Explanation of Rising Multiplicity with Increasing Perpendicular Momentum Transferred to the Leading Proton, Using the Quark Multiple-Scattering Model

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It is shown that the rise in multiplicity with increasing perpendicular momentum transferred to the leading proton seen in recent experiments can be simply explained as due to double quark-quark scattering in the Glauber quark multiple-scattering model. The quark-quark inelastic-scattering amplitude is entirely consistent with the quark-quark elastic amplitudes obtained previously for elastic proton-proton scattering.

In a recent experiment, Ramanauskas *et al.*[1] have studied multiplicity as a function of $p_\perp$, the perpendicular momentum transferred to the leading proton. The reaction specifically studied was

$$p+p \to p + \text{particles},$$

where the leading proton was detected and its momentum determined. All other charged particles were observed and their momenta measured as well. The results, over a range of values for the effective mass of the particles other than the leading proton, show that the multiplicity is roughly constant with $p_\perp$ till a value of $p_\perp = 0.65$ GeV/c at which point it rises sharply by ~ 0.6, followed by indications for a leveling off at $p_\perp \sim 1.0$. We present here an explanation for this in terms of the multiple-quark-scattering model.

Both the low- and high-energy proton-proton elastic scattering data are found to be fitted well using the Glauber model by considering the proton as a composite particle of three quarks. The first calculations for low-energy data were done by Harrington and Pagnamenta,[2] and calculations on data from the CERN intersecting storage rings were done by the authors.[3] In this model, the behavior in the small-momentum-transfer region $|t|<0.7$ (GeV/c)^2 (i.e., the forward slope region) is dominated by the process of a single quark in one proton scattering off of a single quark in the other proton. The behavior in the higher-momentum-transfer regions is determined by higher-order quark scatterings. The various scattering processes to all orders for elastic scattering are shown schematically in Fig. 1(a) where the dots represent quarks and the lines represent quark-quark scatters. Figure 1(b) shows the contributions to the elastic differential cross section $d\sigma/dt$ due to first-, second-, third-, etc. order scattering. It is seen that second-order scattering is equal to first-order scattering at $|t| \simeq 0.7$ (GeV/c)^2 and exceeds it for larger $|t|$. Higher-order quark-quark scattering terms become predominant at even larger $|t|$ values.

We may now consider inelastic scattering to occur in the same way as elastic. Inelastic scattering resulting in single-particle production has previously been treated formally in the Glauber model by Dean.[4] Instead of a line we draw an oval to represent an inelastic process in Fig. 1(a) resulting in pions in the final state. The amplitude for producing pions due to a single quark-quark collision depends primarily on the c.m. energy and not the momentum transfer, since we may think of most of the energy of collision of the quarks as due to motion along the beam direction.

The various lower-order diagrams that contribute to inelastic processes along with a schematic representation of the calculation are shown in Fig. 2. The number of distinct diagrams of each type is given by the number next to each diagram. Notice that first-order pion production processes can occur in what, for elastic scattering, were only second-order diagrams. All the amplitudes for the inelastic pion production are incoherent and therefore the sum of the squares of the amplitudes of each type is taken as giving the differential cross section for proton inelastic scattering. In order to obtain inelastic cross sections for various multiplicities we may multiply each diagram by a product of Poisson distributions which take into account the inelastic pion

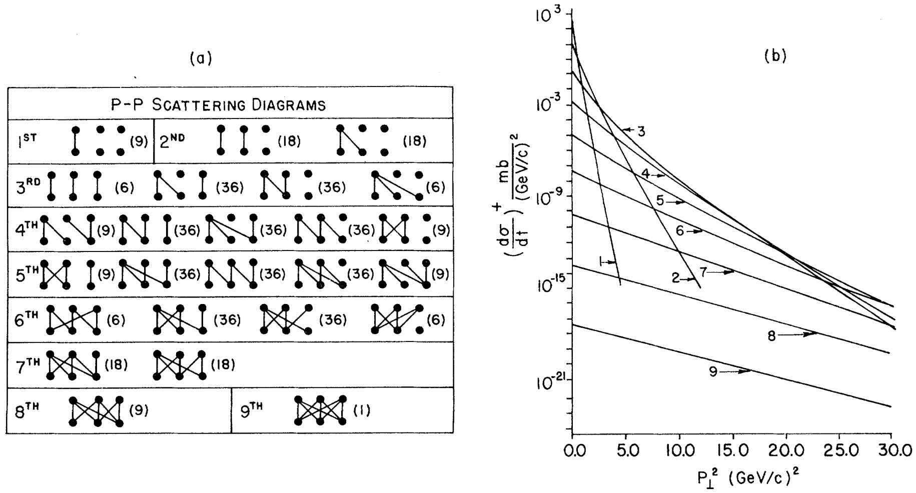

FIG. 1. (a) All possible types of elastic multiple scatters that can occur between two protons composed of three identical quarks. Also shown next to each diagram is the number of distinct ways the scattering can occur. The number of lines in a diagram is the order of the scatter. (b) The contributions of each order of scatter to elastic p-p scattering taken separately is shown for all nine orders.

production distributions due to *single* quark-quark inelastic scattering production.

Here again we parametrize the quark-quark inelastic amplitude in the form $f(0)\exp(-B^2p_\perp^2)$ $\times$(unspecified inelastic factor independent of $p_\perp$) with the same values [in $(\mathrm{GeV}/c)^{-2}$] we used in the elastic case:

$$\mathrm{Im}B^2 = -47 + 13.1\ln(s), \quad \mathrm{Re}B^2 = 0.01,$$

$$\mathrm{Re}f(0) = 0.05, \quad \mathrm{Im}f(0) = 0.9,$$

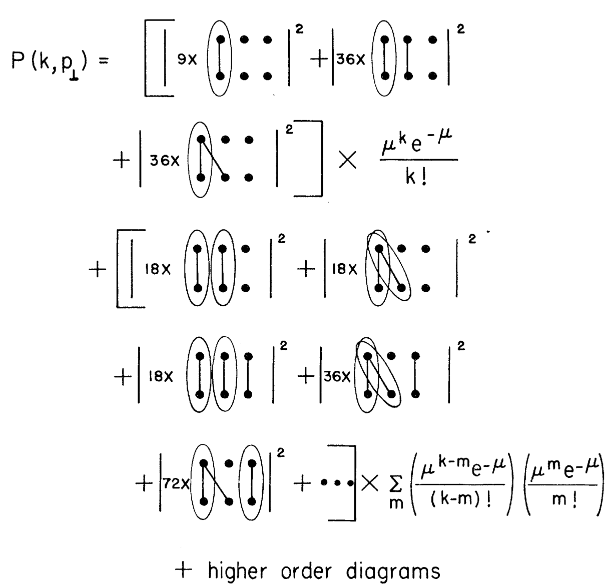

FIG. 2. Inelastic-scattering diagrams along with a schematic representation of the calculation.

where s is the usual c.m. energy squared and is calculated at a proton lab momentum of 28.5 GeV/c, the momentum of the experiment. The proton form factor parameter A_n is also the same as in our previous calculation,

$$A_n^2 = 2.27 + 0.5\ln(s).$$

Any other parametrization which gives a first-order quark-quark scattering term dropping below the second-order term at the appropriate t value could be used to reproduce our final results. If there is only a single inelastic quark-quark scattering process, we obtain the probability for producing k pions due to that process by multiplying by the term $\mu^k e^{-\mu}/k!$, where there is a mean number of μ pions produced in a quark-quark collision. This term takes into account the production mechanism described by the inelastic factor and the phase space available to produce pions in single quark-quark collisions. Consequently, μ should be an increasing function of both the incoming particle energy and the effective mass of the system of particles other than the leading proton. This function can be determined by looking at the multiplicity at small values of $p_\perp$ where only single inelastic scattering occurs.

For the case where there are two inelastic collisions, we must multiply by a term which sums over all the possible ways to produce k pions. In order to produce k pions, we must produce m pions in one collision and k-m pions in the other

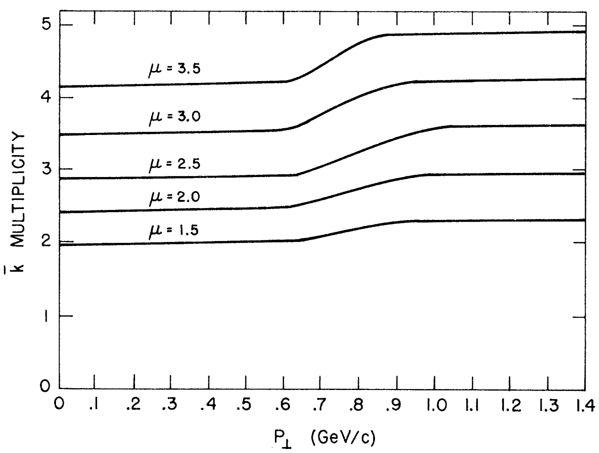

FIG. 3. Multiplicity as a function of $p_\perp$ for the Poisson distribution parameter $\mu = 1.5-3.5$.

collision. The sum of the probabilities for producing k pions in *two* collisions is the double-order Poisson factor

$$\sum_m \frac{\mu^{k-m}e^{-\mu}}{(k-m)!}\frac{\mu^m e^{-\mu}}{m!}.$$

Thus, summarizing what is shown schematically in Fig. 2, to obtain the differential cross sections for producing k pions we (1) multiply the sum of the squares of the amplitudes for first-order inelastic pion scattering by the single-order Poisson factor for producing k pions, (2) multiply the sum of the squares of the amplitudes for second-order scattering by the double-order Poisson factor for producing k pions, (3) neglect higher-order diagrams since we know they are small in the momentum-transfer region covered by the experiment,[5] and (4) add all the contributions for producing k pions together.

The result we call $P(k, p_\perp)$, the probability of producing k pions at momentum transfer $p_\perp$. The multiplicity $\bar{k}$ is then given by

$$\bar{k} = \sum_k kP(k, p_\perp) / \sum_k P(k, p_\perp).$$

In Fig. 3 we show multiplicities as a function of $p_\perp$ for values of μ from 1.5 to 3.5 corresponding to a range of effective masses for the data from Ref. 1. We see that the multiplicities are constant at low $p_\perp$ and then rise to larger values at $p_\perp = 0.65$ GeV/c where double quark scattering begins to be predominant over single quark scattering. There is then a second plateau region for $p_\perp > 1.0$ about 0.6 higher in multiplicity, another characteristic suggested by the data of Ref. 1 and what we expect with only double quark-quark scattering diagrams contributing. For higher momentum transfer, naturally higher-order scattering diagrams will have to be considered. Again we state that we have used the *same* parameter values for inelastic p-p scattering as for elastic p-p scattering,[3] the only addition being the inelastic Poisson factors for pion production which we can obtain from the pion multiplicity at low momentum transfer. Thus we have actually not fitted the data with any free parameters, but have used parameters obtained independent of the data.

In conclusion, the rise in multiplicity at large $p_\perp$ can be simply explained as due to double quark scatters predominating over single quark scatters at large $p_\perp$, and the results are entirely consistent with the elastic p-p quark multiple-scattering model.

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