grows as the beam slows down. This effect characterizes any process that transfers momentum between particle streams of like charge but different effective mass. In an actual experiment in linear geometry, an equal return current is necessary to avoid charging the end plate, but typically this current can flow partially in external conductors.

During the early part of the nonlinear stage, when there are many filaments, it is qualitatively reasonable (and in moderate agreement with simulation) to treat the development statistically as continued growth of modes with long wavelength λ, $L \gtrsim \lambda \gg l$. Thus, the time scale for this stage is a few linear growth periods for modes with $\lambda \sim L$. When only a few filaments remain, such a treatment is obviously inappropriate. If each filament were fully current neutralized, the time required for coalescing of these last few filaments would become exponentially large for filament separation $\gg c/\omega_p$. However, we expect the final recombination, in an actual experiment, to be dominated by the long-range unshielded attractive force due to overall current non-neutrality. The characteristic time, i.e., the time for coalescing under this force of a pair of filaments separated by distance L, is $\tau \approx (\pi\gamma L^2/r_0^2\beta^2)^{1/2} \times (1-\alpha)^{-1}\omega_p^{-1}$, where $\alpha \equiv n_{0p}u_p/n_{0b}u_b$ is the fractional current shielding. Conservation of axial canonical momentum $\vec{p}_z - e\vec{A}/c$ indicates that the change in beam momentum is $\langle \delta p_{bz} \rangle \approx (\omega_b L/4c)^2 \times N^{-1}p_0$, from which u_b and α can be estimated via Eq. (4).

The coalescing of the last few filaments is much slower than this estimate in the present simulations, for two reasons. In the presence of a net current, Maxwell's equations are inconsistent with periodic boundary conditions. Thus, the code must introduce a fictitious, spatially uniform return current chosen to preserve current neutrality, negating the effect of Eq. (4). Furthermore, a periodic simulation system, unlike an actual beam of finite radius, can reach a symmetric, latticelike state. This effect dominates cases *A* and *B*. Further coalescing then depends on symmetry-breaking perturbations, and is much delayed. Therefore, the present simulations give only an upper bound on beam recombination time in a real experiment. Further simulations are in progress, involving beams with finite cross sections, and metallic rather than periodic boundary conditions.

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Phonon Softening in Lanthanum under Pressure

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Superconducting tunneling experiments on double-hexagonal close-packed La show that the ratio $2\Delta_0/kT_c$ increases with pressure. The low-frequency part of the phonon density of states shifts to lower energies. Phonon softening is thus, at least partially, responsible for the strong increase of the superconducting T_c with pressure.

For several years the electronic structure of lanthanum metal has been a matter of unsettled controversy. Up to the present time La is often conventionally regarded as a pure *sd*-band transition metal of the third column of the periodic table, not significantly different from scandium or yttrium.[1] On the other hand, the idea had been advanced a decade ago that a 4*f* band may favorably influence the superconductivity of La.[2]

Experimentally, it has been shown that La is

a medium-coupling superconductor with deviations from the BCS density of states strongly resembling phonon-induced structure.[3-5] Additional structure directly correlated to the f band has never been observed. There is, hence, growing evidence that the electron-phonon interaction is in fact the pairing interaction in La, rather than a special "4f-electron mechanism." Is La therefore a typical sd-band superconductor? This viewpoint has indeed been expressed recently.[6]

There are, however, strong arguments against lanthanum's being an sd superconductor. The T_c of La is fairly high at normal pressure ($T_{c,\mathrm{dhcp}} = 4.9$ K, $T_{c,\mathrm{fcc}} = 6.0$ K) and rises to an extraordinarily high T_c of 13 K at a pressure of 200 kbar.[7] It was noted elsewhere that the very high T_c of La (at any pressure) is an anomalous property of La which should be taken seriously, since it does not fit, by at least an order of magnitude, into the systematic variation of T_c among the sd transition elements.[8] The present study of electron tunneling under pressure discloses yet another anomaly of La which, in our opinion, points to the existence of a 4f band overlapping the Fermi surface.

The tunnel diodes were of the type Al-oxide-La. Thin ($\simeq 200$ Å), granular Al films turned out to be a favorable condition for the diodes to survive in a pressure experiment. The Al films have been oxidized in a glow discharge. To complete the junctions the La films ($\simeq 1000$ Å) were deposited in a vacuum of better than 10^{-6} Torr onto the cooled glass substrate ($\simeq 150$ K), which was surrounded by a shield held at 77 K. The diodes were subsequently coated with a protective film of SiO in order to prevent their oxidation during the time they had to be handled in air. We believe that our La films consist of the homogeneous double-hexagonal close-packed (dhcp) phase. The transition temperatures measured resistively and by the vanishing of the energy gap coincide within 0.02 K. $T_c = 4.95 \pm 0.02$ K, $\Delta_0 = 0.80 \pm 0.03$ meV, and $2\Delta_0/kT_c = 3.75$ agree excellently with values published recently.[5] We succeeded in obtaining junctions which were stable at room temperature and showed no zero-bias anomalies. We have not observed a dependence of T_c on thickness as previously reported.[9]

The pressure was generated in a Teflon high-pressure cell containing a mixture of isoamyl alcohol and n-pentane as the pressure-transmitting medium. A clamp technique was employed. The pressure was determined from the T_c of a Pb sample. The tunneling barrier withstood pressures up to 17.5 kbar without destruction. The derivatives of the I-V characteristic were measured using an ac-bridge technique.

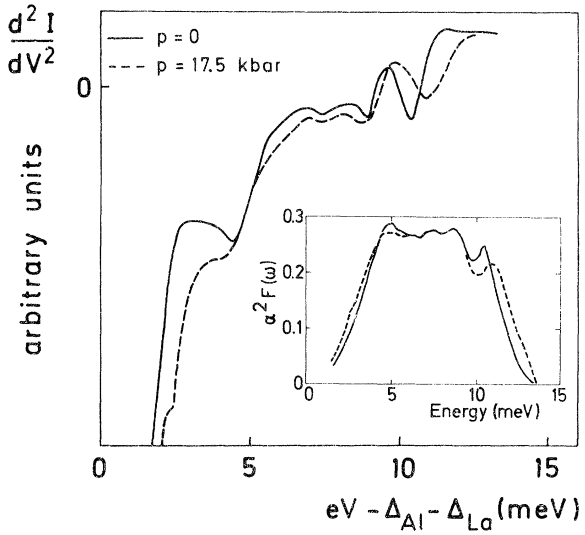

FIG. 1. d^2I/dV^2 data as a function of energy (measured from the gap edge). Inset, spectrum $\alpha^2F(\omega)$. Additional structure occurred at $\simeq 2.3$ meV at pressures above 14 kbar in three experiments.

Six different diodes have successfully been pressurized. Two of them have first been measured at $P = 0$ and then at high pressure. For the other four diodes, the $P = 0$ measurement has been done after the release of pressure. T_c increased from 4.95 to 6.84 K at a pressure of 17.5 kbar, in good agreement with the pressure dependence of T_c for bulk dhcp samples.[7] The coupling strength increases under pressure, the ratio $2\Delta_0/kT_c$ rising from 3.75 to 4.06 at 17.5 kbar. Figure 1 contains d^2I/dV^2 data taken at 1.1 K, the granular Al film being in the superconducting state. The (zero pressure) minima at $\simeq 4.4$ meV, 8.8 meV, and 10.4 meV, which correspond to maxima in the phonon spectrum, agree with previous data.[3-5] It can be seen that a pressure of 17.5 kbar causes a shift of the low-frequency structure to lower energies which is reversible with pressure. The singularity at 8.8 meV stays roughly unchanged. The structure at 10.4 meV related to longitudinal phonons exhibits the regular shift to higher frequencies.

Figure 2 shows the pressure dependence of the three minima. There is some arbitrariness in determining the precise location of the low-frequency minimum (Fig. 1). The sample statistics in Fig. 2, however, convincingly reveal the shift

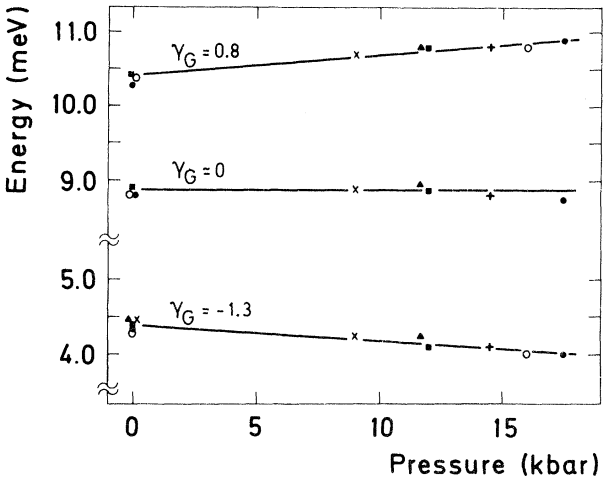

FIG. 2. Pressure dependence of characteristic phonon peaks taken from the minima of d^2I/dV^2 at 1.1 K.

of this minimum, which points to a pressure-induced softening of the transverse phonon modes. Three diodes have shown a slight deterioration after the release of the pressure, leading to broadened, poorly defined minima, particularly at higher energies. Such weak minima have been omitted from this plot.

Average Grüneisen constants $\gamma_G = -d\ln\omega_i/d\ln V = \kappa^{-1} d\ln\omega_i/d\ln P$ (compressibility $\kappa = 35\times10^{-7}$ bar^{-1}) can be assigned and are listed in Fig. 2, where the negative γ_G reflects the phonon softening.

Previous tunneling experiments under pressure on simple metals[10] have revealed a decrease of the coupling strength $2\Delta_0/kT_c$ together with a shift of the whole phonon spectrum to higher frequencies and a corresponding decrease of T_c. Surprisingly, we have observed an almost opposite behavior for La under pressure.

We have computed the function $\alpha^2F(\omega)$ using the McMillan inversion program[11] (see inset of Fig. 1). The lowest peak in $\alpha^2F(\omega)$ diminishes with pressure. The important feature is the marked shift of $\alpha^2F(\omega)$ to lower phonon energies on the low-frequency flank of this peak. The concept of the softening of transverse phonons under pressure provides a simple and consistent interpretation of both these observations. The electron-phonon coupling constant $\lambda = 2\int d\omega\, \alpha^2F(\omega)/\omega$ increases slightly but definitely under pressure. This increase is caused not only by the phonon softening, but also by an overall rise of the electron-phonon interaction. The computed values of μ^* scatter, the highest values for $P = 0$ being of the order of 0.04 ($\lambda = 0.84$), in comparison to $\simeq 0.13$, as is observed for many metals. Occasionally, even negative values of μ^* are found, which are unphysical. Under pressure, μ^* decreases reproducibly. Low μ^* values have also been found by other investigators from tunneling data of transition metals.[3-5] The intrinsic difficulty of electron tunneling into transition metals is related to the short coherence length and leads in the computational process to values of μ^* which are too small. This impedes, at present, the unambigous separation of λ and μ^*. We, therefore, do not wish to present numerical values for the pressure dependence of these quantities.

It has been suggested in a theoretical model by Coqblin and co-workers[12] that a small occupation of the $4f$ band adversely affects the superconductivity because of a strong additional Coulomb repulsion in the narrow band. These authors postulated that pressure raises the $4f$ band, which leads to a decrease of μ^* and an increase of T_c. The present uncertainties in μ^* prevent an unequivocal answer concerning this model. It is clear, however, that phonon softening and an enhancement of the electron-phonon coupling represent another important cause for the strong increase of T_c with pressure. This mechanism has not been previously considered in any model.

From the Grüneisen relation $\gamma_G(T) = \beta V/\kappa C_V$ one may expect that the negative γ_G for the transverse phonons of dhcp La causes an anomalous negative lattice expansion (β) at sufficiently low temperatures when these modes are predominantly excited. The metastable fcc phase of La shows such an anomalous thermal expansion with decreasing temperature below $\simeq 37$ K.[13] The similarity of the two phases of La implies that the negative thermal expansion of fcc La will also be related to a softening of transverse phonons under pressure.

It is puzzling how dhcp La acquires an average room-temperature Grüneisen constant as high as $\simeq 0.8$,[14] since one realizes from the (low-temperature) data of Fig. 2 that only the longitudinal phonons contribute to that extent. There is strong implication that the Grüneisen constants of the transverse modes may be appreciably temperature dependent, becoming negative only at low temperatures. A similar behavior has recently been found for the β-tungsten superconductor V_3Ge for long-wavelength phonons.[15] V_3Ge appears to have much in common with La in that phonon softening is at the root of the rapid increase of T_c with pressure.[16]

Yttrium and lutetium show, like La, a strong increase of T_c at high pressure.[8] Is this behavior generally related to a softening of transverse phonons? The thermal expansion of Y and Lu, contrary to that of La, is "normal" down to liquid-helium temperatures.[17] It seems reasonable to assume that this will also hold at high pressure. Relying on the Grüneisen relation, we expect that the transverse modes will be characterized by positive Grüneisen constants. Thus, there is reason to believe that the pressure-induced softening of the transverse phonons is a unique property of La, rather than an ordinary feature of all trivalent transition metals.

Uranium, contrary to La, is undisputedly an f-band metal, which shows an even stronger increase of T_c with pressure ($P \lesssim 12$ kbar), and, in addition, a negative thermal expansion at low temperatures.[18] It seems quite likely that these effects may also be related to each other by a softening of low-lying phonons under pressure. Based upon this comparison, we suggest that in La the steeply rising density of states of the $4f$ band at the Fermi surface is responsible for the phonon softening under pressure, as well as for the low Debye temperature, the low melting point, and other unusual properties of this metal.

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