where

$$C = \int_{-\infty}^{+\infty} x f(x)\, dx$$

and the primes indicate derivatives with respect to ν_t.

The qualitative features of the experiments may be understood from (2). In Fig. 1(a) the sequence of measurements from high field to low has at any time significantly reduced the original high value of $P(\nu_t)$ for $\nu_t > \nu_s$, where ν_s is the value for the current experiment. Thus $P'(\nu_s)$ is large and negative and the results trace out $G(\nu_s)$. On returning through the resonance from low field to high, the initial value of $P(\nu_t)$ now having been lost for all ν_t, the second term in (2) dominates and the derivative $G'(\nu_s)$ is obtained. In Fig. 1(b) the field-increasing measurements were made first, so $P(\nu_t)$ is expected to be reduced for $\nu_t < \nu_s$; therefore $P'(\nu_s)$ is positive and the results do show a mixture of $-G(\nu_s)$ and $G'(\nu_s)$ as indicated by (2) while, in the return through the line, a mixture of $G(\nu_s)$ and $G'(\nu_s)$ is found. In Fig. 1(c) a preliminary preparation of the system has "burnt a hole" in $P(\nu_t)$ at $\nu_p = 9.32$ GHz. The consequent changes in P' are expected to reduce $R(\nu_s)$ for ν_s somewhat greater than ν_p and increase it for ν_s rather less than ν_p. In agreement with this, it is found that the minimum of the deep hole exhibited by Fig. 1 occurs at a value of ν_s which exceeds ν_p.

Finally, one other result is of interest. If the sample is cooled down at high field and a single measurement made in the center of the resonance, then in agreement with (2), because both P' and G' are zero, almost no induced magnetization is observed. Thus the systematic progress through the line from high field to low has the effect of greatly enhancing the magnitude of the observable effect and was a factor in its discovery.

This work is supported by the Science Research Council to which one of us (T.H.) is indebted for a research studentship.

[1]S. Clough and B. J. Mulady, Phys. Rev. Lett. 30, 161 (1973).

[2]H. Glättli, A. Sentz, and M. Eisenkremer, Phys. Rev. Lett. 28, 871 (1972).

Superfluid Fraction in Thin Helium Films*

Marvin Chester†
School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton, England

and

L. C. Yang
Physics Department, University of California, Los Angeles, California 90024, and Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91103

(Received 14 June 1973)

A shear-mode quartz-crystal microbalance measures only the nonsuperfluid component of the helium film adsorbed on it. From this effect detailed curves of the superfluid content in such films are obtained as functions of the total film mass adsorbed per unit area and of temperature. Besides the characteristic linear growth region of superfluid with thickness, these curves reveal the quantitative details of the onset region. Our results are presented together with some characteristic helium parameters deduced from them.

We have used a thickness shear-mode oscillating quartz crystal as a microbalance to make measurements on the adsorbed helium film at unsaturated vapor pressures. Since the quartz surface oscillates transversely, only the nonsuperfluid component of the adsorbed film loads the crystal. This gives rise to a frequency decrease in direct proportion to the loading mass per unit area, σ_l, with a sensitivity of 3.837×10^{-10} g/cm^2 per hertz of frequency decrease. Both a preliminary note on some of our unprocessed results and basic technical aspects have been reported.[1-3] Our purpose here is to present our quantitative results on the details of the onset of superfluidity in the helium film as functions both of temperature and of the adsorption level σ.

This information bears directly upon a discussion which has appeared recurrently in the recent

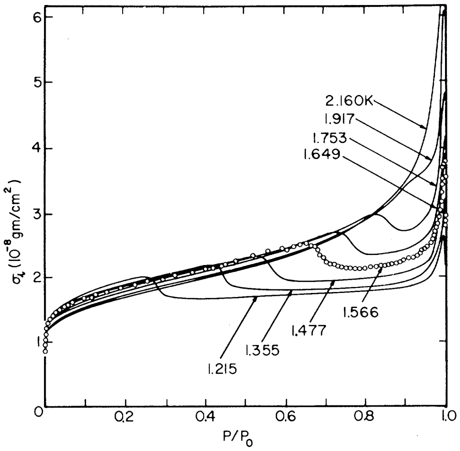

FIG. 1. Experimental adsorption isotherms of liquid helium.

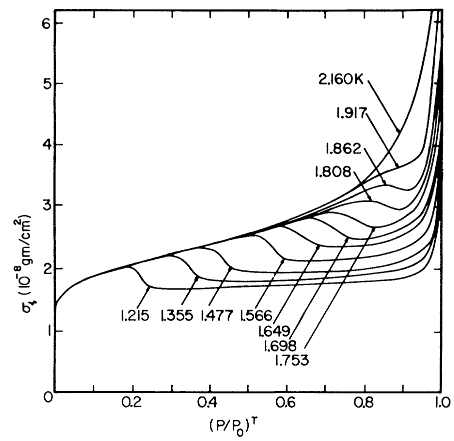

FIG. 2. The same experimental data exhibited in Fig. 1 but replotted employing as abscissa $(P/P_0)^T$.

literature.[4-8] Some experiments[4,5] (buttressed by theory[6]) suggest that with decreasing film thickness the superflow properties fall to zero (onset) while a finite superfluid fraction persists in the film. Other evidence suggests[9] that, in fact, the superfluid fraction itself (and, necessarily, superflow with it) drops relatively precipitously to zero at onset. The essential feature contributed by our measurement technique is that the signal representing the superfluid content has no attenuation component.[9] It is just as strong near onset as it is far from onset. Hence we have detailed data near onset.

Figure 1 represents our "raw" experimental adsorption isotherm data at a representative group of temperatures. For one curve the experimental points are shown. These, being quite dense, have been replaced by the line drawn through them in the other curves. The ordinate has been converted directly from frequency shift, via the sensitivity factor already mentioned, to loading mass adsorbed per unit area σ_l. The abscissa represents the degree of saturation: the measured gas pressure P divided by the saturated vapor pressure $P_0(T)$ for the temperature T at which the isotherm was measured. The adsorbed mass defect, which appears to occur beyond characteristic saturation ratios on each curve, we attribute to the presence of an increasing superfluid content, σ_s, in the film. The superfluid does not register as part of the adsorbed mass because, having no viscosity, it does not "load" the quartz-crystal oscillator.

In order to deduce the superfluid content σ_s quantitatively we must have some means of estimating the total mass adsorbed per unit area, σ, so as to perform the subtraction:

$$\sigma_s = \sigma - \sigma_l. \tag{1}$$

A number of theoretical isotherm equations are extant which relate σ to P. The one most popularly employed for helium is the Frenkel-Halsey-Hill isotherm.[10] Motivated by considerations quite parallel to those underlying the Frenkel-Halsey-Hill theory, we replotted our data of Fig. 1 using as abscissa the Tth power of the saturation ratio P/P_0 instead of this ratio itself. Figure 2 constitutes this plot.

We chose this method of organizing our data in the hope of obtaining a universal curve of adsorption. By this we mean one for which, because of the proper choice of variable (function of T and P), the explicit dependence of $\sigma = \sigma(T, P)$ on two variables is suppressed in favor of an implicit dependence contained in only the one variable. From the figure it is clear that, with the function chosen for the abscissa, one indeed achieves some measure of success. Each of the experimental curves does follow along and then deviate from a single universal curve. This universal curve is essentially coincident with the one labelled $T = 2.160$°K over the range exhibited. At this temperature, the adsorbed film contains no superfluid component ($\sigma_s = 0$) up to values too close to unity to record. Hence the 2.160°K curve can be taken as representing σ, the total amount of helium adsorbed per unit area. And by virtue of its evident universality we may perform the subtraction of Eq. (1) by literally taking differences (vertical ones) on Fig. 2 between the σ

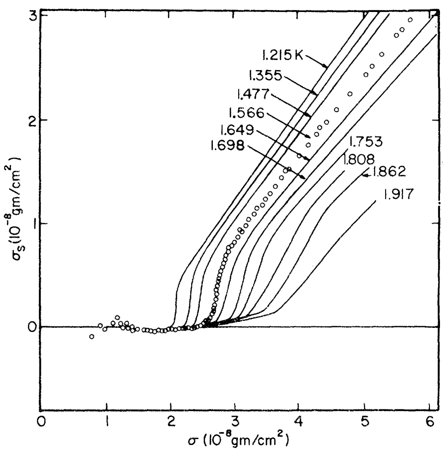

FIG. 3. The superfluid content (mass of superfluid per unit area) of the helium film as a function of the total mass of helium adsorbed per unit area.

curve (2.160°K) and any of the other curves (σ_l) desired. Figure 3 represents the result. The superfluid content, σ_s, is plotted as a function of the total amount of film adsorbed per unit area, σ.

Two remarks should be made on the foregoing procedure. The first is that the universal curve obtained above yields an isotherm which does not reproduce the standard Frenkel-Halsey-Hill reciprocal third power law. Instead the exponent varies[11] between 3 and 4 depending upon adsorption level. Secondly, this procedure eliminates any assumptions about either the form or the strength of the Van der Waals interaction which yields this power law. Rather this information is automatically incorporated into our results both simultaneously and for the identical substrate[12] as that employed for the superfluidity measurements.

Figure 3 represents the essential body of results that we wish to present here. Exhibited there is the detailed onset behavior seen by our apparatus as subjected to a bare minimum of assumptions in the data processing and in terms of directly measurable quantities (i.e., σ instead of thickness). We wish now to compare some of our results with previous work. To do so we must rely on further assumptions, some of which are subjects of debate.

It is commonly assumed that the first layer of the adsorbed film is solid and subsequent layers are liquid of approximately normal liquid density. On this basis we have estimated the density of the solid in the first layer utilizing the conventional technique of plotting Brunauer-Emmett-Teller curves.[13] For helium these have been shown[14,15] to yield the amount in the first two absorbed layers. Taking each layer to be of standard thickness $D_0 = 3.6$ Å, and the second layer to be at normal liquid density, $\rho = 0.145$ g/cm^3, the values of $\sigma_1 = \rho_1 D_0$ are easily deduced. These are exhibited in Table I.

TABLE I. The density in the solid (first) layer of the helium film and some lengths characterizing the superfluid content of thin helium films as deduced from our data.

T (°K)	$\sigma_1/\rho D_0 = \rho_1/\rho$	L/D_0	$2.03 T\rho/T_\lambda \rho_s$	Δ/D_0	$\frac{L}{D_0}\frac{\rho}{\rho_s}$	L_c/D_0
1.215	1.98	1.15	1.17	1.16	1.19	1.95
1.275	2.02	1.15	1.24	1.20	1.20	1.97
1.355	1.92	1.33	1.35	1.43	1.42	2.24
1.424	1.91	1.40	1.45	1.62	1.50	2.52
1.477	1.87	1.52	1.54	1.65	1.68	2.56
1.566	1.84	1.70	1.72	2.06	1.98	2.70
1.649	1.82	2.10	1.93	2.43	2.60	2.94
1.698	1.77	2.21	2.09	2.79	2.87	2.92
1.753	1.74	2.40	2.29	3.31	3.31	3.06
1.808	1.72	2.70	2.54	3.97	4.04	3.00
1.862	1.69	2.96	2.86	4.93	4.82	3.03

Now reverting to Fig. 3 we note that there is a straight-line region. The slope of this region is, in each case, within 15% of the bulk superfluid fraction ρ_s/ρ. This slope should, of course, just be equal to this ratio. We attribute the difference between experiment and theory to the slight inhomogeneity of our crystal surface which was measured to have a surface roughness between 5% and 15% greater than ideal flatness. The intercept of the extended straight-line region with the horizontal axis can be interpreted as defining a characteristic length.[5] If the amount of mass per unit area, σ_1, of the solid first layer is subtracted, this intercept corresponds to the liquid mass, ρL, in the length L. This length has been interpreted[5] in terms of the Ginzberg-Pitaevskii description[16] of superfluidity. More simply, it also may be viewed as that thickness of film which effectively remains normal in the presence of bulk superfluidity in the rest of the liquid film when the straight-line regime beyond onset obtains. Regardless of its interpretation, the experimental value is obtained simply by extrapolation of the straight-line regions of Fig. 3. Our values of L are exhibited in the table. We find

(cf. table)

$$L = (T/T_\lambda)(\rho/\rho_s)\times(7.3\ \text{Å}). \quad (2)$$

This empirical formula is just of the form proposed by Rudnick and Fraser.[5] However, although our measurements yield the same form, the numerical factor is different.

That the superfluid fraction drops relatively precipitously to zero at film thicknesses somewhat larger than L is apparent from Fig. 3. Let us denote by $L+\Delta$ the liquid film thickness at which deviations from straight-line behavior occur. Then Δ represents the transition region of film thickness over which the experimental data clearly deviates from the straight-line, thick-film Ginzburg-Pitaevskii theoretical result. We find empirically (cf. table) that this transition thickness Δ is quite well represented by the formula

$$\Delta = L\rho/\rho_s. \quad (3)$$

As may be evident from Figs. 2 and 3, the point at which the superfluid begins to appear cannot be ascertained with the same precision as can L and Δ. We denote by L_c the thickness of the *liquid* part of the film at which the superfluid just begins to appear. The last column of the table exhibits our estimates of this "onset" thickness, L_c, in standard layers of helium.

We wish to thank the British Science Research Council for support and Professor D. F. Brewer for his hospitality and for useful discussions during the preparation of this paper.

*Work supported in part by a grant from the California Institute of Technology Presidents Fund.

†On leave from, and presently returned to, the Physics Department, University of California, Los Angeles, Calif. 90024.

[1]M. Chester, L. C. Yang, and J. B. Stephens, Phys. Rev. Lett. 29, 211 (1972).

[2]L. C. Yang, M. Chester, and J. B. Stephens, in *Proceedings of the Thirteenth International Conference on Low Temperature Physics, Boulder, Colorado, 1972,* edited by W. J. O'Sullivan *et al.* (Plenum, New York, 1973), Vol. 1, p. 211.

[3]L. C. Yang, thesis, University of California at Los Angeles, 1973 (unpublished).

[4]R. P. Henkel, G. Kukich, and J. D. Reppy, in *Proceedings of the Eleventh International Conference on Low Temperature Physics, St. Andrews, Scotland, 1968,* edited by J. F. Allen, D. M. Finlayson, and D. M. McCall (St. Andrews Univ. Press, St. Andrews, Scotland, 1969), Vol. 1, p. 178.

[5]I. Rudnick and J. C. Fraser, J. Low Temp. Phys. 3, 225 (1970).

[6]M. Revzen, Phys. Rev. Lett. 22, 1102, 1413 (1969).

[7]D. L. Goodstein and R. L. Elgin, Phys. Rev. Lett. 22, 383 (1969).

[8]D. Goodstein, Phys. Rev. 183, 327 (1969).

[9]B. Ratnam and J. Mochel, J. Low Temp. Phys. 3, 239 (1970).

[10]T. L. Hill, Advan. Catal. Relat. Subj. 4, 211 (1952).

[11]E. S. Sabisky and C. H. Anderson, Phys. Rev. A 7, 790 (1973).

[12]E. S. Sabisky and C. H. Anderson, Phys. Rev. Lett. 30, 1122 (1973).

[13]D. M. Young and A. D. Crowell, *Physical Adsorption of Gases* (Butterworths, London, 1972).

[14]L. Meyer, Phys. Rev. 103, 1593 (1956).

[15]W. D. McCormick, D. L. Goodstein, and J. G. Dash, Phys. Rev. 168, 24 (1968).

[16]V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. 34, 1240 (1958) [Sov. Phys. JETP 7, 858 (1958)].

Relativistic Particle Motion in Nonuniform Electromagnetic Waves

G. Schmidt* and T. Wilcox
Department of Physics, University of California, Los Angeles, California 90024†
(Received 25 June 1973)

It is shown that a charged particle moving in a strong nonuniform electromagnetic wave suffers a net acceleration in the direction of the negative intensity gradient of the wave. Electrons will be expelled perpendicularly from narrow laser beams and various instabilities can result.

Particles moving in electromagnetic wave fields strong enough to drive them relativistic have been extensively investigated in recent years, with regard to both cosmic-ray production in pulsars[1] and laser-particle interaction. Linearly polarized plane waves can lead to particle acceleration in the forward direction while particles in a circularly polarized wave are thought to produce dc magnetic fields.[2] Plasma effects[3] including instabilities[4] have also been studied in some cases. Some problems in which the nonuniformity of the wave plays a role have