[5]P. G. Klemens, J. Phys. Soc. Jap. Suppl. 18, 77 (1963).
[6]C. Van Baarle, G. J. Roest, M. K. Roest-Young, and F. W. Gorter, Physica (Utrecht) 32, 1700 (1966).
[7]E. W. Fenton, J. S. Rogers, and S. B. Woods, Can. J. Phys. 41, 2026 (1963).

Gravitationally Induced Electromagnetic Radiation*

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A fully relativistic treatment is given to analyze the electromagnetic radiation induced by gravitational perturbations in extreme relativistic regions. For the sake of clarity, results are presented here for the simplest possible example: an uncharged mass m falling radially into a field of a Reissner-Nordstrøm solution. Explicit results governing the energetics of the process are found. The amount of electromagnetic radiation radiated can be of the order of magnitude of the gravitational one.

The problem of gravitational and electromagnetic radiation under extreme relativistic regimes has been recently examined in a series of researches[1] based on the Regge-Wheeler[2]-Zerilli[3] formalism. The interest in this research stems not least from the clear possibility of observing these effects either during the processes of gravitational collapse,[4] or in collapsed objects in binary systems.[5] The aim of this Letter is to communicate a fully relativistic analysis of processes which can occur in gravitational collapse. We study the feasibility and efficiency of conversion of gravitational radiation into electromagnetic radiation, and other problems related to the coupling between electromagnetic and gravitational fields.

Here the main results of the purely formal investigation are presented.[6] Astrophysical implications will be presented elsewhere. To clarify the phenomena in the simplest possible case we analyze the radial infall of an uncharged system of mass m into a Reissner-Nordstrøm solution of charge Q and mass M.[7] The physical phenomenon we are interested in can be summarized as follows: The infalling neutral object perturbs the background geometry by emission of gravitational waves impinging on the surface of the collapsed object. Electromagnetic radiation is then emitted by the system. In return, the variation in the electromagnetic field will itself generate an emission of gravitational radiation.[8] This coupled effect is here examined and explicit results given. The equations[9] governing the process considered in this paper are

$$\frac{d^2R_{l0}}{dr^{*2}}+(\omega^2-V_l^g)R_{l0}=-\frac{e^{2\nu}}{\lambda r^2+3r-2Q^2}\left(\frac{\lambda r+1}{r^2-2r+Q^2}+\frac{2\lambda r+3}{\lambda r^2+3r-2Q^2}\right)\frac{8Q^2}{i\omega}f_{l0}+S_{l0}{}^1, \quad (1)$$

with

$$S_{l0}{}^1\equiv\frac{8\gamma_0(l+\frac{1}{2})^{1/2}re^{\nu}e^{i\omega T}}{\lambda r^2+3r-2Q^2}\left[\frac{1}{i\omega}\left(\frac{1}{r}-\frac{2\lambda r+3}{\lambda r^2+3r-2Q^2}\right)-\frac{1}{2}\left(\frac{e^{-2\nu}}{dT/dr}+\frac{dT}{dr}\right)\right];$$

$$\frac{d^2f_{l0}}{dr^{*2}}+\left(\omega^2-e^{\nu}\left[\frac{l(l+1)}{r^2}+\frac{4e^{\nu}Q^2}{r^2(\lambda r^2+3r-2Q^2)}\right]\right)f_{l0}$$
$$=-\frac{i\omega e^{\nu}}{r^2}\left[\left(\frac{l(l+1)}{2r}-\frac{e^{\nu}(3r-4Q^2)}{r(\lambda r^2+3r-2Q^2)}\right)R_{l0}+\frac{dR_{l0}}{dr^*}\right]+S_{l0}{}^2, \quad (2)$$

with

$$S_{l0}{}^2 \equiv 4e^{\nu}\gamma_0(l+\tfrac{1}{2})^{1/2}e^{i\omega T(r)}/r(\lambda r^2+3r-2Q^2),$$

where

$$e^{\nu}=1-2/r+Q^2/r^2, \tag{3}$$

$$dr/dr^* = e^{\nu}, \quad \lambda=(l-1)(l+2)/2, \tag{4}$$

$T(r)$ indicates the time coordinate of the trajectory of the test particle, and

$$V_l{}^g = e^{\nu}\left(\frac{l(l+1)}{r^2}-\frac{6}{r^3}+\frac{8Q^2}{r^4}+\frac{4[(3r-4Q^2)^2(r-Q^2)-e^{\nu}(3r-5Q^2)r^2]}{(\lambda r^2+3r-2Q^2)r^4}+\frac{2e^{\nu}(3r-4Q^2)^2}{r^2(\lambda r^2+3r-2Q^2)^2}\right). \tag{5}$$

Diagrams of the effective potential of Eqs. (1) and (2) are given in Fig. 1 for a value of $l=2$. The radial coordinate, the charge of the black hole, and the frequency of the radiation, as well as the function $T(r)$, are measured in units of the black-hole mass, while $\gamma_0=e^{\nu}\,dT/ds$, where s is the arc length along the geodesic. The radial function f_{l0} is related to the components of the electromagnetic tensor $F_{\mu\nu}$ needed to compute the radial flux of electromagnetic energy by

$$F_{12}=e^{-\nu}f_{l0}Y_{l0,\theta},\quad F_{13}=e^{-\nu}f_{l0}Y_{l0,\varphi},\quad F_{02}=(df_{l0}/dr^*)(1/i\omega)Y_{l0,\theta},\quad F_{03}=-(df_{l0}/dr^*)(1/i\omega)Y_{l0,\varphi}. \tag{6}$$

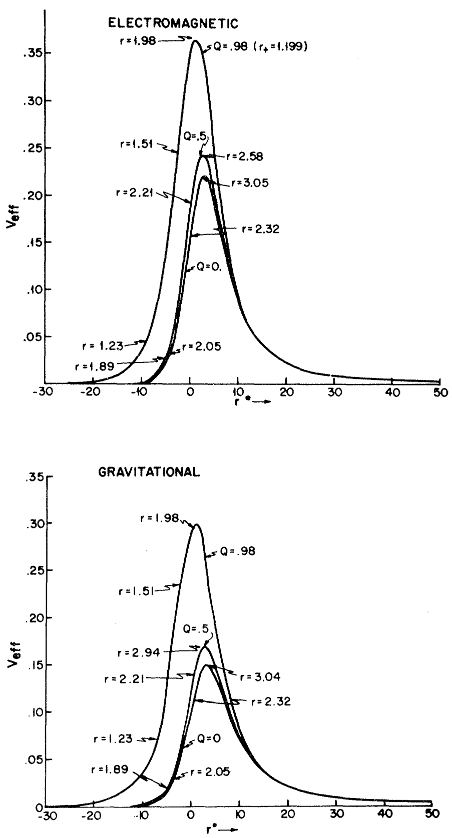

FIG. 1. Effective potentials for the gravitational and electromagnetic perturbations, for $l=2$, as a function of the coordinate r^*; for a direct comparison we have given on the curve of the effective potential the corresponding value of the coordinate r.

The radial function R_{l0} is related to the function K_{l0} given in Ref. 3 by

$$K_{l0}=\left(\frac{l(l+1)}{2r}-\frac{e^{\nu}(3r-4\theta^2)}{r(\lambda r^2+3r-2Q^2)}\right)R_{l0}+\frac{dR_{l0}}{dr^*}. \tag{7}$$

As usual[1] we have imposed the conditions of purely ingoing waves at the black-hole surface and purely outgoing waves at infinity. We then have for the expression of the gravitational energy emitted, averaged over all angles,

$$\left\langle\frac{dE}{d\omega}\right\rangle = m^2\omega^2 R_{l0}{}^*R_{l0}\,l(l+1)(l-1)(l+2)/32\pi, \tag{8}$$

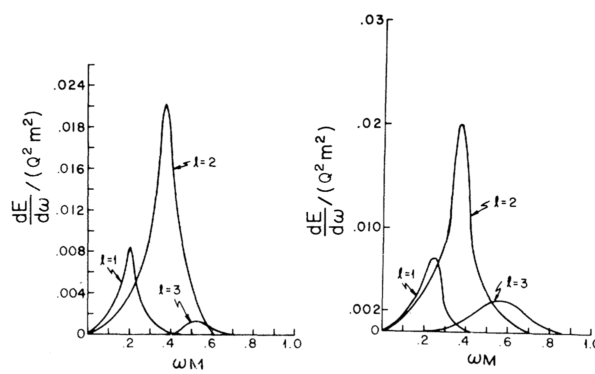

FIG. 2. Electromagnetic radiation (measured in units of the rest mass m of the particle) emitted by a radially infalling particle starting at rest at infinity. The intensity is here given for selected values of the multipoles as a function of the frequency (measured in units of the black-hole mass). The diagram on the left (right) refers to a black hole with $Q=0.5$ ($Q=0.8$).

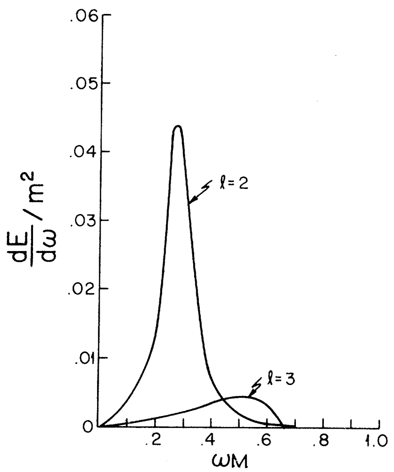

FIG. 3. Gravitational radiation emitted by radially infalling particle starting its motion at rest at infinity in the field of a Reissner-Nordstrøm black hole with $Q = 0.8$.

and for the electromagnetic energy,

$$\left\langle\frac{dE}{d\omega}\right\rangle = Q^2 m^2 f_{l0} f_{l0}^* \, l(l+1)/2\pi, \qquad (9)$$

where Q is the charge of the black hole measured in units of the black-hole mass and m is the mass of the test particle. Details on the integration techniques are given elsewhere.[10] Some of the main results are presented in Figs. 2 and 3.

We can then summarize the main results as follows:

(1) The gravitationally induced electromagnetic radiation originated by a radially infalling uncharged test particle in a field of a Reissner-Nordstrøm metric is

$$\Delta E \sim 0.03\, m Q^2 (m/M),$$

which can clearly be of the same order of magnitude as the radiated gravitational energy, namely,

$$\Delta E \sim 0.010\, m (m/M)$$

(see Fig. 2).

(2) The total amount of gravitational radiation emitted, as well as its multipole distribution, is not affected by the presence of a charge on the black hole (compare and contrast Fig. 3 in this manuscript with Fig. 1 of Davis *et al.*[11]).

(3) The dipole ($l = 1$) electromagnetic radiation term is consistently lower, as a consequence of the "feedback" terms in Eqs. (1) and (2), than the quadrupole ($l = 2$) term. As the value of Q increases there is a slow enhancement of the higher multipoles (compare and contrast the $l = 3$ curves in Fig. 2 in the two cases $Q = 0.5$ and $Q = 0.8$).

A detailed analysis of the results here presented as well as the gravitationally induced electromagnetic radiation from more general trajectories are given by Ruffini and Zerilli.[12]

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[1]See R. Ruffini, in *Black Holes*, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1973), p. 453.

[2]T. Regge and J. A. Wheeler, Phys. Rev. 108, 1063 (1957).

[3]F. Zerilli, Phys. Rev. D 2, 2141 (1970), and Phys. Rev. Lett. 24, 737 (1970), and J. Math. Phys. (N.Y.) 11, 2203 (1970).

[4]See R. Ruffini, Trans. N.Y. Acad. Sci. 35, 196 (1973), or R. B. Partridge and R. Ruffini, "Gravitational Waves and a Search for the Associated Microwave Radiation," Gravity Research Foundation Essay, 1970 (unpublished).

[5]See, e.g., H. Gursky, in *Black Holes*, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1973).

[6]The derivation of the equations, as well as a detailed analysis of the results, is given elsewhere by R. Ruffini and F. Zerilli, to be published.

[7]The case in which the infalling particle is itself endowed with a charge has also been treated by M. Johnston, R. Ruffini, and F. Zerilli, to be published.

[8]It is relevant to remark here that this phenomenon is totally different from the one recently investigated by I. B. Khriplovich and O. P. Sushkov, Institute of Nuclear Physics, Novosibirsk, Report No. 51/73 (to be published). Their analysis deals with the radiation emitted by a charged particle in an external electromagnetic field. In that case the fact that there is a zero-order electromagnetic field implies that the test charge q gives a contribution to the stress-energy tensor which is first order in the value of the charge. They, however, neglect the effects of gravitationally induced electromagnetic radiation here presented which should be expected to be, in many cases of physical interest, of much greater importance. The fully relativistic treatment of these two combined effects (gravitationally induced electromagnetic field and the charge of the particle) has been analyzed in a Reissner-Nordstrøm geometry and the results presented by M. Johnston, R. Ruffini, and F. Zerilli, to be published.

[9]F. Zerilli, to be published.

[10]M. Johnston, M. Peterson, and R. Ruffini, to be published.

[11]M. Davis, R. Ruffini, W. H. Press, and R. H. Price, Phys. Rev. Lett. 27, 1466 (1971).

[12]R. Ruffini and F. Zerilli, to be published.