For the Crab pulsar ($L=10^{38}$ erg/sec, $\Omega=200$ rad/sec), $J_c\approx 1.7\times 10^{33}$ particles/sec and $\epsilon_{max}\approx 2.8\times 10^{16}(J_c/J)$ eV. Thus, for a given L, a larger flux of lower energy or smaller flux of higher-energy cosmic rays may be obtained, depending on J. In either case, 50% of the pulsar wave luminosity is converted into cosmic rays of energy less than ϵ_{max}. Within the superrelativistic approximation, these conclusions may be valid for the relativistic injection of neutral plasma of any composition.

Care must be exercised in drawing far-reaching quantitative conclusions from our plane-wave model. Dependences on the azimuthal and polar angles in spherical geometry have been ignored. Furthermore, for a plane wave to be a good approximation, the radial excursion D of a particle during a half-period $\eta=\pi$, $D=(\pi c/\Omega)\beta/(\beta^2-1)$, should be smaller than the radial scale length. This is only marginally satisfied at the injection radius and even less well at larger radii. Clearly, investigation of the spherical wave is called for.

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Real Part of the Proton-Proton Forward-Scattering Amplitude from 50 to 400 GeV

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From measurements of proton-proton elastic scattering at very small momentum transfers where the nuclear and Coulomb amplitudes interfere, we have deduced values of ρ, the ratio of the real to the imaginary forward nuclear amplitude, for energies from 50 to 400 GeV. We find that ρ increases from -0.157 ± 0.012 at 51.5 GeV to $+0.039\pm 0.012$ at 393 GeV, crossing zero at 280 ± 60 GeV.

We have determined the ratio $\rho(E)$ of the real to the imaginary part of the forward proton-proton elastic nuclear scattering amplitude for incident energies from 50 to 400 GeV. The measurements were performed at the National Accelerator Laboratory by observing wide-angle recoil protons from an internal hydrogen gas-jet target. Elastic scattering was studied in the range from $|t|=0.001$ (GeV/c)^2, which is well inside the Coulomb region, to $|t|=0.04$ (GeV/c)^2, where the nuclear interaction dominates. The ratio ρ was determined from the strength of the interference between the nuclear and Coulomb amplitudes in the t region where they are comparable, $|t|\sim 0.002$ (GeV/c)^2.

Previously, ρ was measured at energies up to 70 GeV at Serpukhov in an experiment similar to the one reported here.[1] Recently, the CERN-Rome collaboration at the CERN intersecting storage rings (ISR) reported[2] ρ to be $+0.02\pm 0.05$ at 290 GeV and $+0.03\pm 0.06$ at 500 GeV (lab equivalent energy). We find that ρ increases from -0.157 ± 0.012 at 51.5 GeV to $+0.039\pm 0.012$ at 393 GeV, crossing zero at 280 ± 60 GeV.

Our experimental method makes use of the fact that the kinetic energy T of the recoil proton from elastic p-p scattering is directly related to the momentum transfer through $|t|=2mT$, where

m is the proton mass. In order to reach small t values, the target consisted of a pulsed hydrogen gas jet[3] with a density of about 5×10^{-7} g/cm^3. The density distribution of the jet at beam height was approximately Gaussian with a full width at half-maximum of ~12 mm. The energy of the incident protons is directly related to the time during the acceleration cycle at which the jet is pulsed and could be selected between 50 and 400 GeV.

Since, for fixed t, the angle of the recoil proton with respect to the incident beam is practically independent of the beam energy, the same detection apparatus could be used for measurements at any desired energy. Thus, the jet was pulsed more than once during a single acceleration cycle, typically twice. The duration of each pulse was 200 msec which corresponds to an energy bite of ±10 GeV.

Recoil protons emerging from the jet were recorded by an array of ten silicon solid-state detectors, each of an area ~100 mm^2, placed at a distance of 2.48 m from the target. The thickness of the detectors ranged from 0.1 to 5 mm in order to stop the elastic recoils and thus provide a precise measurement of their kinetic energy. The detectors were mounted at equal 9.3-mrad intervals on a movable carriage. Data were taken at six carriage positions separated by 1.86 mrad. At one position, adjacent detectors overlapped, providing a cross calibration of detector efficiency and acceptance. Two detectors mounted at fixed positions and two scintillator telescopes were used as monitors. This apparatus is similar to the one used for a measurement of the slope of the forward diffraction peak of p-p scattering.[4]

Recoil protons from elastic scattering gave distinct peaks in the energy spectrum of each detector. The width of the peak was essentially due to the width of the jet. Background under these peaks was small and primarily caused by scattering from residual gas (associated with the jet) in the accelerator vacuum system. This background contribution was estimated by running five out of every fifteen pulses with the detector carriage 64 mrad closer to 90°, where the elastic peaks were either completely eliminated or shifted toward much lower energies. After this background was subtracted, a very small residual background remained on the low-energy side of the elastic peak.

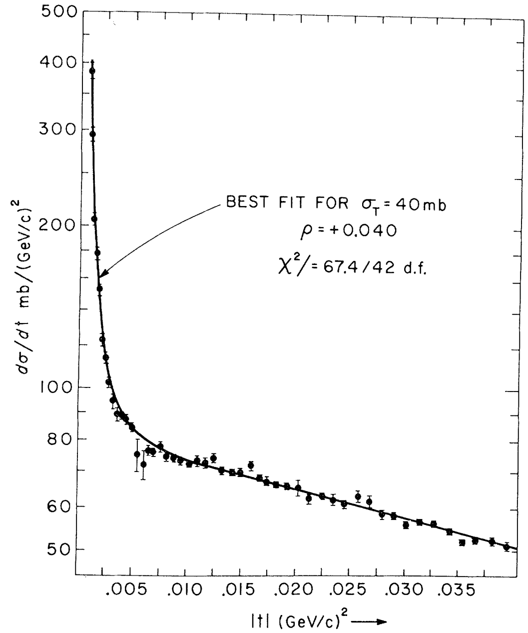

FIG. 1. The measured differential cross section $d\sigma/dt$ for p-p small-angle scattering at 400 GeV. The curve is the best fit to the data which are normalized as discussed in the text.

This was attributed to recoil protons from inelastic collisions and was subtracted both by an empirical fit as well as by using known data on resonance excitation[5]; both methods yielded similar results, the correction to the data being approximately 1 to 2%.

From the elastic peaks we obtained the differential cross section $d\sigma/dt$ by two different methods which yielded the same result within experimental error. In one case, the peaks were fitted with a distorted Gaussian function; in the other, the number of counts in the elastic peak was determined by summing over a fixed width of the jet.[6] The vairation of t and of $d\sigma/dt$ over the elastic peak was taken into account in the calculation of the observed data points. Corrections were made for counting losses due to dead time ($\lesssim 2\%$) and to nuclear interactions in the detectors ($\lesssim 0.5\%$). Typical differential cross sections obtained through this procedure are shown in Fig. 1 for 400-GeV incident energy. The absolute values of $|t|$ were determined by a simultaneous fit to the observed energies of the elastic peaks, making use of the precisely known spacing between detectors and carriage positions.

The observed differential cross sections were fitted by the Bethe interference formula[7]

$$\frac{1}{\pi}\frac{d\sigma}{dt}=K\left[\left(\frac{2\alpha}{t}\right)^2 G^4(t)-(\rho+\alpha\varphi)\frac{\alpha}{\pi}\sigma_T\frac{G^2(t)}{|t|}e^{bt/2}+\left(\frac{\sigma_T}{4\pi}\right)^2(1+\rho^2)e^{bt}\right], \tag{1}$$

where K is an overall normalization factor, α is the fine-structure constant, $G(t)$ is the proton form factor $=(1+q^2/0.71)^{-2}$, $\alpha\varphi$ is the phase of the Coulomb amplitude {we used the phase calculated by Yennie and West,[8] where $\alpha\varphi=\alpha[\ln(t_0/|t|)-C]$, $t_0=0.08$ (GeV/c)^2, and $C=0.577$}, and b is the nuclear slope parameter.

In fitting the data with Eq. (1), the slope parameter b was allowed to float with a Gaussian error of ± 0.2 (GeV/c)$^{-2}$ about our recently reported values[4]

$$b=8.23+0.556\ln s. \tag{2}$$

The values of the total cross section were fixed according to the expression

$$\sigma_T=38.4+0.49\ln^2(s/122). \tag{3}$$

Equation (3), given by Leader and Maor[9] for proton energies $E>50$ GeV, fits well the Serpukhov[10] and ISR data.[11,12] The overall normalization K and the ratio ρ of the real to imaginary nuclear amplitude were treated as free parameters. The results of this fit are given in Table I.

The error on ρ contains contributions from the following effects:

(a) The error on the measured values of $d\sigma/dt$. This includes the statistical error of the data[13] and of the background subtraction, as well as an uncertainty of 1% for our knowledge of the detector area. The resulting error on ρ ranges typically from 0.008 to 0.012. The χ^2 for the fits ranges from 45 to 85 for 56 degrees of freedom.

(b) An error due to variations in the position and shape of the gas jet. This contributes to the error in ρ between 0.008 and 0.012.

The two errors (a) and (b) have been added in quadrature and the result is the error quoted in Table I. In addition, the values of ρ are subject to an overall systematic shift of ± 0.015 due to an uncertainty of ∓ 0.4 mrad in the angular position of the detectors.[14] The values of the slope parameter b used in the fit are shown in Table I; an uncertainty of ± 0.2 (GeV/c)$^{-2}$ in b has been included in the fit and its effect on ρ is contained in the quoted error $\Delta\rho$. The effect of b on ρ is typically $\Delta\rho/\Delta b=+0.04$. These effects are summarized in Table II. The dependences of ρ and σ_T are strongly correlated. For this reason we have included in Table I the value of σ_T used at each energy as well as $\Delta\rho/\Delta\sigma_T$. It is important to stress that a decrease in σ_T results in an increase in ρ. Thus, *if σ_T were constant with energy, then the values of ρ obtained from these measurements would be even higher (more positive).*

Our results are plotted in Fig. 2 together with the Serpukhov data[1] and ISR data.[2] Our data show that $\rho(E)$ crosses zero at 280 ± 60 GeV and becomes positive. It is inconsistent with the possibility that $\rho(E)$ approaches zero asymptotically from below in this energy range.

Dispersion relations provide a connection between the behavior of $\rho(E)$ and the energy dependence of the proton-proton and antiproton-proton total cross sections.[15] This integral relation is such that ρ measured at energy E_0 has a certain

TABLE I. The ratio $\rho(s)=\mathrm{Re}F(E)/\mathrm{Im}F(E)$ of the p-p forward scattering amplitude.

$E^{(a)}$ (GeV)	s (GeV^2)	ρ	$\Delta\rho^{(b)}$	$b^{(c)}$ (GeV^{-2})	$\sigma_T^{(d)}$ (mb)	$\Delta\rho/\Delta\sigma_T$ (mb^{-1})
51.5	98	−0.157	± 0.012	10.80	38.44	−0.033
94.5	178	−0.098	± 0.012	11.13	38.46	−0.029
145.0	273	−0.064	± 0.010	11.36	38.71	−0.026
174.6	329	−0.039	± 0.012	11.46	38.88	−0.025
185.4	349	−0.038	± 0.014	11.48	38.94	−0.025
215.5	405	−0.020	± 0.012	11.55	39.11	−0.024
244.1	459	−0.013	± 0.010	11.62	39.27	−0.023
269.2	506	+0.022	± 0.015	11.69	39.43	−0.022
348.7	656	+0.025	± 0.015	11.86	39.79	−0.020
393.0	739	+0.039	± 0.012	11.90	39.98	−0.020

[a]The energy bins are centered at the value indicated and are typically 20 GeV wide.

[b]The data are subject to an overall energy independent systematic uncertainty of ± 0.015.

[c]These values are obtained from the fit when b is constrained with a Gaussian error of ± 0.2 (GeV/c)$^{-2}$ about the value given by Eq. (2) of the text.

[d]The values of σ_T used as an input to the fit. These are obtained from Eq. (3) of the text.

TABLE II. Errors in the determination of $\rho(s)=\mathrm{Re}F(E)/\mathrm{Im}F(E)$.

Source of error	Typical contribution to $\Delta\rho$
Statistical uncertainty and uncertainty in detector area	± 0.008 to 0.012
Error due to variations in jet width and position	± 0.008 to 0.012
Error in b $\Delta\rho/\Delta b\simeq +0.04$ (GeV^2)	Less than ± 0.008
Error in detector angular position $\Delta\rho/\Delta\theta\simeq -0.04$ (mrad^{-1})	Possible overall systematic shift by ± 0.015
Error due to σ_T $\Delta\rho/\Delta\sigma_T\simeq -0.025$ (mb^{-1})	Not included

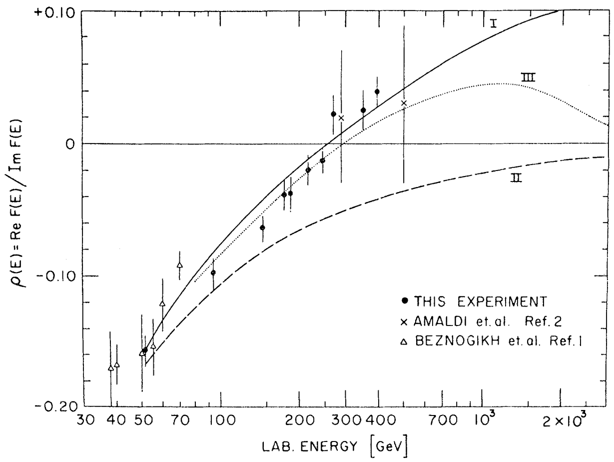

FIG. 2. The ratio ρ of the real to the imaginary part of the forward p-p nuclear amplitude as a function of energy. The curves are dispersion relation calculations assuming (I) $\sigma_T(pp)$ and $\sigma_T(\bar{p}p)$ increase as $\ln^2(s/122)$ to E_∞; (II) $\sigma_T(pp)$ is constant for $E > 120$ GeV at 38 mb; (III) $\sigma_T(pp)$ becomes constant above 2000 GeV at 43.5 mb. In each case it is assumed that $\sigma_T(\bar{p}p)$ approaches $\sigma_T(pp)$ as $E^{-0.602}$.

sensitivity to the behavior of $\sigma_T(pp)$ and $\sigma_T(\bar{p}p)$ at energies above E_0. This question has been studied under various assumptions concerning the extrapolation of $\sigma_T(pp)$ and $\sigma_T(\bar{p}p)$ to energies above those at which measurements now exist. As an illustration, we have plotted on Fig. 2 dispersion-relation curves for the *spin-independent* amplitudes calculated by using the form quoted by Söding[16] and using the parametrization of the total cross sections given by Eq. (3). It is assumed that $\sigma_T(pp)$ and $\sigma_T(\bar{p}p)$ obey the Pomeranchuk theorem, approaching equal values at infinite energy as $E^{-0.602}$, with the following energy dependence[17]: Curve I, $\sigma_T(pp)$ and $\sigma_T(\bar{p}p)$ increase as $\ln^2(s/122)$ to E_∞; Curve II, $\sigma_T(pp)$ is constant for $E > 120$ GeV at 38 mb; Curve III, $\sigma_T(pp)$ is constant for $E > 2000$ GeV at 43.5 mb. If dispersion relations are valid, and if the Pomeranchuk theorem holds and the high-energy total cross sections vary monotonically with energy, and approach each other as a power of energy, our data are consistent with an increase in $\sigma_T(pp)$ at least up to 2000 GeV as reported[11,12] and inconsistent with a constant $\sigma_T(pp)$ above 120 GeV.

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[14]The angular position of the detectors was determined from the energy of the elastic peaks as mentioned in the text. In addition, the fit to $d\sigma/dt$ is very sensitive to the initial angle (i.e., to the t values) that is used. Both methods gave the same result and provide an estimate of our uncertainty in the angle.

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