[1]E. W. Müller, Naturwissenschaften 29, 533 (1941).
[2]E. W. Müller, Phys. Rev. 106, 618 (1956).
[3]R. Gomer, *Field Emission and Field Ionization* (Harvard Univ. Press, Cambridge, Mass., 1961).
[4]E. W. Müller and T. T. Tsong, *Field Ion Microscopy, Principles and Applications* (Elsevier, New York, 1969).
[5]M. Vesely and G. Ehrlich, Surface Sci. 34, 547 (1973).
[6]E. W. Müller, J. A. Panitz, and S. B. McLane, Rev. Sci. Instrum. 39, 83 (1968), also in *Proceedings of the Fourth European Regional Conference on Electron Microscopy,* edited by O. S. Bocciarelli (Tipografisa Poliglotta Vaticana, Rome, Italy, 1968), Vol. 1, p. 135.
[7]S. S. Brenner and J. T. McKinney, Appl. Phys. Lett. 13, 29 (1968).
[8]E. W. Müller, Quart. Rev. Chem. Soc. 23, 177 (1969).
[9]E. W. Müller, S. B. McLane, and J. A. Panitz, Surface Sci. 17, 430 (1969).
[10]T. T. Tsong and E. W. Müller, Phys. Rev. Lett. 25, 911 (1970).
[11]E. W. Müller and S. V. Krishnaswamy, Surface Sci. 36, 29 (1973).
[12]J. T. McKinney and S. S. Brenner, in Abstracts to the Sixteenth Field Emission Symposium, Pittsburgh, September 1969 (unpublished).
[13]E. W. Müller, J. Less-Common Metals 28, 37 (1972).
[14]T. T. Tsong and W. W. Müller, J. Chem. Phys. 41, 3279 (1964).
[15]W. Schmidt, T. Reisner, and E. Krautz, Surface Sci. 26, 297 (1971).
[16]K. D. Rendulic, Surface Sci. 28, 285 (1971), and 34, 581 (1973).
[17]S. B. McLane, E. W. Müller, and S. V. Krishnaswamy, Surface Sci. 27, 367 (1971).
[18]D. A. Nolan and R. M. Herman, Phys. Rev. B (to be published).
[19]E. W. Müller, S. V. Krishnaswamy, and S. B. McLane, Surface Sci. 23, 112 (1970).
[20]E. W. Müller, Ber. Bunsenges. Phys. Chem. 75, 979 (1971).
[21]E. W. Müller, S. V. Krishnaswamy, S. B. McLane, T. Sakurai, and R. Walko, in Abstracts to the Nineteenth Field Emission Symposium, Urbana, Illinois, August 1972 (unpublished).
[22]E. W. Müller, Labex Lecture, London, April 1973 (unpublished), and Lab. Pract. 22, 408 (1973).
[23]P. J. Turner, private communication.
[24]P. J. Turner, B. J. Regan, and M. J. Southon, Vacuum 22, 443 (1972).
[25]E. W. Müller, S. V. Krishnaswamy, and S. B. McLane, Rev. Sci. Instrum. 44, 84 (1973).
[26]T. T. Tsong and W. W. Müller, J. Chem. Phys. 55, 2884 (1971).
[27]J. A. Panitz, Rev. Sci. Instrum. 44, 1034 (1973).

Finite-Geometry Effects and Onset of Superfluidity in ^{3}He-^{4}He Mixtures near T_λ

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The amplitude and velocity of second sound near T_λ and the onset temperature $T_o(X)$ for superfluidity have been measured for ^{3}He-^{4}He mixtures in resonators equipped with superleak transducers. The reduced temperature $\epsilon_o = 1 - T_o(X)/T_\lambda(X)$ and the quantity $l\rho_s/T$ at ϵ_o vary by factors of about 6 for $X = 0$ to 0.4. The exponents λ' for $\epsilon_o = (d/d^*)^{-\lambda'}$ and ν' for the onset length $l = l^*\epsilon^{-\nu'}$ are independent of X to within $\pm 3\%$. Contrasting with this behavior is a 20% change of the effective exponent ζ of $\rho_s/\rho = k\epsilon^\zeta$.

The pressure P and the ^{3}He impurity concentration X are inert variables for the superfluid transition. We have shown recently that for pure ^{4}He the pressure has no influence on the reduced shift $\epsilon_o = 1 - T_o(P)/T_\lambda(P)$ of the onset temperature T_o for superfluidity in finite geometries.[1] It was concluded that the coefficients and exponents of $\epsilon_o = (d/d^*)^{-\lambda'}$ and of the onset length $l = l^*E^{-\nu'}$, as well as the quantity $l\rho_s/T$, are independent of pressure (d, channel diameter).[1] A constant $l\rho_s/T$ is expected if the onset of superfluidity is determined by a characteristic length l, which is related by a constant multiplicative factor to the phase coherence length ξ defined by Ferrell *et al.*[2] and Fisher, Barber, and Jasnow.[3]

In this Letter we report on the effects of concentration on these quantities for ^{3}He-^{4}He mixtures. We find for the mixtures that ϵ_o increases by a factor of about 6 between $X = 0.0$ and 0.4. This observation is consistent with an even larger value of ϵ_o measured by Ahlers and Greywall near the tricritical point.[4] In addition, we find that the product $l\rho_s/T$ at ϵ_o depends strongly upon the concentration and changes by a factor of about 7 between $X = 0.0$ and 0.4. We therefore conclude that the characteristic length l which determines the onset of superfluidity in the mixtures is not related by a constant multiplicative factor to the

phase coherence length ϵ as defined in Refs. 2 and 3. We find also that the exponents λ' and ν' are independent of X to within $\pm 3\%$, whereas the coefficients d^* and l^* increase with X by almost a factor of 4.

Our results for the velocity of second sound near $T_\lambda(X)$ agree with previous measurements by others.[5] They yield a superfluid fraction $\rho_s/\rho = k\epsilon^\zeta$ with an amplitude k changing by a factor of 26 and an effective exponent ζ changing by 20% for $0 \leq X \leq 0.55$. The strong dependence of k upon X has been emphasized recently by Ahlers.[6]

The onset temperatures $T_o(X)$ for superfluidity of the mixtures and the velocities of second sound have been determined by measuring the vanishing amplitude of the sound and its frequency in resonators equipped with superleak transducers. These measurements were performed for $X = 0.000$, 0.096, 0.204, 0.396, and 0.555,[7] and for superleak channels with $d = 0.2$ and 0.6 μm. The experimental method was similar to the one described in detail in Ref. 1. Two cylindrical copper resonators (0.5 cm long, 0.8 cm diam) were mounted inside a copper cell, which was suspended in a vacuum can in a helium bath. The resonators were terminated at both ends by identical superleak condenser transducers.[8] The vibrating elements of the transducers were gold-coated Nuclepore filter papers[9] used to excite and detect second-sound plane-wave modes of the cavities. One cavity was equipped with filter paper permeated by 0.2-μm-wide channels, the other one was equipped with 0.6-μm channel paper. After filling the copper cell and resonators at $T < T_\lambda(X)$ with a mixture, a low-temperature valve in the vacuum can was closed and isolated the sample from the bath. A very small gas ballast volume at low temperature assured that the data were taken under saturated vapor pressure. A wave analyzer was used to sweep the frequency of the second sound through at least five harmonics. Generator and detector transducers were biased with 250 V. The generators were always driven at an ac voltage of 0.695 V rms, giving signals of 100 to 300 μV at the receivers for temperatures away from onset. The efficiency of the transducers and the Q factor of the resonances were largely independent of concentration. The second-sound velocities and amplitudes were determined at each temperature from the wave-analyzer output. Temperatures were measured and regulated with a heater wound on the copper cell and a germanium thermometer mounted in the wall of the cell. Thermal equilibrium was established at each temperature before the data were recorded. The temperature resolution and stability during taking of a datum point was better than 1 μK. The λ temperatures of the mixtures were determined from the inflection point of the velocity of first sound.[1,10] Independent checks of these measured $T_\lambda(X)$ were obtained from least-squares fits of $u_2 = u_2^* \epsilon^\varphi$ and of $\rho_s/\rho = k\epsilon^\zeta$ to our data with T_λ as a variable parameter. These calculated values for T_λ are in excellent agreement with the measured values. For the analysis of our data we used the measured λ temperatures, and concentrations X determined from them and the phase diagram.[11]

In approaching T_λ the onset length increases, and eventually becomes comparable to the channel diameter of the filter paper in the transducers. Then, superfluidity vanishes in the channels, and the transducers are not suitable for the production or detection of second sound.[1,4,8] For a given channel the onset occurs at the same value of the onset length for each mixture. For the determination of $T_o(X)$ we plotted the amplitude of a particular second-sound harmonic (always between the 27th and the 30th) against $\epsilon = 1 - T/T_\lambda$. The onset temperature T_o was defined as occurring when the sound signal vanished into the 0.03-μV noise level of our electronics. Shifting the level to 0.1 μV did not change the concentration dependence of our data and had only a slight effect on the absolute values. The influence of the cutoff level and other possible sources for errors are discussed in detail in Ref. 1. Reduced shifts $\epsilon_o = 1 - T_o(X)/T_\lambda(X)$ of the onset temperature are plotted as a function of X in Fig. 1 for channels with $d = 0.2$ and 0.6 μm. For both filter papers ϵ_o increases by about a factor of 6 from $X = 0$ to 0.4. This behavior is in striking contrast to the pressure independence of ϵ_o in pure ^{4}He.[1] The reduced shift ϵ_o is expected to vary as $\epsilon_o = (d/d^*)^{-\lambda'}$.[3,12] Therefore, λ' is proportional to $\ln(\epsilon_{o,1}/\epsilon_{o,2})$, where $\epsilon_{o,i}$ is the reduced shift of the onset temperature in a channel with diameter d_i. With this equation and our data for $\epsilon_{o,i}$ in $d_1 = 0.2$-μm and $d_2 = 0.6$-μm channels, we calculated the relative change of λ' with concentration. The results are shown in Fig. 2(b). The diameter d of a channel is proportional to the onset length $l = l^* \epsilon^{-\nu'}$, implying $\lambda' = 1/\nu'$. Therefore, a change of $\Delta\lambda'/\lambda'$ with concentration is identical to the behavior of $\Delta\nu'/\nu'$. Even though ϵ_o changes drastically with concentration, the exponents λ' and ν' stay constant to within our resolution of $\pm 3\%$ for $0 \leq X \leq 0.4$. The concentration dependence of ϵ_o and

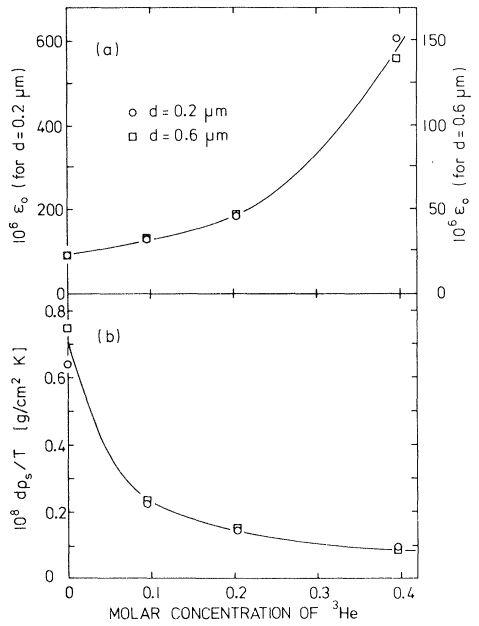

FIG. 1. Finite-geometry effects in ^{3}He-^{4}He mixtures. (a) Reduced shift $\epsilon_o = 1 - T_o/T_\lambda$ of the onset temperature T_o as a function of ^{3}He concentration; left-hand scale, ϵ_o measured in $d = 0.2$-μm channels; right-hand scale, ϵ_o measured in $d = 0.6$-μm channels; (b) the quantity $d\rho_s/T$ calculated at ϵ_o as a function of ^{3}He concentration for the two channel diameters. The lines are only a guide for the eye.

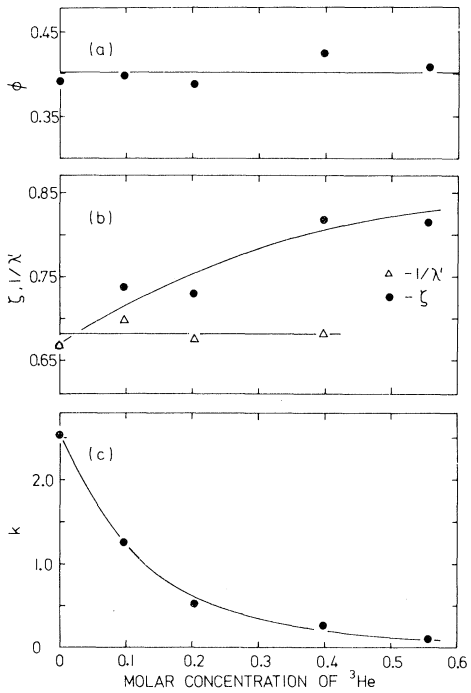

FIG. 2. Parameters of the second-sound velocity u_2, of the superfluid fraction ρ_s/ρ, and of the onset temperature ϵ_o in ^{3}He-^{4}He mixtures near T_λ. (a) Effective exponent φ of $u_2 = u_2^* \epsilon^\varphi$. (b) Effective exponents ζ of $\rho_s/\rho = k\epsilon^\zeta$ and λ' of $\epsilon_o = (d/d^*)^{-\lambda'}$. The values of $1/\lambda'$ were normalized to $1/\lambda' \equiv \zeta = 0.670$ at $X = 0$. (c) Amplitude k of $\rho_s/\rho = k\epsilon^\zeta$. The lines are only a guide for the eye.

the concentration independence of λ' and ν' imply that the coefficients l^* and d^* as well as the onset length l at constant ϵ increase by almost a factor of 4 from $X = 0$ to 0.4.

The phase coherence length ξ defined in Refs. 2 and 3 is expected to satisfy the relation $\xi\rho_s/T = \text{const}$. For pure ^{4}He this quantity was found to be independent of temperature[13] and pressure,[1] if $\xi \propto$ onset length l. Because $d \propto l = \text{const}$ at the onset temperature, we expect $d\rho_s/T = \text{const}$ (or $\rho_s/T = \text{const}$ for constant d) at ϵ_o from the above relation if $l \propto \xi$. For our small temperature range we can set $T = T_o(X)$ in the above relation. From the measured ϵ_o we have calculated ρ_s at onset with the equations for $\rho_s(\epsilon, X)$ to be discussed below. The data shown in Fig. 1 demonstrate that $d\rho_s/T$ or $l\rho_s/T$ at ϵ_o decreases by a factor of about 7 between $X = 0$ and 0.4. We therefore conclude that the onset length l is not simply proportional to the phase coherence length ξ in the mixtures. We draw attention to the consistency of the results for $d\rho_s/T$ from the two filter papers.

Our results for the velocity u_2 of second sound were fitted by the equation $u_2 = u_2^* \epsilon^\varphi$. The values for u_2 and u_2^* at small X and the values for φ at all X (see Fig. 2) are remarkably independent of concentration, in agreement with the results of others.[5] With the relation between u_2 and ρ_s/ρ,[4,14] and thermodynamic data for the mixtures,[14] ρ_s/ρ was calculated from our measured u_2 and ϵ. The resulting values were fitted to $\rho_s/\rho = k\epsilon^\zeta$. Both the coefficient k as well as the effective exponent ζ are strong functions of concentration as shown in Fig. 2. The remarkably strong concentration dependence of the coefficient k has been discussed for dilute mixtures recently.[6] The change of the exponent ζ with concentration is in contrast to the behavior of the exponents λ' and ν'. It has to be kept in mind, that these numbers for ζ result from a fit of a pure power law to data in a limited temperature range ($10^4\epsilon = 0.23$-2.9, 0.33-2.9, 0.53-4.8, 1.2-19, 2.8-140 for $X = 0$, 0.096, 0.204, 0.396, 0.555, respectively). However, the results for pure ^{4}He, e.g., are in very good agreement with data[15] taken in a more extended temperature

range. The necessity of higher-order correction terms[15] for the representation of ρ_s/ρ could only be decided from much more accurate data.

In contrast to the effect of pressure,[1] the adding of ^{3}He has a strong influence on geometry effects as demonstrated here for ϵ_o, l, l^*, d^*, and $l\rho_s/T$. The critical exponents λ' and ν' seem to withstand the influence of ^{3}He as they did the application of pressure,[1] in agreement with the universality concept.[16] The effect of ^{3}He on the amplitude and on the effective critical exponent of the superfluid fraction is much stronger than the effect of pressure.

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[1]G. G. Ihas and F. Pobell, to be published.

[2]R. A. Ferrell, N. Menyhárd, H. Schmidt, F. Schwabl, and P. Szépfalusy, Ann. Phys. (New York) 47, 565 (1968); B. I. Halperin and P. C. Hohenberg, Phys. Rev. 117, 952 (1969).

[3]M. E. Fisher, M. N. Barber, and D. Jasnow, Phys. Rev. A 8, 1111 (1973).

[4]G. Ahlers and D. S. Greywall, Phys. Rev. Lett. 29, 849 (1972), and in *Proceedings of the Thirteenth International Conference on Low Temperature Physics, Boulder, Colorado, 1972,* edited by W. J. O'Sullivan et al. (Plenum, New York, 1973).

[5]S. M. Noble and D. J. Sandiford, J. Phys. C: Proc. Phys. Soc., London 3, L123 (1970); G. Terui and A. Ikushima, Phys. Lett. 43A, 255 (1973); G. Ahlers and D. S. Greywall, to be published, and private communication.

[6]G. Ahlers, to be published.

[7]For $X = 0.555$ only reliable frequency data were obtained.

[8]R. Williams, S. E. A. Beaver, J. C. Fraser, R. S. Kagiwada, and I. Rudnick, Phys. Lett. 29A, 279 (1969); R. A. Sherlock and D. O. Edwards, Rev. Sci. Instrum. 41, 1603 (1970).

[9]Nuclepore Filters, General Electric Co., Pleasonton, Calif.

[10]W. C. Thomlinson and F. Pobell, Phys. Rev. Lett. 31, 283 (1973).

[11]T. R. Roberts and S. G. Sydoriak, Phys. Fluids 3, 895 (1960).

[12]A. E. Ferdinand and M. E. Fisher, Phys. Rev. 185, 832 (1969); M. A. Moore, Phys. Lett. 37A, 345 (1971); M. E. Fisher and M. N. Barber, Phys. Rev. Lett. 28, 1516 (1972).

[13]I. Rudnick and J. C. Fraser, J. Low Temp. Phys. 3, 225 (1970); E. S. Sabisky and C. H. Anderson, Phys. Rev. Lett. 30, 1122 (1973).

[14]G. Ahlers, in "The Physics of Liquid and Solid Helium," edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, to be published), Vol. I, Chap. 5.

[15]D. S. Greywall and G. Ahlers, Phys. Rev. A 7, 2145 (1973).

[16]L. P. Kadanoff, in *Proceedings of the International School of Physics "Enrico Fermi," Course LI,* edited by M. S. Green (Academic, New York, 1973); R. B. Griffiths, Phys. Rev. Lett. 24, 1479 (1970).

Particle Trapping in Magnetic Line Cusps

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A computer study of the magnetic line cusp with an internal field shows that a large fraction of the particles in the field-plasma boundary move in definite guiding-center orbits. In the absence of microinstabilities such particles will remain trapped between Coulomb collisions. Since only boundary particles can escape from a "stuffed" line cusp, this result gives an optimistic assessment of line cusps for fusion.

Cusp geometries have not been extensively investigated in the last decade because theory predicted a particle loss rate high enough to preclude the use of cusps in a fusion reactor. However, the experiments of Gallagher and Levine[1] on a toroidal line cusp produced an $n\tau$ of 10^{12} sec/cm^3 in a 10^{16}/cm^3 hydrogen plasma. Though their temperature of 15 eV was still in the collisional regime, the results sparked a re-examination of the theory of collisionless line cusps.

Recently, it was pointed out[2] that a certain fraction of the particles in velocity space cannot