cause the profiles to smear out in the line centers, then such effects should be strong functions of the impressed electrical field, but should not depend on the reduced mass. Thus the H_β and D_β line profiles emitted from the seeded argon plasma should not differ since the properties of the plasma, including the impressed field strength, are essentially that of a pure argon plasma. Nevertheless, significant differences have been observed which are only readily explainable in terms of the changed reduced mass of the radiator-perturber system. On the other hand, the quite different D-D and H-Ar arc plasmas with approximately the same reduced mass are in very close agreement.

Furthermore, inspection of other recently published experimental profiles reveals, for similar plasma conditions, relative dips of about 15% for pure H plasmas,[4,5] and 18% for H-Ar mixtures,[6,7] in excellent agreement with our results.

In conclusion we may state that all our results can be very plausibly explained in terms of ion dynamic effects: (1) We observe a pronounced variation in the relative peak-dip difference of H_β, which is correlated to the reduced mass of the radiator-ion-perturber system. (2) The relative dip at a fixed electron density (and, approximately, fixed temperature) appears to scale linearly with $1/\sqrt{\mu}$. This suggests that the magnitude of the ion dynamic effect depends on the relative velocity of the perturber with respect to the radiator. (3) The slope of the dependence of the relative dip on electron density approaches that predicted by the various current theories as the reduced mass is increased. (4) Extrapolation of the measured values to infinite reduced mass indicates that the major portion of the discrepancy between experiment and theory in the line center of H_β may be attributed to ion motion.

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Optical Properties of a Plasma Sphere

R. Ruppin
Soreq Nuclear Research Centre, Yavne, Israel
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The classical theory for the scattering and absorption of electromagnetic radiation by a homogeneous isotropic sphere is extended so as to apply to a sphere in which dispersive plasma oscillations exist. Computations performed for small metallic spheres predict the appearance of an absorption structure in the frequency region just above the plasma frequency.

The optical properties of a plasma sphere are of great interest in view of the rapid development of the technique of producing plasmas by laser irradiation of small pellets. Most of the theoretical effort is being directed towards the understanding of nonlinear properties and other effects which contribute to the heating of the plasma. However, in experiments of light scattering for diagnostic purposes, the linear optical properties play a fundamental role. These properties are calculated in the present work.

The classical theory of the scattering and absorption of electromagnetic radiation by a sphere, usually referred to as the Mie theory,[1] has found numerous applications in various branches of physics and chemistry.[2,3] The Mie theory yields the optical properties of spheres of arbitrary size in terms of the transverse dielectric constant of the sphere material. Although the original Mie theory referred to homogeneous spheres only, it can be extended to inhomogeneous cases[3,4] as long as the spherical symmetry is maintained. A case in which deviations from the Mie theory can be expected is that of a sphere which can support dispersive longitudinal polarization waves. A warm plasma sphere and a metal sphere are two examples of such a case. An extension of the Mie theory is presented here which applies to these situations, and which reduces to the classical theory when the sphere material cannot support dispersive longitudinal modes.

The incident field, the scattered field, and the transverse electromagnetic field are expanded inside the sphere in terms of the transverse vector spherical wave functions[5,6]

$$\vec{M}_{mn}(\vec{r}) = \mathrm{curl}[\vec{r} y_{mn}(\theta,\varphi) z_n(kr)],$$

$$\vec{N}_{mn}(\vec{r}) = k^{-1}\,\mathrm{curl}[\vec{M}_{mn}(\vec{r})].$$

The propagation constant k is equal to $k_0=\sqrt{\epsilon_M}\,\omega/c$ outside the sphere, where ϵ_M is the dielectric constant of the medium surrounding the sphere, and ω is the frequency of the incident plane wave. Inside the sphere it is given by the root k_T of the equation

$$k_T = [\epsilon_T(k_T,\omega)]^{1/2}\omega/c,$$

where ϵ_T is the transverse dielectric constant of the sphere material. The radial function $z_n(kr)$ is given by $j_n(k_T r)$ inside the sphere, $j_n(k_0 r)$ for the incident wave, and $h_n(k_0 r)$ for the scattered wave, where j_n and h_n are spherical Bessel and Hankel functions, respectively.

The longitudinal modes, which are excited inside the sphere, are expanded in terms of the longitudinal vector spherical functions[5,6]

$$\vec{L}_{mn}(\vec{r}) = k_l^{-1}\,\mathrm{grad}[y_{mn}(\theta,\varphi) j_n(k_l r)],$$

where k_l is obtained from the dispersion law of the longitudinal modes

$$\epsilon_L(k_l,\omega)=0,$$

where ϵ_L is the longitudinal dielectric constant of the sphere material.

The expansion coefficients of all the fields follow from the boundary conditions at the surface of the sphere. Because one allows for the excitation of longitudinal modes inside the sphere, it is necessary to augment the usual continuity conditions on the tangential components of $\vec{E}$ and $\vec{H}$ by an additional boundary condition. The requirement that the normal displacement current is continuous is adequate for this purpose, as has been shown by Melnyk and Harrison.[7] Using the boundary conditions one obtains the following expressions for the expansion coefficients of the scattered field, i.e., the generalized Mie coefficients:

$$a_n = -\frac{j_n(k_TR)[k_0R\,j_n(k_0R)]' - j_n(k_0R)[k_TR\,j_n(k_TR)]'}{j_n(k_TR)[k_0R\,h_n(k_0R)]' - h_n(k_0R)[k_TR\,j_n(k_TR)]'},$$

$$b_n = -\frac{c_n j_n(k_0R) + j_n'(k_lR)\{\epsilon_M[k_TR\,j_n(k_TR)]' j_n(k_0R) - \epsilon_T j_n(k_TR)[k_0R\,j_n(k_0R)]'\}}{c_n h_n(k_0R) + j_n'(k_lR)\{\epsilon_M[k_TR\,j_n(k_TR)]' h_n(k_0R) - \epsilon_T j_n(k_TR)[k_0R\,h_n(k_0R)]'\}}.$$

Here

$$c_n = n(n+1)[j_n(k_lR)/k_lR]\,j_n(k_TR)(\epsilon_T-\epsilon_M),$$

R is the radius of the sphere, and the prime denotes differentiation with respect to the radial functions. The optical properties follow from the coefficients a_n and b_n in the usual way.[1-3,5] Thus, the extinction cross section of the sphere (which is the sum of the scattering and the absorption cross sections) is given by

$$\sigma = -2(k_0R)^{-2}\sum_{n=1}^{\infty}(2n+1)\,\mathrm{Re}(a_n+b_n)$$

in units of the geometric cross section πR^2. The coefficients a_n are exactly the same as in the classical Mie theory. The coefficients b_n reduce to those of the classical theory when c_n is equal to zero. This happens in the case of a sphere which cannot support longitudinal polarization waves, in which case the imaginary part of k_l becomes infinitely large.

I have computed the extinction cross section of

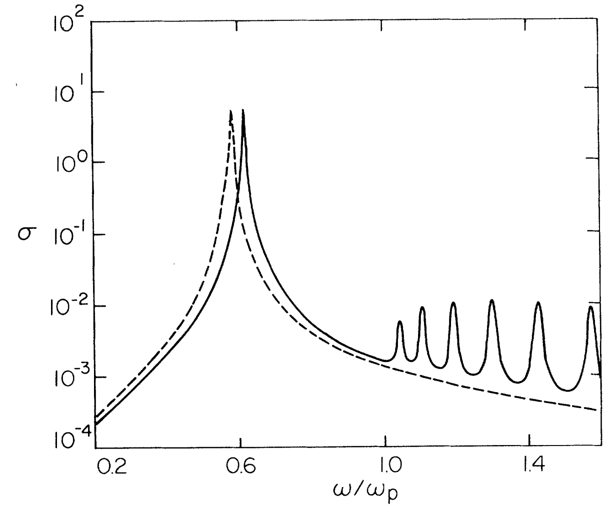

FIG. 1. Calculated extinction cross section, in units of the geometric cross section, of a potassium sphere of radius 20 Å, with a damping factor $\gamma = 0.01$. Solid curve, exact calculation; broken curve, classical Mie theory.

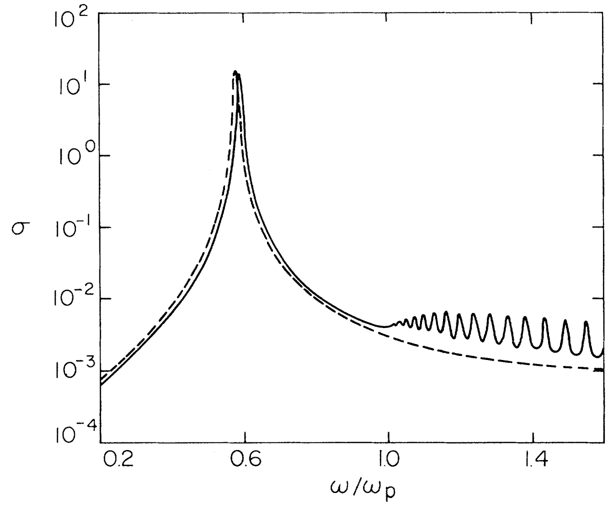

FIG. 2. Same as for Fig. 1, but with a radius of 50 Å.

small metallic spheres, using for the transverse and the longitudinal dielectric constants the well-known expressions[8]

$$\epsilon_T = 1 - \frac{\omega_p^2}{\omega(\omega + i/\tau)} \frac{3}{2a^2}\left[\frac{1+a^2}{a}\tan^{-1}(a) - 1\right],$$

$$\epsilon_L = 1 - \frac{\omega_p^2}{\omega(\omega + i/\tau)} \frac{3}{a^2}\left[1 - \frac{\tan^{-1}(a)}{a}\right] \left[1 + \frac{i}{\omega\tau}\left(1 - \frac{\tan^{-1}(a)}{a}\right)\right]^{-1},$$

where

$$a^2 = -k^2 v_F^2(\omega + i/\tau)^{-2},$$

ω_p is the plasma frequency, v_F is the Fermi velocity, and τ is the relaxation time.

The computed extinction cross section of a potassium sphere of radius 20 Å is shown in Fig. 1. The values $v_F = 8.52\times10^7$ cm/sec, $\omega_p = 6.5\times10^{15}$ sec^{-1}, and $\gamma = (\omega_p\tau)^{-1} = 0.01$ have been employed in this calculation. In the frequency region above ω_p, in which plasma modes exist, a series of absorption maxima appears. These peaks, which are not predicted by the classical theory, are due to the excitation of bulk oscillations and appear at frequencies for which the condition

$$j_n'(k_l R) = 0$$

is satisfied. In this example only the first term, $n = 1$, of the generalized Mie series contributes to the optical properties since the sphere size is much smaller than the wavelength of the incident beam. Below ω_p there appears only one absorption peak, as in the classical theory. This peak is due to the resonant excitation of a transverse electromagnetic mode and appears classically at the frequency $\omega_p/\sqrt{3}$. The exact calculation predicts a slight shift of this peak towards the high-frequency side. In the frequency range below ω_p no oscillatory longitudinal polarization fields can be excited. Only evanescent longitudinal modes exist, i.e., modes for which k_l is essentially imaginary. These modes modify the internal electric field, thus causing a shift of the main absorption maximum from its classical position. This shift decreases as the sphere radius is increased, as examplified in Fig. 2, in which the extinction cross section of a sphere of radius 50 Å is shown. As to the absorption peaks above ω_p, their number increases but their intensity decreases as the radius is increased. It is found that for spheres with $R \gtrsim 200$ Å the exact extinction curve hardly deviates from the classical one.

The dependence of the absorption structure on the damping factor γ is such that the peaks become smaller and broader as γ increases, as shown in Fig. 3. It should be noted that the value of γ in very small particles is indeed expected to increase beyond its value in bulk samples, since the mean free path of the electrons is reduced as a result of collisions with the surfaces.

Metallic particles less than 100 Å in size can be prepared[9,10] but as there always exists some

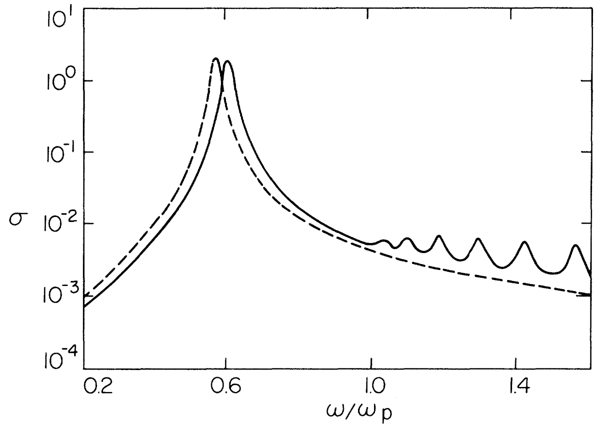

FIG. 3. Same as for Fig. 1, but with $\gamma = 0.03$.

size distribution, it will hardly be possible to observe the individual peaks above ω_p, since their position varies with the sphere radius. It can, however, be expected that a broad absorption band, roughly corresponding to the envelope of these peaks, will appear. Such an effect seems to have been observed by Duthler, Johnson, and Broida,[10] who have measured the scattering of light from sodium spheres of about 30–50 Å in diameter. They detected a broad maximum just above the plasma frequency, which could result from an absorption structure which is smeared out because of the size distribution of the spheres. They have also observed the main maximum, which in the classical theory appears at the frequency $\omega_p/\sqrt{3}$, but it was shifted towards the high-frequency side. This is in qualitative agreement with our results, which indeed predict such a shift.

With classical plasmas, unlike the case of metallic plasmas, the absorption structure above ω_p will be observable even for spheres of macroscopic size.[11] It might therefore provide a diagnostic tool for plasmas produced by laser irradiation of small pellets, though the extension of the theory to the inhomogeneous case has first to be worked out.

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Soundlike Collective Excitations in Superfluid ^{3}He

P. Wölfle
Max-Planck-Institut für Physik und Astrophysik, Munich, Germany
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I discuss soundlike collective excitations in anisotropic superfluid Fermi liquids in both the collisionless and the hydrodynamic regimes. Expressions for the velocities of zero, first, second, and fourth sound are derived in terms of the Landau parameters of the normal state. The second- and fourth-sound velocities are found to vary appreciably with orientation.

The recently discovered[1-3] low-temperature phases of liquid ^{3}He have raised interest in the theory of (possibly anisotropic) superfluid Fermi liquids. In this Letter I present a theory of the collective behavior of a superfluid Fermi liquid, which is on the level of description of Landau's theory of normal Fermi liquids[4,5] As essential results I obtain expressions for the velocities of zero, first, second, and fourth sound in terms of the interaction parameters F_0 and F_1 for the normal Fermi liquid (which are known for ^{3}He) and the equilibrium gap parameter.

In **principle**, the nonequilibrium physics of superfluid Fermi systems can be described by a matrix