Exact Finite-Range Distorted-Wave Born-Approximation Calculation for the Reactions ^{208}Pb(^{16}O, ^{15}N)^{209}Bi and ^{88}Sr(^{16}O, ^{15}N)^{89}Y†

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We develop a straightforward, fast approach to exact finite-range distorted-wave Born-approximation calculations, and apply it to the analyses of the reactions ^{208}Pb(^{16}O, ^{15}N)-^{209}Bi and ^{88}Sr(^{16}O, ^{15}N)^{89}Y.

It has long been believed that to carry out exact finite-range (EFR) distorted-wave Born-approximation (DWBA) calculations[1] is rather impractical, since they are very time consuming. Thus for light-ion reactions usually the zero-range approximation is made, while for heavy-ion reactions the no-recoil (NR) approximation[2,3] has often been made. However, in most practical experimental situations the criterion for the validity of the NR approximation, which may be written as $(x/a)k_aR_a \ll 1$, is violated. Here we consider a reaction of the type $A(a, b)B$, where $a = b + x$ and k_a is the wave number in the incident channel, while R_a is the radius of the nucleus a; in the statement of the criterion, x and a represent the corresponding masses.

Recent work[4,5] in which EFR calculations were applied to cases with comparatively light targets showed that indeed the recoil effects cannot be neglected. Unfortunately, it appears that these calculations required rather lengthy computing time as was expected, and thus it did not seen easy to carry out such calculations extensively, particularly with heavy targets. We have, however, recently discovered a technique[6] which allows one to carry out EFR calculations rather fast. A corresponding computer program SATURN-MARS has also been completed, and the purpose of the present article is to present some of the results of our calculations.

Recently Larsen *et al.*[7] and Kovar *et al.*[8] reported experiments on one-nucleon transfer reactions with ^{208}Pb as target bombarded by ^{16}O and/or ^{12}C, and in particular the latter authors analyzed their own data on (^{16}O, ^{15}N) as well as (^{12}C, ^{11}B) reactions by using a NR computer program.[3] The spectroscopic factors S_{NR} they extracted are reproduced in our Table I, and as is seen (i) for $j = l - \frac{1}{2}$ $(= l + \frac{1}{2})$ they are much larger (smaller) then unity, the value one expects to obtain,[9] and (ii) they also are strongly dependent on E_L, the incident energy of ^{16}O in the laboratory system.

Although it was further shown[8] that this unsatisfactory situation was remedied to a large extent by carrying out calculations[10] which included the recoil effect to the first order in the ratio x/a, still a rather strong E_L dependence remained. Even if this type of calculation had given $S \cong 1$ all the way, the result would still be suspicious be-

TABLE I. Results of the EFR calculations for the reaction ^{208}Pb(^{16}O, ^{15}N)^{209}Bi.

			E_L = 104 MeV				E_L = 140 MeV				
Ex (MeV)	n&j	L,L'	S_{EFR}	S_{NR}	$\sigma_{L'}/\sigma_L$	time for DWBA(sec)	S_{EFR}	S_{NR}	$\sigma_{L'}/\sigma_L$	time for DWBA(sec)	time for form factor (sec)
0.00	$1h_{9/2}$	4,5	1.05 (1.50)	3.04	0.14	77(+43)	1.00	3.84	0.20	87(+43)	71(+63)
0.90	$2f_{7/2}$	4,3	0.82 (1.03)	0.80	0.03	65(+37)	0.70	0.48	0.08	79(+37)	47(+57)
2.84	$2f_{5/2}$	2,3	0.75 (0.88)	3.20	0.10	50(+30)	0.57	3.20	0.17	60(+30)	39(+57)
3.12	$3p_{3/2}$	2,1	0.65 (0.68)	0.92	0.02	28(+26)	0.55	0.48	0.06	30(+26)	31(+45)
3.64	$3p_{1/2}$	0,1	0.50 (0.50)	2.80	0.15	16(+20)	0.58	4.80	0.34	20(+20)	25(+40)

cause the first-order correction to the zeroth-order NR calculation is seen to be so large that higher-order corrections would further modify the theoretical cross section significantly. In other words, calculations that include the correction to infinite order are needed, which means one has to carry out EFR calculations, as we did.

The EFR cross sections we obtained are shown as solid lines in Fig. 1 and are compared with experiment.[8] The optical and bound-state parameters used are given in the caption of this figure, and are the same as in Ref. 8. As is seen, the angular distribution agrees well with experiment for $E_L = 140$ MeV, but not for 104 MeV, the discrepancy getting larger the lower the spin j, i.e., the higher the excitation energy of the final state in ^{209}Bi. Note that this feature is very much the same as in the NR calculations.[8] However, the extracted spectroscopic factor S_{EFR} differs greatly from S_{NR}, as is seen in Table I. Our S_{EFR} is essentially E_L independent, is close to unity for $j = \frac{9}{2}$, and decreases smoothly with decreasing j to 0.5 for $j = \frac{1}{2}$. This feature is at variance with a previous (^{3}He, d) study[9] which gave $S \cong 1$ for all j. However, it should be noted that lower-j states are highly excited states, and thus are likely to lose more of their single-particle nature. In fact a recent structure study predicted $S(\frac{9}{2}) \cong 1$ and $S(p_{1/2}) \cong 0.5$.[11] It thus appears that our results are very reasonable, showing that an EFR calculation removes all the difficulties of the NR calculations as enumerated above.

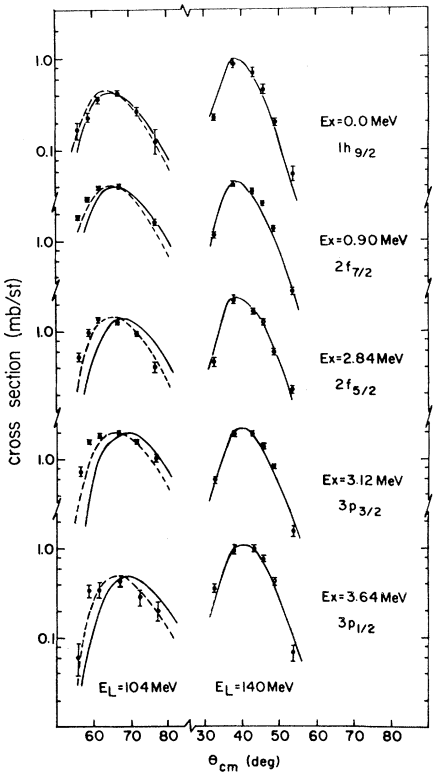

FIG. 1. Comparison of the EFR calculation with the data of the reaction ^{208}Pb(^{16}O, ^{15}N)^{209}Bi of Ref. 8. Optical potentials (Woods-Saxon) used are $V = -40$ MeV, $W = -15$ MeV, $R = 1.31(A_1^{1/3} + A_2^{1/3})$ fm, $a = 0.45$ fm, except for 140 MeV where $V = -30$ MeV. Bound-state wave functions are obtained using separation energy with $r_0 = 1.28$ fm, $a = 0.76$ fm, and $V_{s.o.} \sim 6$ MeV for ^{209}Bi, and $r_0 = 1.2$ fm, $a = 0.65$ fm and $V_{s.o.} = 0$ for ^{16}O.

In comparing EFR and NR calculations, it should be noted that the EFR cross section σ(EFR) can be written as a sum of σ_L and $\sigma_{L'}$, the contributions of components with transferred orbital angular momentum L and L' of a natural- and unnatural-parity nature, respectively. The values of L and L' are listed in Table I, which also gives the ratio $\sigma_{L'}/\sigma_L$. Since this ratio,[12] which vanishes for NR calculations, is comparatively small even for EFR calculations, it is seen that the large difference between S_{EFR} and S_{NR} must arise from the difference between σ_L(EFR) and σ_L(NR); and in fact, we get, e.g., for the 104-MeV case, σ_L(EFR)/σ_L(NR) = 5.0, 1.4, 3.8, 1.0, and 2.6 for $j = \frac{1}{2}$ through $\frac{9}{2}$. This example shows how dangerous the NR calculation can be, even when $\sigma_{L'}/\sigma_L$ as estimated with EFR calculations is rather small.

For $E_L = 104$ MeV, we show in Fig. 1 as dashed lines the cross sections obtained by taking $r_0 = 1.35$ fm for the ^{15}N channel, every other parameter being the same as in Ref. 8. The angular distribution is now seen to agree with experiment very well. However, the corresponding S_{EFR}, given in parentheses in Table I, deviates from the values obtained previously, the more so the larger is j. In order to make an unambiguous analysis, it is highly desirable to have data for the scattering of ^{15}N.

The computer time needed in performing the above calculations on the IBM 360/195 computer at Argonne National Laboratory is given in Table I. The first figure is the central processing unit time, while the figure in parentheses is the accumulated access time to the internal input/output devices. As is seen, the calculations have been done rather fast. For calculations with smaller targets and lower E_L the needed machine time is still further reduced, being in many cases a small fraction of a minute.

We also analyzed data for the reaction[13] ^{88}Sr(^{16}O,

^{15}N)^{88}Y since in this experiment the cross sections were taken with a large number of different E_L, as is seen in Fig. 1 of Ref. 13. The extracted spectroscopic factor in the NR calculations[13] showed a sizable increase with increased E_L for the $p_{1/2}$ final state at $E_x = 0.91$ MeV, though it was essentially independent of E_L for the $g_{9/2}$ ground state. From the fit to the data, which is very much the same as in Fig. 1 of Ref. 13, we extracted S_{EFR} values which were found to be independent of E_L. The obtained values of S_{EFR}, 1.66 and 1.75, respectively, for the $g_{9/2}$ and $p_{1/2}$ states, are too large compared with the corresponding values of 0.88 and 0.90 obtained from (^{3}He, d) work,[14] but this is likely a reminder of the need for further search of bound-state geometry.

The examples we chose and analyzed above are those for which the spectroscopic information had been known from light-ion reactions.[9,14] Therefore our emphasis has so far been to test whether heavy-ion reaction data can in fact be used as a spectroscopic tool, rather than to extract new information on nuclear spectroscopy. In this sense our results are very encouraging, particularly because it was confirmed that S_{EFR} can be extracted independent of E_L. Although there clearly exists a need for more parameter searches, our results are again encouraging since it was also confirmed that EFR calculations can be made rather fast, though certainly more involved than NR calculations.

From the point of view of nuclear spectroscopy, the real importance of the heavy-ion reaction lies in its capability of inducing multinucleon transfer reactions. Our technique can of course be applied to those cases too. Another important feature is that the projectile and/or outgoing particle can be excited with comparative ease, which will make it more important than in light-ion reactions to treat the process in terms of coupled-channel Born approximation (CCBA) rather than of DWBA.[6] Our program can also carry out an EFR CCBA. Results of various DWBA and CCBA calculations now under way along these lines, together with details of the present calculations, will be reported elsewhere in the near future.

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