Evidence for Scaling of Semi-inclusive Cross Sections between 13 and 205 GeV/c

Z. Ming Ma
Michigan State University, East Lansing, Michigan 48823*

and

R. Englemann† and R. Singer
Argonne National Laboratory,‡ Argonne, Illinois 60439

and

L. Voyvodic and J. Whitmore
National Accelerator Laboratory,‡ Batavia, Illinois 60510

(Received 22 August 1973)

Semi-inclusive invariant cross sections for π^- produced in pp interactions at 205 GeV/c are compared, at fixed $n_-/\langle n_-\rangle$, with data at lower energies ranging from 13 to 28.4 GeV/c. Except in the central region ($x \simeq 0$), the data are found to be in good agreement with the predictions of Koba, Nielsen, and Olesen for semi-inclusive scaling.

Since the intersecting storage rings at CERN and the 300-GeV synchrotron at the National Accelerator Laboratory came into operation, a great deal of interest and effort have been concentrated on the search for systematics in high-energy collisions. It is now commonly accepted that for inclusive reactions such as

$$p + p \to \pi^- + \text{anything}, \tag{1}$$

the scaling of invariant cross sections in the fragmentation region seems to be a valid principle and that in the pionization region ($x \simeq 0$), the invariant cross sections are approaching asymptotic scaling[1] at a rate of roughly $p_{\text{lab}}^{-1/4}$. When a 4π solid-angle detector such as a bubble chamber is used to study Reaction (1), one of the most accurately determined quantities is the total number n of charged particles in a given event. This quantity is not an explicit variable in most models for inclusive reactions.

It was proposed about a year ago by Koba, Nielsen, and Olesen[2] (KNO) that there exist scaling laws applicable to the so-called "semi-inclusive reaction." A semi-inclusive reaction is an inclusive reaction, such as Reaction (1), for a particular topology. An example would be

$$p + p \to \pi^- + (n-1)\text{ charged} + \text{anything neutral}. \tag{2}$$

Assuming that Feynman's scaling function

$$f^{(q)}(x_1, p_{\perp 1}, x_2, p_{\perp 2}, \dots, x_q, p_{\perp q}) \equiv \frac{1}{\sigma_{\text{inel}}} \frac{d^{3q}\sigma}{(d^3p_1/2\omega_1)(d^3p_2/2\omega_2)\cdots(d^3p_q/2\omega_q)} \tag{3}$$

is nonsingular at $x_1 = x_2 = \cdots x_q = 0$, and that it approaches scaling rapidly as $s \to \infty$, KNO proved that

$$P_n(s) \equiv \frac{\sigma_n}{\sigma_{\text{inel}}} = \frac{1}{\langle n\rangle} \Psi\left(\frac{n}{\langle n\rangle}\right) + \varphi(\ln^{-2}s) \tag{4}$$

to the highest order in $\ln s$. In Eq. (4), Ψ is an unknown function of $n/\langle n\rangle$ only, and $\varphi(\ln^{-2}s)$ represents a correction which varies, at most, as fast as the inverse of $\ln^2 s$. In spite of the explicit asymptotic nature of Eq. (4), KNO's first semi-inclusive scaling law (hereafter referred to as KNO-I) appears to be satisfied in the 50- to 303-GeV/c region. Slattery[3] has shown that the product $\langle n\rangle P_n$ when plotted versus $n/\langle n\rangle$ follows a universal curve independent of s.

KNO further predicted[4] that the semi-inclusive invariant cross sections should also scale asymptotically. For Reaction (2), the semi-inclusive π^- invariant cross section, defined by

$$g_n^-(s, x_-, p_\perp^-) \equiv \frac{1}{\sigma_n} \frac{d\sigma}{d^3p_-/2\omega_-}, \tag{5}$$

should scale as

$$g_n^-(x_-, p_\perp^-) = h(x_-, p_\perp^-, n_-/\langle n_-\rangle) \times [1 + \varphi(1/\ln s)], \tag{6}$$

where the minus subscript and superscript stand for π^-'s, and h is an unknown function independent of s. Equation (6) is referred to as KNO-II.

In this Letter, we report a test of this scaling law using pp interactions at 205 GeV/c as compared to data from similar reactions between 13.0 and 28.4 GeV/c. The high-energy data were obtained from an exposure of the 30-in. hydrogen bubble chamber at the National Accelerator Laboratory to a proton beam of 205-GeV/c incident momentum,[5] and the lower-energy data were obtained using the Brookhaven alternating-gradient synchrotron and the 80-in. hydrogen bubble chamber. Measurements for the 13.0- to 28.4-GeV/c events were done at Lawrence Berkeley Laboratory and processed through the TVGP system at Michigan State University. Since K^- and $\bar{p}$ contaminations below 30 GeV/c are believed[6] to be no more than ~3%, all negative tracks were interpreted to be due to π^-'s. Partially integrated invariant cross sections for six through fourteen prongs from the 205-GeV/c data are shown in Fig. 1 as open circles.

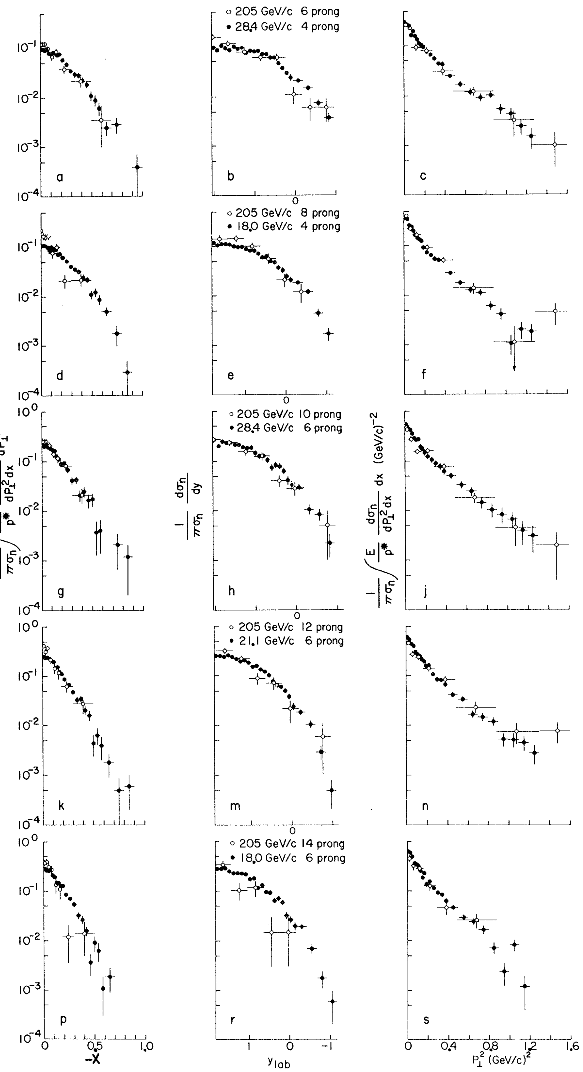

FIG. 1. Comparisons of semi-inclusive invariant cross sections at fixed $n_-/\langle n_-\rangle$. Data shown in the x and $y_{\rm lab}$ distributions have been folded and averaged about 90° in the c.m. system. In the $p_\perp^2$ distributions, data have been integrated over x. Good agreement with KNO-II is seen in the data except near $x=0$.

According to KNO-II [Eq. (6)], these cross sections should scale with the reduced negative multiplicity, $n_-/\langle n_-\rangle$, that is, should be identical to those at a different s, and a different topology but the same $n_-/\langle n_-\rangle$ ratio. Table I shows the averaged negative multiplicity for pp interactions at 13.0, 18.0, 21.1, 24.2, 28.4, and 205 GeV/c. Holding $n_-/\langle n_-\rangle$ constant, six prongs at 205 GeV/c are to be compared with four prongs at 28.4 GeV/c, eight prongs at 205 GeV/c with 18-GeV/c four prongs, and so on. Figures 1(a), 1(d), 1(g), 1(k), and 1(p) show the π^- semi-inclusive cross sections integrated over $p_\perp^2$ and folded and averaged about 90° in the c.m. system. Here one finds good agreement between the high- and lower-energy data except in the central region ($x \simeq 0$). This is similar to discrepancies observed[5] in comparing π^- inclusive cross sections at 28.5 and 205 GeV/c. Figure 1 indicates that the non-scaling behavior in the central region is not restricted to particular topologies. In the fragmentation region, the good agreement between these two sets of data is remarkable in view of the fact that comparisons are being made between different energies and different topologies. It should be noted that comparisons made between semi-inclusive invariant cross sections at fixed n show poor agreement. Comparisons of the laboratory rapidity distributions are given in Figs. 1(b), 1(e), 1(h), 1(m), and 1(r). The cutoff on the left-hand side of the figure is at 90° in the c.m. for the lower-energy data. Again, discrepancies in the central region exist, while good agreement is observed in the fragmentation region. In Figs. 1(c), 1(f), 1(j), 1(n), and 1(s), semi-inclusive cross sections integrated over x are plotted versus $p_\perp^2$. The KNO scaling prediction is well satisfied for all values of this variable.

TABLE I. Averaged negative nultiplicity $\langle n_-\rangle$.

Energy (GeV/c)	$\langle n_-\rangle$
13.0	0.83 ± 0.02
18.0	1.02 ± 0.02
21.1	1.15 ± 0.02
24.2	1.24 ± 0.02
28.4	1.30 ± 0.02
205	2.83 ± 0.09

In view of the fact that Feynman's scaling hypothesis fails in the central region and one of the basic assumptions of the KNO predictions is the rapid approach of asymptotic scaling as $s \to \infty$, it is interesting to ask if the apparent success of the two KNO semi-inclusive scaling laws can be induced by physical pictures other than Feynman's scaling hypothesis. KNO[2] have pointed out that the energy independence of the ratios $\langle n^q\rangle/\langle n\rangle^q$, $q = 2, 3, 4, \ldots$, is equivalent to KNO-I [Eq. (4)]. Published data[3] from 19 to 303 GeV/c indicate that the ratios $\langle n^q\rangle/\langle n\rangle^q$ are approximately constant above 50 GeV/c for all q values. Arnold[7] has argued that the success of KNO-I in the 50- to 303-GeV/c region is only a low-energy phenomenon and the asymptotic form of the scaling function is that of a δ function. The energy dependence of Ψ can be seen in Fig. 2. Open points are from the low-energy data, and the solid curve is from Slattery's fit to the 50- to 303-GeV/c data. Values from 50 to 69 GeV/c are shown as black circles. It is obvious that 50- and 69-GeV/c data are well described by the curve, whereas the lower-energy points indicate a narrower distribution. Therefore one may conjecture, on the basis of data below 303 GeV/c, that the width of Ψ grows larger for increasing s. Such a trend has been shown[8-10] to be consistent with a two-component picture, that is, an energy-independent diffractive component plus an energy-dependent nondiffractive part. However, within the spirit of such a model, the asymptotic form of Ψ is expected to be two δ functions, one at $n/\langle n\rangle = 0$ for the diffractive component and one at $n/\langle n\rangle = 1$ for the nondiffractive component. Therefore, according to the two-component picture, the shape of the Ψ function shown in Fig. 2 merely indicates that the asymptotic region for semi-inclusive scaling is still far off. Turning to KNO-II, Eq. (6) is a mathematical consequence of KNO-I being true and the energy independence of the ratio

$$\frac{\tilde{f}^{(q+1)}(x_-, p_\perp^-, 0, 0, \ldots, 0)}{[\tilde{f}^{(1)}(0)]^q} \to \text{constant for all } q, \qquad (7)$$

where $\tilde{f}^{(q+1)}(x_-, p_\perp^-, 0, 0, \ldots, 0)$ stands for the Feynman scaling function [Eq. (3)] integrated over transverse momenta for all particles other than the π^- and evaluated at $x_1 = x_2 = \ldots = x_q = 0$. Since KNO-I is known not to be valid below 50 GeV/c, the agreement with KNO-II from 13 to 205 GeV/c can be induced by the function Ψ and the ratio given in Eq. (7) having similar energy dependence. The apparent success of KNO-II should be viewed as an empirical fact and may serve as a constraint to theoretical models such as the two-component model.

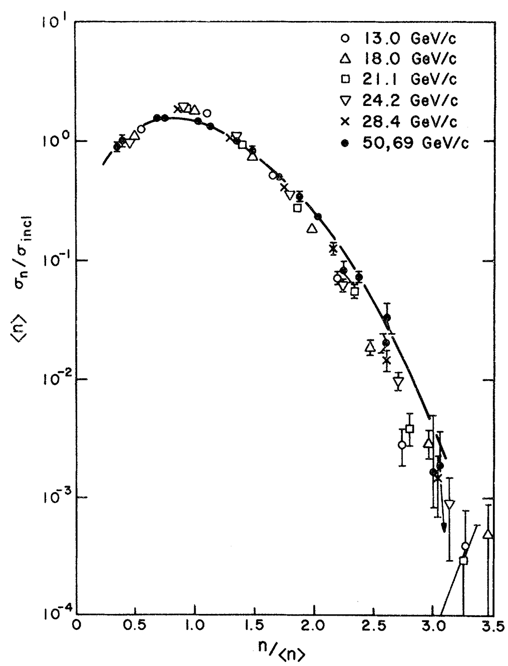

FIG. 2. Energy dependence of the KNO-I scaling function. The solid curve is from a fit to 50–303-GeV/c data by Slattery. A trend for the distribution to become broader with increasing s is observed.

One of us (Z.M.M.) would like to thank E. Yen for informative discussions on theoretical models.

*Work supported in part by the National Science Foundation.

†Work performed under the auspices of the U.S. Atomic Energy Commission.

‡Present address: State University of New York at Stony Brook, Stony Brook, N.Y. 11790.

[1]T. Ferbel, Phys. Rev. Lett. 29, 448 (1972).

[2]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).

[3]P. Slattery, Phys. Rev. Lett. 29, 1624 (1972), and Phys. Rev. D 7, 2073 (1973).

[4]Z. Koba, H. B. Nielsen, and P. Olesen, Phys. Lett. 38B, 25 (1972).

[5]Y. Cho *et al.*, Phys. Rev. Lett. 31, 413 (1973).

[6]J. V. Allaby *et al.*, CERN Report No. 70-12, 1970 (unpublished).

[7]R. C. Arnold, ANL Report No. ANL/HEP 7255, 1972 (unpublished).

[8]K. Fiałkowski, Phys. Lett. 41B, 379 (1972); K. Fiałkowski and H. I. Miettinen, Phys. Lett. 43B, 61 (1973).

[9]H. Harari and E. Rabinovici, Phys. Lett. 43B, 49 (1973).

[10]L. Van Hove, Phys. Lett. 43B, 65 (1973).