ceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, New York, 1972), p. 264. We ignore the possible ϵ dependence of the virtual-photon cross section over the range of this experiment.

[3]The experimental aperture covers photon azimuths φ within about 30° of the electron scattering plane on the side of the virtual-photon line opposite from the outgoing electron. In a separate experiment (to be described in detail elsewhere), using a scintillator hodoscope to detect recoil protons on both sides of the beam, we have established that the azimuthal asymmetry is zero within 15%. We will therefore integrate over φ assuming no φ dependence.

[4]The zero-missing-mass yields are consistent with known radiative ep scattering cross sections and estimates of π^0 electroproduction. An estimate of the two-photon radiative tail of the one-photon peak is included in the fitting, but is generally too small to be significant.

[5]The continuum is fitted with $a(m_x^2 - 4m_\pi^2)^{1/2}$; higher-order terms do not affect the vector-meson results. The continuum yield (Fig. 1 is typical) has a generally smooth dependence on the kinematic variables and a flatter t dependence than ρ^0 and ω production. It is similar to the proton missing-mass continuum in photoproduction (G. E. Gladding *et al.*, to be published; we are indebted to M. J. Tannenbaum for sending us the data before publication), and is presumably a mixture of $\Delta\pi$ and nonresonant $p\pi\pi$ and $p\pi\pi\pi$ final states.

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Transverse-Momentum Distribution of Pions in High-Energy pp Collisions*

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The inclusive π^0 transverse-momentum distribution at 90° in pp collisions of c.m. energy 53.4 GeV is shown to be described over 10 orders of magnitude by a Gaussian distribution in the transverse rapidity variable $y_\perp = \tanh^{-1}(p_\perp/E)$.

The slow falloff[1] of the inclusive cross section for $pp \to \pi^0 X$ for extremely high-momentum pions at 90° in the c.m. frame has recently attracted considerable interest. The purpose of this Letter is to show that these cross sections, which vary by 10 orders of magnitude in the range $0 < p_\perp < 10$ GeV/c at total c.m. energy $\sqrt{s} = 53.4$ GeV/c, are described by the simple formula

$$E(d^3\sigma/d^3p)_{90°} = A \exp(-y_\perp^2/2L_\perp) \quad \text{mb}/(\text{GeV}/c)^2\ \text{sr}, \tag{1}$$

where $y_\perp$ is the transverse rapidity

$$y_\perp = \tfrac{1}{2}\ln[(E+p_\perp)/(E-p_\perp)], \tag{2}$$

and the parameters A and $2L_\perp$ are equal to 300 and 1.028, respectively. This formula is intended to apply at 90° in the c.m. frame, i.e., at $x = 0$, with x the usual kinematic variable $x = 2p_\parallel/\sqrt{s}$. Figure 1 shows the data of Ref. 1 plotted as a function of $y_\perp^2$. We have supplemented these results by some lower $p_\perp$ results[2,3] at a nearly identical energy and small nonzero x. The agreement over such an enormous range of values of

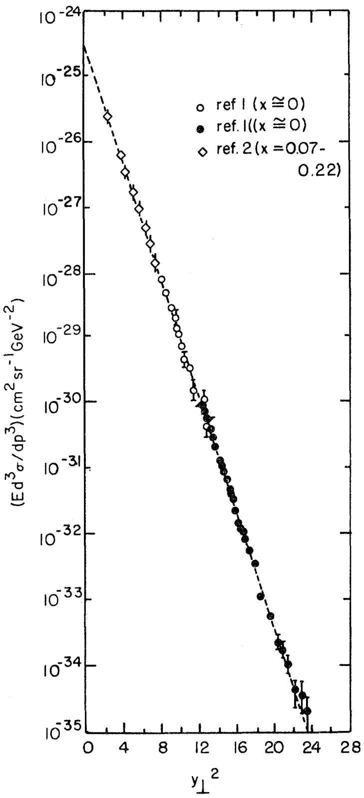

FIG. 1. The inclusive cross section for $pp \to \pi^0 X$ is shown as a function of $y_\perp$ at $\sqrt{s} = 53.4$ GeV. The dashed curve is the prediction of Eq. (1) for $A = 300$, $2L_\perp = 1.028$.

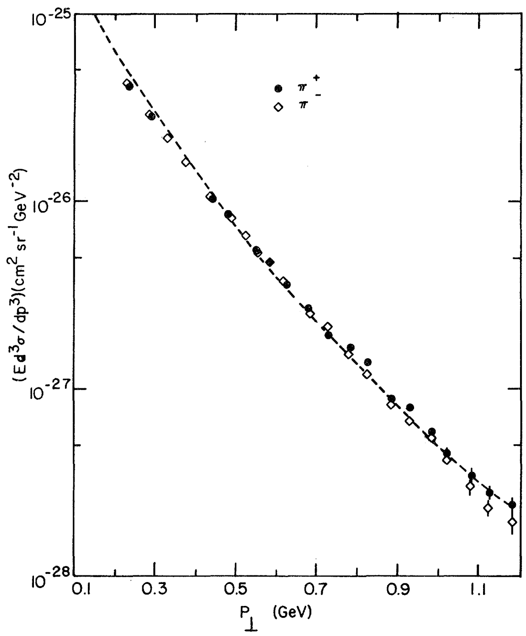

FIG. 2. The inclusive cross section (Ref. 2) for $pp \to \pi^\pm X$ is shown as a function of $p_\perp$ at $\sqrt{s} = 52.7$ GeV and $x = 0$. The dashed curve is the prediction of Eq. (1) for $A = 200$, $2L_\perp = 1.203$.

the cross section is impressive.

In previous papers[4,5] we have suggested that the ordinary (longitudinal) rapidity $y_\parallel = \frac{1}{2}\ln[(E+p_\parallel)/(E-p_\parallel)]$ distributions are Gaussians whose widths are given by Landau's hydrodynamical model.[6] That width, $2L_\parallel = \ln(s/4m_p^2)$, is determined by the thickness of the Lorentz-contracted proton. Symmetry considerations led us to guess formula (1) for the 90° case with a width determined by a constant or very weakly energy-dependent (uncontracted) proton radius R. $L_\parallel$ may be written as $L_\parallel \cong \ln(R/\Delta)$, where Δ is of the order of R/γ, $\gamma = E_{\text{c.m.}}/m_p$. In the transverse direction we expect Δ to be order $\frac{1}{2}R$, giving $2L_\perp \approx 2\ln 2 \approx 1.4$ in contrast with the empirical value 1.028. These heuristic remarks are not intended to be a substitute for a genuine derivation of Eq. (1).

Since $y_\perp^2$ increases rapidly with increasing $p_\perp$, the cross-section formula (1) is exceedingly sensitive to the parameter $L_\perp$ for large $p_\perp$. A change of $L_\perp$ by a few percent changes the cross section by perceptible amounts, e.g., by a factor of 7 for $y_\perp^2 = 20$ and a 10% change in $L_\perp$. Accurate data at various energies are needed to determine the possible energy dependence of $L_\perp$.

The rapidity Gaussians are characteristic of the hydrodynamic model. At very small values of $p_\perp$ it is possible that thermal fluctuations will predominate and that Eq. (1) will fail for $p_\perp \lesssim m_\pi$. [Note that (1) is Gaussian in $p_\perp$ for very small $p_\perp$.] It will be of interest to see how well our formula works for moderate $p_\perp$ since (1) has some interesting fine structure as a function of $p_\perp$ or $p_\perp^2$. Apparently there are no 90° results for $p_\perp < 0.1$; for $p_\perp < 1$ GeV/c we have had to use slightly noncentral data[2] having $x = 2p_\parallel/\sqrt{s}$ in the range $0.07 < x < 0.22$.

We now turn to the available charged-pion data at CERN'S intersecting-storage-ring energies. Figure 2 shows the $\pi^\pm$ data (combined) from Banner *et al.*[7] as a function of $p_\perp$. The dashed curve is the prediction of Eq. (1) for the $p_\perp$ distribution, but with somewhat different parameters: $A = 200$, $2L_\perp = 1.203$. This difference from the π^0 case could be due to either (a) a genuine difference between charged and neutral pions, (b) some energy or $p_\perp$ dependence not described by (1), (c) experimental inaccuracies, or (d) our problems in transcribing data from log plots of small size. A point to note is that there is a slight change of slope around $p_\perp = 0.6$ GeV/c, which is the first sign of the slow falloff at large $p_\perp$. Perhaps it is worth mentioning that such behavior was observed some time ago[8,9] in the $p_\perp$ distributions of γ rays produced in cosmic-ray jets.

Further experimental work is necessary to validate the remarkably simple formula (1) in the small-$p_\perp$ region. It is also important to study the energy dependence of the very large-$p_\perp$ distributions. Our results suggest that a single mechanism, possibly hydrodynamical in nature, is in operation over the entire range of momentum transfers investigated.

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Light-Cone Dominance in Inclusive e^-e^+ Annihilation*

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It is argued that light-cone dominance ideas are compatible with canonical scaling in the process $e^-e^+ \to$ hadron + anything and a logarithmic increase in hadronic multiplicity.

Motivated by Bjorken scaling[1] and by Wilson's short-distance expansions,[2] light-cone expansions of products of pairs of operators were introduced.[3-5] These were also generalized to products of more than two operators,[6,7] in order to discuss inclusive e^-e^+ annihilation and coincidence electroproduction. There has been some discussion whether light-cone dominance in $e^-e^+ \to \gamma \to$ hadron + anything is compatible with scaling and a logarithmic increase in hadronic multiplicity. Callan and Gross[8] showed that if the term multiplying the leading light-cone singularity were regular at short distances, then the e^-e^+ multiplicity would be finite. It was then shown that the regularity does not occur in superrenormalizable or softened field theories,[9] but that the short-distance singularity is not sufficient to make the multiplicity logarithmic. Recently Fritzsch and Minkowski[10] have argued that if the term multiplying the leading light-cone singularity were sufficiently singular at short distances to make the multiplicity logarithmic, then it would also violate scaling. In fact it would also violate the spectral conditions. On the other hand, the parton model[11] can accommodate both a logarithmic multiplicity and scaling.

In this paper we argue that light-cone dominance yields scaling in one-particle inclusive e^-e^+ annihilation and is also compatible with a logarithmic increase in multiplicity. Let p be the four-momentum of the observed hadron, and x the space-time distance between the coordinates of the two electromagnetic currents in the expression for the cross section. Then the logarithmic increase is obtained by a certain singularity in $p \cdot x$. The scaling, however, is not