in order to find conditions under which unstable modes can be found and derive significant asymptotic limits[4,8] where variational forms leading to precise evaluations of ω can be extracted from it. On this basis we can identify two kinds of modes which can be unstable with $\mathrm{Im}(\omega) \gtrsim \mathrm{Re}(\omega)$ and therefore do not require consideration of the term in J_2. One kind of mode corresponds to the case where the terms in J_1 and J_3 are the largest within Eq. (9) and does not depend on the relative sign of the impurity-ion density gradient versus that of the main-ion density. The second kind corresponds to the terms in J_1 and ω_{*I} being the largest within Eq. (9). In this latter case a necessary condition for instability is $\omega_{*I}/\omega_{*i} < 0$, implying that impurity ions and the main ions have opposite density gradients as can be realized in the outer region of the plasma column closer to the container walls. Instabilities involving mode-particle resonances are also found from Eq. (9) and in this case the growth rate $\gamma = \mathrm{Im}(\omega)$ is proportional to the term in J_2.

If we include, in the equilibrium distribution function, terms representing the temperature gradients of both the main (i) and impurity (I) population, we obtain[8] instead of Eq. (9)

$$\left(\frac{T_i}{T_e}+\frac{n_i}{n_e}\right) - Z_I{}^2\frac{n_I}{n_e}\frac{T_i}{m_I\omega^2R_0{}^2q_0{}^2}J_3 + Z_I\frac{n_I}{n_e}\frac{\omega_{*I}}{\omega} - \left(\frac{n_i}{n_e}\right)\left(\frac{r_0}{R_0}\right)^{1/2}\left[(\omega_{*i}-\omega_{Ti})\frac{\omega}{\hat{\omega}_{bi}{}^2}J_1 + i(\omega_{*i}-\tfrac{3}{2}\omega_{Ti})\frac{\omega^2}{\hat{\omega}_{bi}{}^3}J_2\right] = 0, \tag{10}$$

where $\omega_{Ti} = [d\ln(T_i)/d\ln(n_i)]\omega_{*i}$. Now we can see that in the limit where the two terms before the last prevail a necessary condition for instability is

$$\omega_{*I}/(\omega_{*i}-\omega_{Ti})<0,$$

and indicates that in the presence of the ion temperature gradient the relevant impurity mode can be excited even if the sign of the impurity density gradient is the same as that of the main ion population. In the same limit we have

$$\omega \approx \hat{\omega}_{bi}\left(\frac{Z_I n_I}{J_1 n_i}\right)^{1/2}\left(\frac{R_0}{r_0}\right)^{1/4}\left(\frac{\omega_{*I}}{\omega_{*i}-\omega_{Ti}}\right)^{1/2}. \qquad (11)$$

If we consider $\omega_{*I}\sim\omega_{*i}\sim\omega_{Ti}$, $J_1\sim J_3\sim 1$, and $T_i \sim T_e$, the conditions $\omega<\hat{\omega}_{bi}$ and $Z_I n_I \omega_{*I}/n_e\omega>1$, that are necessary for the validity of Eq. (11), require

$$\frac{\omega_{bi}^2}{\omega_{*i}^2}\frac{R_0}{r_0}<Z_I\frac{n_I}{n_i}\left(\frac{R_0}{r_0}\right)^{1/2}<1,$$

while

$$\frac{\omega_{*i}}{\hat{\omega}_{bi}}>\frac{m_i}{m_I}\left(\frac{R_0}{r_0}\right)^{3/4}\left(\frac{n_i Z_I}{n_I}\right)^{1/2}$$

corresponds to $Z_I T_i < m_I R_0^2 q_0^2 \omega\omega_{*I}$.

We also notice that $\tilde{n}_{em}$ and $\tilde{\varphi}_m$ are in phase, as indicated by Eq. (2). Therefore no net transport of electrons[10] across the magnetic field is found to be produced by these modes when the evolution of the average distribution function is estimated by the well-known quasilinear theory. On the basis of the same theory, we also expect that a rearrangement of the impurity- and main-ion spatial distributions as well as transport of ion thermal energy across the magnetic field result.

In conclusion, we see that in regimes where a finite portion of the ion population becomes magnetically trapped, new "macroscopically" unstable modes can be found which have the necessary characteristics to produce a strong convection of impurity ions and, in the limit where $\omega\sim\hat{\omega}_{bi}$, amplification and nonlinear scattering of the trapped ion orbits of the type discussed in Ref. 8 and by Coppi and Taroni.[11] The latter effect may be favorable, such as in hindering the onset of lower-frequency interchange trapped particle modes.[5] As for the fluidlike modes which are found in the collisional regime and are associated with finite ion thermal conductivity, they may play a role within the evolution of the so-called "disruptive" instability (of the toroidal plasma column) which has been observed in all tokamak experiments. For instance, in conditions of relatively high density and low temperature in the outer region of the plasma column, a violent convection of impurities toward the center can be produced when a macroscopic perturbation enhances the interaction of the plasma column with the container walls and the influx of impurities. As a consequence the average Z of the plasma and its electrical resistivity may be increased, in the central region, to the point where a positive radial gradient of the longitudinal current density is generated. The development of electromagnetic instabilities, such as of tearing modes, leading to a disruption of the magnetic confinement configuration, could then be favored.

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