times are of the order of magnitude of the times for material to fall into the star, so a more detailed model is necessary to assess the effect of nuclear burning. Free-fall times are so short as to allow very little nuclear burning to occur. It is likely therefore that the limit on the accretion rates from nuclear burning are much weaker than the one estimated above, and that much larger accretion rates are indeed allowed.

We can then summarize our results as follows:

(1) Neutrino emission could allow accretion rates in neutron stars much larger than the critical value obtained from a straightforward application of the Eddington limit $[(dM/dt)_{\rm acc} \gg (10^{-7}-10^{-8})M_\odot/{\rm yr}$ for a neutron star of 1 solar mass].

(2) Following Davis and Evans,[5] we assume a minimum detectable flux in the range 1–5 MeV of $2\times10^9-1.5\times10^7$ neutrinos/cm^2 sec^1. If we also assume that all the known binary x-ray sources are at a distance from Earth larger than 10^3 parsec, we can then conclude that their neutrino fluxes appear to be too small for a direct detection.

(3) Some of the x-ray sources have large variations in the intensity of their x-ray flux[4] and discontinuities are observed in their accretion rate.[1] It is conceivable that the emission of neutrinos be larger at the time of maximum x-ray activity and accretion rate. A correlation between these events would be extremely important for the identification of the source.

(4) Quite apart from the steady accretion processes, a sharp increase of activity both in x-ray and radio waves has been noticed in some x-ray sources (Cygnus X-3). During these periods the neutrino emission could be higher than the one estimated here.

After this work was submitted for publication, Ya. B. Zel'dovich kindly informed one of us (R.R.) of similar results obtained by him and coworkers also pointing out the possible basic influence of neutrino production on the accretion processes in binary x-ray sources. The main issue, still to be answered, has to do with the analysis of the instabilities originated from nuclear burning.

*On leave of absence from Princeton University.

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Cosmic-Ray Generation by Pulsars*

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Electromagnetic and particle energy fluxes are equipartitioned in a superrelativistic plasma wave. The consequences for cosmic-ray acceleration by pulsars are investigated.

Pulsar electromagnetic fields may be a cosmic-ray source.[1] Large-amplitude plasma waves differ significantly from the vacuum wave used in Ref. 1 to estimate particle acceleration. We investigate a plasma wave of such large amplitude that both electrons and ions are superrelativistic, which transports both species parallel to the propagation direction with the same average velocity $V_\parallel = c^2/v_p$, where v_p is the phase velocity. This, plus equality of electron and ion perpendicular currents, permits small parallel electric fields and a simple dispersion relation which defines a largest particle flux that can be carried by the wave. Below this cutoff, the electromagnetic and particle energy fluxes are equal.

Consider a linearly polarized plane wave with all quantities functions of $\eta = \Omega(t - z/\beta c)$. β must be determined from the dispersion relation. Maxwell's equations, the equations of motion for each particle species, and the equations of continuity become

$$\hat{k}\times\vec{E}'=\beta\vec{B}', \quad (1a)$$

$$\hat{k}\times\vec{B}'=-\beta\vec{E}'-(4\pi e\beta c/\Omega)(n_i\vec{u}_i-n_e\vec{u}_e), \quad (1b)$$

$$\hat{k}\cdot\vec{B}=0, \quad (1c)$$

$$\hat{k}\cdot\vec{E}'=-(4\pi e\beta c/\Omega)(n_i-n_e), \quad (1d)$$

$$(\beta-\vec{u}_{i,e}\cdot\hat{k})P_{i,e}'=\pm(e\beta/\Omega)(\vec{E}+\vec{u}_{i,e}\times\vec{B}), \quad (1e)$$

$$n_{i,e}'(\beta-\vec{u}_{i,e}\cdot\hat{k})=n_{i,e}\vec{u}_{i,e}'\cdot\hat{k}, \quad (1f)$$

where the prime denotes $d/d\eta$. Coupled nonlinear particle equations follow from eliminating $\vec{E}$ and $\vec{B}$ assuming no static fields:

$$\frac{d^2}{d\eta^2}\vec{p}_\perp{}^{i,e}\pm\frac{4\pi e^2}{\Omega^2 m_{i,e}}\frac{\beta^2}{\beta^2-1}[n_i\vec{u}_\perp{}^i-n_e\vec{u}_\perp{}^e]=0, \quad (2a)$$

$$(d^2/d\eta^2)[\beta p_{\parallel\, i,e}-(1+p_{i,e}{}^2)^{1/2}\pm\psi)=0, \quad (2b)$$

where $\vec{p}_{i,e}$ is the ion (electron) momentum in units of mc, $u=v/c=p(1+p^2)^{1/2}$, $n_{i,e}$ is the rest-frame ion (electron) density, and ψ, the dimensionless electrostatic potential, satisfies

$$\nabla^2\psi=(4\pi e^2/m_e c^2)(n_i-n_e). \quad (2c)$$

Integrating (1f) once,

$$n_{i,e}=n_{i,e}{}^0(\beta-1/\beta)/(\beta-u_{i,e\parallel}), \quad (2d)$$

where $n_{i,e}{}^0$ is chosen near the rest-frame ion (electron) density in the wave. $n_0=n_i{}^0=n_e{}^0$ is a necessary condition that the charge density and parallel current vanish, averaged over a period in η. We further assume (1) $n_i\simeq n_e$—charge separations and $E_\parallel$ are small [$E_\parallel\ll E_\perp$ and $\psi(\eta)\cong 0$]; (2) ion and electron perpendicular currents are almost equal. Equations (1a) and (1b) then become

$$\frac{d^2p_\perp}{d\eta^2}+\frac{2\omega_p{}^2}{\Omega^2}\frac{\beta p_\perp}{\beta(1+p^2)^{1/2}-p_\parallel}=0, \quad (3a)$$

$$\beta p_\parallel-(1+p^2)^{1/2}=\gamma(\beta u_\parallel-1)=\text{const}=C, \quad (3b)$$

where $\omega_p\equiv(4\pi e^2 n_0/m_e)^{1/2}$. We define $\gamma_{\text{inj}}\equiv\gamma(p_\perp=0)$ and assume that $1\ll\gamma_{\text{inj}}\ll\gamma_w$, where γ_w denotes an average γ in the wave field. It follows from (3b) that

$$\frac{(\beta u_\parallel-1)_w}{(\beta u_\parallel-1)_{\text{inj}}}=\frac{\gamma_{\text{inj}}}{\gamma_w}\ll 1. \quad (4)$$

Since for pulsars (4) is satisfied over most of the period η, and, as we will see, $\beta>1$ while $u_{\parallel\,\text{inj}}\approx 1$, $u_\parallel\approx\beta^{-1}$. Thus all charged particles have the same $u_\parallel\simeq\beta^{-1}$. This partially justifies our charge neutrality assumption. Also, since $u\approx 1$, $u_\perp{}^\pm\approx\pm(1-\beta^{-2})^{1/2}$, and current equality is satisfied. Since $p\gg 1$ for most of η, Eq. (3a) becomes independent of rest mass (for the real momentum $mcp_\perp$). The wave equation for $\vec{E}$ may now be integrated:

$$\frac{\partial\vec{E}}{\partial\eta}=-\frac{8\pi enc}{\Omega}\frac{\beta^2}{\beta^2-1}\vec{u}_\perp$$

$$\approx-\frac{8\pi enc}{\Omega}\left(\frac{\beta^2}{\beta^2-1}\right)^{1/2}\hat{e}_\perp\,\text{sgn}p_\perp, \quad (5)$$

where $\vec{u}_\perp\approx e_\perp(1-\beta^{-2})^{1/2}\,\text{sgn}p_\perp$ has been used. This solution is sketched by Max and Perkins.[3] Since $|\partial\vec{E}_\perp/\partial\eta|\approx 2E_{\max}/\pi$, Eq. (5) yields the dispersion relation

$$\nu^2\Omega^4/\pi^2\omega_p{}^4=\beta^2/(\beta^2-1), \quad (6)$$

where $\nu\equiv eE_{\max}/mc\Omega\gg 1$. Equation (6) written in terms of the particle flux $\omega_c{}^2=\omega_p{}^2/\beta$ is

$$\beta^2=\tfrac{1}{2}\lambda^{-2}[1\pm(1-4\lambda^2)^{1/2}];\quad \lambda=\pi\omega_c{}^2/\nu\Omega^2. \quad (7)$$

There is a maximum particle flux $E_{\max}\Omega/8\pi^2 e$ above which no electromagnetic wave can propagate. From Eq. (3a) one finds $p_{\perp\max}=\frac{1}{4}\nu\pi$; hence $p_{\max}=\frac{1}{4}\nu\pi[\beta^2/(\beta^2-1)]^{1/2}$, while from Eq. (5)

$$E_{\max}=(4\pi^2 enc/\Omega)[\beta^2/(\beta^2-1)]^{1/2}. \quad (8)$$

The particle energy flux is

$$\varphi_p\approx\frac{2n\langle p\rangle mc^3}{\beta}=\frac{4}{3}\frac{np_{\max}mc^3}{\beta}=\frac{\Omega^2m^2c^3\nu^2}{12\pi e^2\beta}, \quad (9)$$

where Eq. (6) has been used. The electromagnetic Poynting vector is

$$\varphi_{em}=(c/4\pi\beta)\langle E^2\rangle=(c/12\pi\beta)E_{\max}{}^2=\varphi_p=\varphi. \quad (10)$$

Hence in this approximation the electromagnetic and particle energy fluxes are equipartitioned.

We attempt now to justify neglect of charge separation, by estimating a pessimistic upper bound. Consider the integrated form of (1f), which may be written using (2b) as

$$n_e=\frac{N_0}{1-(\beta^2-1)^{-1}[-\psi(\eta)+K^-][\gamma_e(\eta)]^{-1}}, \quad (10a)$$

$$n_i=\frac{N_0}{1-(\beta^2-1)^{-1}[(m_e/m_i)\psi+K^+][\gamma_i(\eta)]^{-1}}, \quad (10b)$$

where $K^\pm\equiv\gamma_{\text{inj}}{}^{i,e}(u_{\parallel\,\text{inj}}{}^{i,e}-1)$. $\psi(\eta)$ has period π, and $\psi(0)=\psi(\pi)$ can be chosen to vanish. By symmetry, $E_\parallel$ vanishes at $\eta=0,\pi$ $[(d\psi/d\eta)_{\eta=0,\pi}=0]$. Any residual $E_\parallel$ within $0<\eta<\pi$ therefore arises from charges near the nodes. There are charge-boundary layers within the intervals $0<\eta<\Delta\eta$ and $\pi-\Delta\eta<\eta<\pi$, and compensating charges in $\Delta\eta<\eta<\pi-\Delta\eta$ and the maximum possible charge density can be estimated from (10). The maximum ion density at the nodes corresponds to relativistic injection, or $u_{\parallel \text{inj}}{}^{i} \cong 1$, and $n(\eta = 0) = N_0(\beta + 1)/\beta$. If n_e is taken everywhere uniform, $\delta n_{\max} = N_0/\beta$. $\Delta\eta$ can be defined, using (10b), as the point where the charge increment falls to some fraction of its injection value, say

$$\frac{1}{\beta^2 - 1}[\psi(\Delta\eta) + K^+]\frac{1}{\gamma(\Delta\eta)} = \frac{1}{2}\frac{1}{\beta^2 - 1}\frac{K^+}{\gamma_{\text{inj}}} = \frac{1}{2}\frac{1}{\beta + 1}.$$

It is readily shown (*a posteriori*) using Poisson's equation that $|\psi(\eta)| \ll |K|$ in this region, and $\Delta\eta$ can therefore be determined from the equation $\gamma(\Delta\eta) \cong 2\gamma_{\text{inj}}$. Near $\eta = 0$,

$$\gamma(\eta \cong 0) \cong \gamma_{\text{inj}}\left[1 + \frac{1}{2}\frac{\beta}{\beta - 1}\frac{\nu^2}{\gamma_{\text{inj}}{}^2}\eta^2 + O(\eta^4)\right];$$

therefore,

$$\Delta\eta \cong [2(\beta - 1)/\beta]^{1/2}\gamma_{\text{inj}}/\nu.$$

If we assume a constant maximum charge density N_0/β near the nodes, and the compensating density $(2\Delta\eta/\pi\beta)N_0$ elsewhere, then $\psi(\eta)$ is specified by matching $\psi(\Delta\eta)$ and $d\psi(\Delta\eta)/d\eta$ at the boundaries. The net result is

$$\psi(\eta) \cong \psi_0 - \frac{\beta}{\pi}\frac{\omega_e{}^2}{\Omega^2}\Delta\eta\left(\eta - \frac{\pi}{2}\right)^2,$$

where $\psi_0 \equiv (\pi/4)\beta\,\omega_e{}^2/\Omega^2$. In the central region $\Delta\eta < \eta < \pi - \Delta\eta$,

$$\gamma \simeq \left(\frac{\beta^2}{\beta^2 - 1}\right)^{1/2}|p_\perp| \simeq \left(\frac{\beta^2}{\beta^2 - 1}\right)^{1/2}\left[\frac{\pi}{4}\nu - \frac{\nu}{\pi}\left(\eta - \frac{\pi}{2}\right)^2\right],$$

$$\left|\frac{\psi(\eta)}{\gamma(\eta)}\right| \simeq \left(\frac{\beta^2}{\beta^2 - 1}\right)^{1/2}\frac{\omega_e{}^2}{\Omega^2\nu} = \frac{\beta^2 - 1}{\pi\beta}\left[\frac{2(\beta - 1)}{\beta}\right]^{1/2}\frac{\gamma_{\text{inj}}}{\nu}.$$

In other words the scale $\Delta\eta$ of the nodes, where particle energies are near their injection values, determines the magnitude of charge separation effects. Similarly, the equality of ion and electron current magnitudes fails only within $\Delta\eta$ near the nodes. Since $\nu \simeq 10^{11}$ for pulsars, our approximations seem justified.

For pulsars, we assume that scaling plane-wave solutions to spherical geometry yields reasonable dimensional estimates. Since the average total energy of a particle in an electromagnetic wave with an intensity gradient is an adiabatic invariant,[4] particles ejected by a complicated wave source will probably convert their total energy into direct motion. Because the superrelativistic approximation used above *requires* a net transport by the wave, the wave solutions are naturally expressed in parameters pertinent to pulsar physics, the total energy-loss rate L, which may be estimated from the observed rates of increase of pulsar periods, and the total particle injection number flux J, which is determined by conditions at the pulsar surface and is essentially unknown. Physically, we expect particles to be injected into a complex near wave zone beyond the light cylinder at $r = c/\Omega$, whose structure is difficult to understand, but that a few light-cylinder radii away, the wave will settle into a self-consistent form. Thereafter, the wave luminosity $L = 4\pi R^2(c/\Omega)^2(\varphi_p + \varphi_{\text{em}})$ and injection flux $J = 4\pi R^2(c/\Omega)^2 j$ parametrize the wave. R is the radial distance in units of c/Ω.

Our basic results are simple. If the plasma density in the wave is too large, the wave will not propagate. The critical density for cutoff near the light cylinder is roughly that given by Goldreich and Julian[5] for the onset of hydromagnetic behavior. More explicitly, there is maximum J_c, for a given L, which can be transported by the wave,

$$J^2 < \frac{3\sqrt{2}}{2\pi^2}\frac{cR^2}{e^2}L = J_c{}^2. \tag{11}$$

For $J > J_c$, a hydromagnetic solution must be found; for $J < J_c$ a wave solution can apply. Depending upon injection conditions, two distinct wave modes are possible, corresponding to the two branches of the dispersion curve, where

$$(2\varphi)^{1/2} = \lambda[1 \pm (1 - 4\lambda^2)^{1/2}]^{1/2},$$

$$\varphi = 2\pi^2 e^2 cJ^2/\Omega^2 R^2 L.$$

For the upper sign, β scales asymptotically with R; the wave energy and particle densities also increase with radial distance, and the transport velocities approach zero. On the lower branch, the wave approaches a vacuum state ($\beta \to 1$) as $R \to \infty$. Here, equipartition of particle and energy fluxes implies the maximum particle energy $\epsilon_{\max} = p_{\max}mc^2$ attained in the wave is independent of R. Since $p_{\max} = \nu(\pi/4)[\beta^2/(\beta^2 - 1)]^{1/2}$ and $E_{\max} = (8\pi^2 enc/\Omega)[\beta^2/(\beta^2 - 1)]^{1/2}$, $p_{\max} = (1/32\pi)E_{\max}{}^2/nmc^2$. The number density in the wave n is related to the total particle injection flux density, $j = 2n_i c$, by $n = j\beta/2c$ whereupon

$$p_{\max} = \frac{1}{8\pi mc^2}c\frac{E_{\max}{}^2}{jmc^2} = \frac{3}{4}\frac{\varphi_p + \varphi_{\text{em}}}{jmc^2},$$

$$\epsilon_{\max} = \tfrac{3}{4}L/J = 4.7 \times 10^{11} L/J \text{ eV}.$$

For the Crab pulsar ($L = 10^{38}$ erg/sec, $\Omega = 200$ rad/sec), $J_c \approx 1.7\times10^{33}$ particles/sec and $\epsilon_{max} \approx 2.8 \times10^{16}(J_c/J)$ eV. Thus, for a given L, a larger flux of lower energy or smaller flux of higher-energy cosmic rays may be obtained, depending on J. In either case, 50% of the pulsar wave luminosity is converted into cosmic rays of energy less than ϵ_{max}. Within the superrelativistic approximation, these conclusions may be valid for the relativistic injection of neutral plasma of any composition.

Care must be exercised in drawing far-reaching quantitative conclusions from our plane-wave model. Dependences on the azimuthal and polar angles in spherical geometry have been ignored. Furthermore, for a plane wave to be a good approximation, the radial excursion D of a particle during a half-period $\eta = \pi$, $D = (\pi c/\Omega)\beta/(\beta^2 - 1)$, should be smaller than the radial scale length. This is only marginally satisfied at the injection radius and even less well at larger radii. Clearly, investigation of the spherical wave is called for.

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Real Part of the Proton-Proton Forward-Scattering Amplitude from 50 to 400 GeV

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From measurements of proton-proton elastic scattering at very small momentum transfers where the nuclear and Coulomb amplitudes interfere, we have deduced values of ρ, the ratio of the real to the imaginary forward nuclear amplitude, for energies from 50 to 400 GeV. We find that ρ increases from -0.157 ± 0.012 at 51.5 GeV to $+0.039\pm0.012$ at 393 GeV, crossing zero at 280 ± 60 GeV.

We have determined the ratio $\rho(E)$ of the real to the imaginary part of the forward proton-proton elastic nuclear scattering amplitude for incident energies from 50 to 400 GeV. The measurements were performed at the National Accelerator Laboratory by observing wide-angle recoil protons from an internal hydrogen gas-jet target. Elastic scattering was studied in the range from $|t| = 0.001$ (GeV/c)^2, which is well inside the Coulomb region, to $|t| = 0.04$ (GeV/c)^2, where the nuclear interaction dominates. The ratio ρ was determined from the strength of the interference between the nuclear and Coulomb amplitudes in the t region where they are comparable, $|t| \sim 0.002$ (GeV/c)^2.

Previously, ρ was measured at energies up to 70 GeV at Serpukhov in an experiment similar to the one reported here.[1] Recently, the CERN-Rome collaboration at the CERN intersecting storage rings (ISR) reported[2] ρ to be $+0.02\pm0.05$ at 290 GeV and $+0.03\pm0.06$ at 500 GeV (lab equivalent energy). We find that ρ increases from -0.157 ± 0.012 at 51.5 GeV to $+0.039\pm0.012$ at 393 GeV, crossing zero at 280 ± 60 GeV.

Our experimental method makes use of the fact that the kinetic energy T of the recoil proton from elastic p-p scattering is directly related to the momentum transfer through $|t| = 2mT$, where