(cf. table)

$$L = (T/T_\lambda)(\rho/\rho_s)\times(7.3\ \text{Å}). \tag{2}$$

This empirical formula is just of the form proposed by Rudnick and Fraser.[5] However, although our measurements yield the same form, the numerical factor is different.

That the superfluid fraction drops relatively precipitously to zero at film thicknesses somewhat larger than L is apparent from Fig. 3. Let us denote by $L+\Delta$ the liquid film thickness at which deviations from straight-line behavior occur. Then Δ represents the transition region of film thickness over which the experimental data clearly deviates from the straight-line, thick-film Ginzburg-Pitaevskii theoretical result. We find empirically (cf. table) that this transition thickness Δ is quite well represented by the formula

$$\Delta = L\rho/\rho_s. \tag{3}$$

As may be evident from Figs. 2 and 3, the point at which the superfluid begins to appear cannot be ascertained with the same precision as can L and Δ. We denote by L_c the thickness of the *liquid* part of the film at which the superfluid just begins to appear. The last column of the table exhibits our estimates of this "onset" thickness, L_c, in standard layers of helium.

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Relativistic Particle Motion in Nonuniform Electromagnetic Waves

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It is shown that a charged particle moving in a strong nonuniform electromagnetic wave suffers a net acceleration in the direction of the negative intensity gradient of the wave. Electrons will be expelled perpendicularly from narrow laser beams and various instabilities can result.

Particles moving in electromagnetic wave fields strong enough to drive them relativistic have been extensively investigated in recent years, with regard to both cosmic-ray production in pulsars[1] and laser-particle interaction. Linearly polarized plane waves can lead to particle acceleration in the forward direction while particles in a circularly polarized wave are thought to produce dc magnetic fields.[2] Plasma effects[3] including instabilities[4] have also been studied in some cases. Some problems in which the nonuniformity of the wave plays a role have recently been investigated by Kaw and Kulsrud[5] and Vittitoe and Wright.[6]

Here a general treatment is presented of particles moving in weakly nonuniform waves where the wave intensity varies slowly on the space and time scale of the particle oscillation period. It has long been known that nonrelativistic particles suffer a net acceleration in such a wave in the direction of the negative intensity gradient.[7] This behavior is characterized by a "ponderomotive force" and leads to mode coupling and instabilities in plasmas.[8] While the lowest-order particle motion in the nonrelativistic case is that of a harmonic oscillator, the relativistic case is more involved; but it will be shown that field nonuniformities lead to a generalized ponderomotive acceleration tending to expel particles from regions of strong field intensity.

The equation of motion of an electron in an electromagnetic field may be written as

$$\frac{dU_\mu}{d\tau}=\frac{e}{m}\left(\frac{\partial A_\mu}{\partial x_\nu}-\frac{\partial A_\nu}{\partial x_\mu}\right)\frac{dx_\nu}{d\tau}, \tag{1}$$

where U_μ is the four-velocity and A_μ the four-vector potential. This equation may be cast in the more convenient form

$$\frac{d}{d\tau}\left(U_\mu-\frac{e}{m}A_\mu\right)=-\frac{e}{m}\frac{\partial A_\nu}{\partial x_\mu}U_\nu. \tag{2}$$

Consider first in the lowest order the motion of the particle in a uniform plane wave. Adopting the Coulomb gauge we have $A_\mu=(a\vec{A}_\perp, 0, 0)$, $a=\text{const}$; $\vec{A}_\perp(x_\parallel-ct)=\vec{A}_\perp(\eta)$. Equation (2) reads now in component form

$$\frac{d}{d\tau}\left(\vec{U}_\perp-\frac{e}{m}a\vec{A}_\perp\right)=0, \tag{3a}$$

$$\frac{d}{d\tau}U_\parallel=-\frac{e}{m}a\frac{\partial\vec{A}_\perp}{\partial x_\parallel}\cdot\vec{U}_\perp, \tag{3b}$$

$$\frac{d}{d\tau}\gamma=\frac{e}{mc^2}a\frac{\partial\vec{A}_\perp}{\partial t}\cdot\vec{U}_\perp, \tag{3c}$$

and from Eqs. (3b) and (3c)

$$(d/d\tau)(U_\parallel-c\gamma)=0. \tag{4}$$

One may integrate Eqs. (3a) and (4) to obtain constants of motion.

Now we let $A_\mu=(a\vec{A}_\perp, bB(\eta), 0)$, where a and b vary slowly on scale lengths much larger than the particle and wave oscillation periods. From $\nabla\cdot\vec{A}=0$,

$$\vec{A}_\perp\cdot\frac{\partial a}{\partial\vec{x}_\perp}+\frac{\partial b}{\partial x_\parallel}B+b\frac{\partial B}{\partial\eta}=0. \tag{5}$$

The two dominant terms are the first and the last ones, giving $bB\sim aA_\perp\lambda/L$, where λ is the wavelength and L the scale length of variation of the slowly varying a and b.

Equation (2) yields now

$$\frac{d}{d\tau}\left(\vec{U}_\perp-\frac{e}{m}a\vec{A}_\perp\right)=-\frac{e}{m}\vec{A}_\perp\cdot\vec{U}_\perp\frac{\partial a}{\partial\vec{x}_\perp}, \tag{6}$$

$$\frac{d}{d\tau}\left(U_\parallel-c\gamma-\frac{e}{m}bB\right)=-\frac{e}{m}\vec{A}_\perp\cdot\vec{U}_\perp\left(\frac{\partial}{\partial x_\parallel}+\frac{1}{c}\frac{\partial}{\partial t}\right)a, \tag{7}$$

where terms on the right-hand sides represent small perturbations due to field nonuniformities and some longitudinal bB terms have been neglected.[9] These two equations plus the exact equation $c\gamma=(c^2+U_\parallel^2+U_\perp^2)^{1/2}$ (arising from $U_\mu U_\mu=\text{const}$) are the complete set of equations to be solved.

Substituting now the lowest-order solution for $\vec{U}_\perp$ from the integral Eq. (3a) to the right-hand side of Eqs. (6) and (7) and integrating over a period of particle oscillation yields

$$\Delta\left(\vec{U}_\perp-\frac{e}{m}a\vec{A}_\perp\right)=-\frac{e^2}{2m^2}\int A_\perp^2\,d\tau\frac{\partial a^2}{\partial\vec{x}_\perp}, \tag{8}$$

$$\Delta(U_\parallel-c\gamma)=-\frac{e^2}{2m^2}\int A_\perp^2\,d\tau\left(\frac{\partial}{\partial x_\parallel}+\frac{1}{c}\frac{\partial}{\partial t}\right)a^2, \tag{9}$$

where Δ represents the change of a quantity over an oscillation period. This period can be characterized by the time it takes for the particle to complete an oscillation between equal values of $\vec{A}_\perp$. The integral in Eqs. (8) and (9) is easily evaluated:

$$\int A_\perp^2\,d\tau=\int A_\perp^2(d\eta/d\tau)^{-1}d\eta=(\lambda/K)\langle A_\perp^2\rangle, \tag{10}$$

where $d\eta/d\tau=U_\parallel-c\gamma=-K$ is constant to lowest order from Eq. (4), and $\langle A_\perp^2\rangle$ is half of the amplitude squared for linear polarization and the amplitude squared for circular polarization.

Consider first a light pulse broad in the direction perpendicular to its propagation such that $\partial a^2/\partial\vec{x}_\perp\approx 0$. Since for such a long pulse

$$\left(\frac{\partial}{\partial x_\parallel}+\frac{1}{c}\frac{\partial}{\partial t}\right)a^2=0,$$

all perturbation quantities vanish. As the pulse moves through an originally stationary particle, after the pulse passed the particle is left stationary ($\Delta\vec{A}_\perp=0$, $\Delta\vec{U}_\perp=0$, $\Delta U_\parallel=0$).

In the following we will focus our attention to time independent fields, $\partial a/\partial t=0$. Since $\Delta(c\gamma)$

$=(c\gamma)^{-1}(U_\perp \Delta U_\perp + U_\parallel \Delta U_\parallel)$, one may combine Eqs. (8), (9), and (10) to find the change of particle energy during an oscillation period:

$$\Delta(c\gamma) = -\frac{\lambda}{K^2}\frac{e^2}{2m^2}\langle A_\perp{}^2\rangle\frac{da^2}{d\tau} + \frac{e^2}{2m^2K}A_\perp{}^2\Delta a^2. \qquad (11)$$

A term linear in $A_\perp$ arising from the zero-order constant of motion in $U_\perp \Delta U_\perp$ which averages to zero over an oscillation period has been dropped from Eq. (11).

For $K=\text{const}$ $(\partial a/\partial x_\parallel = 0)$ one may integrate Eq. (11) over an oscillation period to find $\langle\Delta\gamma\rangle = 0$, where the angular brackets signify the average of a quantity over an oscillation period. The integration of Eq. (7) shows that $K = K_0 + O(\lambda \partial a^2/\partial x_\parallel)$; hence corrections in Eq. (11) due to the variation of K give higher-order terms. Thus one finds in general the interesting result that the average particle energy is an adiabatic invariant.

Introduce now the (proper) time of an oscillation, $\Delta\tau = \int (d\eta/d\tau)^{-1} d\eta = \lambda K^{-1}$, and use Eqs. (8) and (9) to calculate the particle acceleration over the slow time scale:

$$\langle\Delta\vec{U}_\perp/\Delta\tau\rangle = -(e^2/2m^2)\langle A_\perp{}^2\rangle\frac{\partial a^2}{\partial\vec{x}_\perp}, \qquad (12)$$

and

$$\langle\Delta U_\parallel/\Delta\tau\rangle = -(e^2/2m^2)\langle A_\perp{}^2\rangle\,\partial a^2/\partial x_\parallel, \qquad (13)$$

or

$$d\vec{U}_d/d\tau = -(e^2/2m^2)\nabla\langle a^2 A_\perp{}^2\rangle, \qquad (14)$$

where $\vec{U}_d$ is the drift velocity of the particle, whose acceleration is due to the intensity gradient of the wave.[10] Hence a particle oscillating in a strong wave field suffers an acceleration toward the weaker-field region and is ultimately ejected from the beam. In the process the oscillatory particle energy turns into directed energy, keeping γ constant on the average. One may multiply Eq. (14) by $\vec{U}_d$ to find

$$U_d{}^2 + (e^2/m^2)\langle a^2 A_\perp{}^2\rangle = \text{const}, \qquad (15)$$

which expresses again the constancy of the sum of oscillatory and drift energies. Equation (15) may be regarded as an energy equation with $\varphi = (e^2/m^2)\langle a^2 A_\perp{}^2\rangle$ playing the role of a potential. A particle when injected from outside the beam will be reflected as from a potential barrier.

Finally, we wish to point out some consequences of this acceleration. Particles in a laser beam will be accelerated and ejected sideways. The presence of a plasma will modify our equations (e.g., the phase velocity of waves is no longer c), but the basic effect of ponderomotive acceleration is still there. The ejection of particles leads now to a change of dielectric function along the beam path and results in self-focusing and filamentation as in the nonrelativistic case.

For a circularly polarized wave the lowest-order particle motion is gyration with $\vec{A}_\perp \cdot \vec{U}_\perp = \text{const}$, with the electric force providing the centripetal acceleration and $\vec{v}\times\vec{B}=0$. In the presence of $\partial a/\partial\vec{x}$ the guiding center will be accelerated toward the weaker field, as can be seen from Eq. (6) with the right-hand side providing a constant acceleration. The physical reason for this acceleration is easily seen; the electric force acting on the particle is stronger along part of its orbit where the intensity is larger than on the weaker field side, providing a net accelerating force. The presence of a background plasma is known to lead to an axial dc magnetic field. This may be easily incorporated in the calculation, and one finds that the outward ponderomotive force leads to an azimuthal drift in a cylindrical beam. A perturbation of such an equilibrium can lead to flute-type instabilities. Details of this problem will be published elsewhere.

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[10]A comparable result has been obtained by T. W. B. Kibble [Phys. Rev. 150, 1060 (1966)]. His analysis treats the average motion of a charged particle in an electromagnetic field in a weak-field limit [$(e/m)A_\mu$ is an expansion parameter]. The present perturbation scheme treats arbitrarily strong electromagnetic fields; our perturbation parameters are the quantities Λ_i/L, as described above.

Observation of Temperature Oscillations in Quasi-isothermal Superfluid Film Oscillations*

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We report the first observations of the small temperature oscillations which accompany the *nearly* isothermal oscillations in the levels of two reservoirs of He II which are connected only by the mobile superfluid film. We show that our experimental results are in accord with a nearly isothermal approximation to the general theory first elucidated by Robinson for capillary flow. The amplitude of the temperature oscillations is in agreement with expectations based on the measured thermal conductance and time constant for our apparatus.

Consider two reservoirs each partially filled with He II. Let the reservoirs be coupled only by means of the mobile superfluid film. As is well known, any chemical potential difference between the fluid surfaces in the reservoirs will tend to be reduced by mass transport through the superfluid film. As explained by Robinson[1] in his treatment of He II flow in narrow channels, the kinetic energy of the moving superfluid may result in an oscillation in the level difference between the two baths. In particular, Robinson showed that the two cases of weak and strong thermal contact between the reservoirs resulted in oscillatory solutions to the equations of motion. He called these two cases adiabatic and isothermal. Atkins[2] was the first to observe the isothermal oscillations in a film-flow experiment and the adiabatic oscillations were first seen in bulk flow through superleaks by Manchester and Brown.[3] More recently, Hammel, Keller, and Sherman,[4] Hoffer *et al.*,[5] and Hallock and Flint[6] have studied various aspects of the "isothermal" film oscillations. Here we wish to report the first observation of the small-amplitude temperature oscillations which accompany the level oscillations described by the nearly isothermal limit of the theory due to Robinson.[1]

The apparatus, Fig. 1, consists of two cylindrical reserviors each of which makes up the annular region of a coaxial capacitor. The reservoirs are sealed except for a superfluid film path. A capillary (not shown) which has a cross sectional area which is 6% of the area of the annular region enters from the bottom and is used to fill the assembly to about the midpoint of the annular region. A superfluid valve[7] is used to seal the

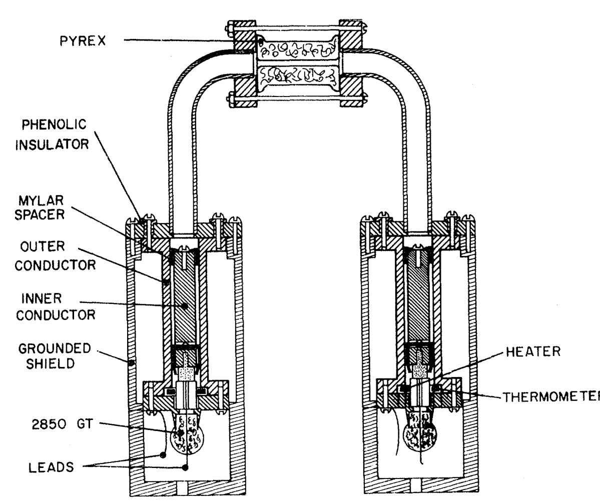

FIG. 1. Apparatus used for the measurements. The gap between the inner and outer conductors is 100 μm. A Mylar gasket (not shown) insulates the grounded upper arm from the capacitors. Superfluid integrity is accomplished through the use of indium seals.