Commission, the National Science Foundation, and the French Centre National de la Recherche Scientifique.

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[4]In the calculation of transverse momentum imbalance (Δp_t) a momentum of 205 GeV/c was assigned to the outgoing π^-, and its π^- direction at the primary vertex was recalculated. The resulting resolution in the component of Δp_t perpendicular to the lens axis, $\Delta p_{t\varphi}$, is ± 50 MeV/c. The reaction $\pi^- p \to \pi^- \pi^0 \pi^0 p$ with all π's forward is expected to be the main source of the small inelastic background under the elastic peak in $\Delta p_{t\varphi}$. (Events with backward π^0's yield a missing mass to the proton that is easily distinguishable from the π^- mass, and unless G-parity exchange processes are unexpectedly large, few events should have one or three forward π^0's.) We study this $\pi^-\pi^0\pi^0 p$ background by dropping two π's from four-prong events and subjecting these pseudo two-prong events to the same analysis used for real two-prong events. The broad $|\Delta p_{t\varphi}|$ distribution from these events agrees in shape and approximate magnitude with that observed in the real two-prong events outside the elastic peak ($|\Delta p_{t\varphi}| > 250$ MeV/c). Thus we have used the pseudo two-prong background distribution to subtract an $\sim 5\%$ background under the elastic peak.

[5]This correction was made by extrapolating $d\sigma_{el}/dt$ from $|t| = 0.03$ to 0.0 GeV^2 with the same slope b. The inelastic two-prong events for $-t < 0.03$ GeV^2 were corrected by assuming the same fraction of lost events as for elastics.

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[11]When we fit separate straight lines to $\langle n_{ch}\rangle$ versus Q for pions and protons between 40 and 205 GeV/c we obtain a 2σ difference between the two lines.

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Effective Angular-Momentum Barrier in the SU(3) Test of Reactions Involving Pseudoscalar Mesons*

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SU(3) in particle reactions involving pseudoscalar mesons is discussed in the framework of chiral SU(3)⊗SU(3) charge algebra, partial conservation of axial-vector current, and asymptotic SU(3).

Naive application of SU(3) to particle reactions has met many difficulties.[1] Meshkov, Snow, and Yodh[2] first compared experiments with the remarkable *exact* SU(3) predictions

$$\tfrac{1}{3}\sigma(\pi^- p \to \pi^+\Delta^-) = \sigma(\pi^- p \to K^+ Y^{*-}) = \sigma(K^- p \to \pi^+ Y^{*-}) = \sigma(K^- p \to K^+ \Xi^{+-}). \tag{1a}$$

With the hope of accounting for the effect of mass breaking, comparison of the cross sections at the same Q value was suggested.[2] As shown in Fig. 1, while $\frac{1}{3}\sigma(\pi^+\Delta^-) \approx \sigma(\pi^+ Y^{*-})$ and $\sigma(K^+ Y^{*-}) \approx \sigma(K^+\Xi^{*-})$ are found to hold, there is a vast discrepancy (of an order of magnitude) between these two groups.

For another SU(3) prediction,

$$\sigma(K^-p \to \pi^+\Sigma^-) = \sigma(K^-p \to K^0\Xi^0), \quad (1b)$$

$$\sigma(\pi^-p \to K^+\Sigma^-) = \sigma(K^-n \to K^0\Xi^-), \quad (1c)$$

it was also found[3] that whereas Eq. (1c) is approximately satisfied, Eq. (1b) is drastically violated.

As a possible broken SU(3) relation between the lth partial wave amplitudes f_l and f_l' of two SU(3)-related reactions, Trilling[1] and Rosenfeld[4] proposed the following one with a prescribed angular-momentum barrier effect:

$$f_l(1/p_f^{\,l}) = [\text{SU(3) factor}]f_l'(1/p_f'^{\,l}), \quad (2)$$

where p_i and p_f denote the initial and final c.m. momentum, respectively. If a partial wave analysis is not available, one may represent the effect of the barrier and phase space by the effective angular momentum $l_{\text{eff}} \equiv L$ appearing in the total-cross-section relations $\sigma/p_f^{2L+1} = [\text{SU(3) factor}]^2\sigma'/p_f'^{2L+1}$. It has been argued[1,4] that the above prescriptions tend to remove the difficulties encountered and, in particular, the phenomenological choice $l_{\text{eff}} \equiv L = 1$ can resolve the vast discrepancies met in Reaction (1a).

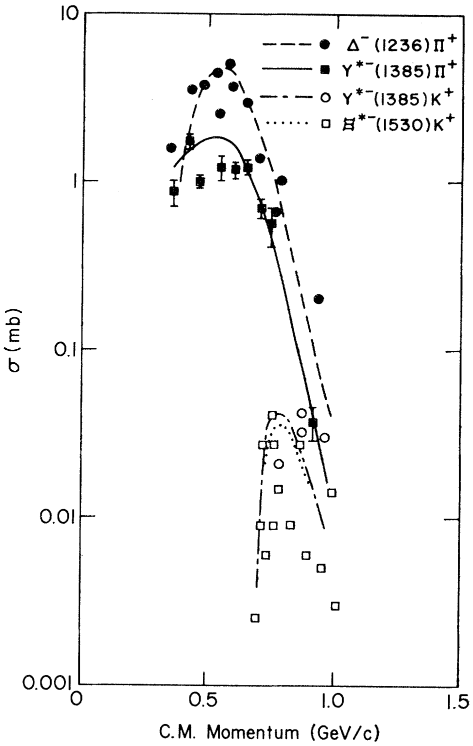

FIG. 1. Cross sections for the reactions $\pi^-p \to \pi^+\Delta^-(1236)$, $K^+Y^{*-}(1385)$, $K^-p \to \pi^+Y^{*-}(1385)$, $K^+\Xi^{*-}(1530)$. The three curves for $\pi p \to \pi\Delta$, $\pi p \to KY^*$, and $Kp \to K\Xi^*$ are the predictions with the input reaction $Kp \to \pi Y^*$ given by a hand-drawn solid curve.

The purpose of this paper is to demonstrate that the effective angular-momentum barrier $l_{\text{eff}} \equiv L = 1$ can have a reasonable theoretical basis in the framework of (a) chiral SU(3)⊗SU(3) charge algebra, (b) partial conservation of axial-vector current (PCAC) for Q_j^5, and (c) asymptotic SU(3).

We first note that when combined with asymptotic SU(3), the imposition of the algebras $[Q_i, Q_j] = if_{ijk}Q_k$ and $[Q_i, Q_j^5] = if_{ijk}Q_k^5$ implies the following general result[5]: (A) The matrix elements of Q_j^5, taken only between the states of particles every one of which has infinite momentum, allow the usual parametrization in terms of exact SU(3) plus mixing.

Therefore, (a) and (c) prescribe for us where one can use exact SU(3) (including mixing) parametrization in broken SU(3). We neglect SU(3) mixing in this paper.

Consider the following special case of our general result (A):

$$\lim_{\vec{p}_A, \vec{p}_B, \vec{p}_D \to \infty} [\langle D(\vec{p}_D)|Q_C^5|A(\vec{p}_A)B(\vec{p}_B)\rangle - f_{A'B';C'D'}^{AB;CD}\langle D'(\vec{p}_D)|Q_{C'}^5|A'(\vec{p}_A)B'(\vec{p}_B)\rangle] = 0. \quad (3)$$

Here A', B', C', and D' are the SU(3) counterparts of A, B, C, and D, respectively, and by (c) each of the SU(3) partners has the same momentum. $f_{A'B';C'D'}^{AB;CD}$ is a unique number determined by the SU(3) Clebsch-Gordan coefficients. We can now relate Eq. (3) to physical processes thanks to PCAC.

By using the PCAC relation $\dot{Q}_C^5 = f_C m_C^2 \int \varphi_C(x)\,d^3x$ and defining $J_C = -(\Box - m_C^2)\varphi_C$, we obtain (suppressing helicity indices)

$$\langle D|Q_C^5|AB\rangle \propto \delta(\vec{p}_D - \vec{p}_A - \vec{p}_B) f_C m_C^2 (E_A + E_B - E_D)^{-1}(m_C^2 - p_C^2)^{-1}\langle D|J_C(0)|AB\rangle. \quad (4)$$

Define

$$(E_D)^{1/2}\langle D|J_C|AB\rangle(E_AE_B)^{1/2} = F_{CD}^{AB} \text{ and } K_{CD}^{AB}(s,t,p_C^2) = \sum_{\text{spin}}|F_{CD}^{AB}|^2.$$

Then $K_{CD}{}^{AB}$ is a Lorentz scalar and depends only on the invariant variables $s=(p_A+p_B)^2$, $t=(p_D-p_B)^2$, and $p_C{}^2=(p_A+p_B-p_D)^2$. We apply the limit $|\vec{p}_A|, |\vec{p}_B|, |\vec{p}_D| \to \infty$ with the conditions $\vec{p}_i \cdot \vec{p}_D > 0$ and $|\vec{p}_i \times \vec{p}_D| / |\vec{p}_D| < \infty$, where $i=A$ and B. Then, s and t become finite and $p_C{}^2 \to 0$. In this limit[6] $(E_A+E_B-E_D)^{-1}/(E_A E_B E_D)^{1/2} = 2(s-m_D{}^2)^{-1}(|\vec{p}_D|/|\vec{p}_A||\vec{p}_B|)^{1/2}$ and we obtain

$$\lim_{\vec{p}_A,\vec{p}_B,\vec{p}_D\to\infty} \sum_{\text{spin}} |\langle D|Q_C{}^5|AB\rangle|^2 \propto [f_C{}^2/(s-m_D{}^2)^2] K_{CD}{}^{AB}(s,t,0). \tag{5}$$

Therefore, Eqs. (3) and (5) lead to

$$\frac{f_C{}^2}{(s-m_D{}^2)^2} K_{CD}{}^{AB}(s,t,m_C{}^2=0) = (f_{A'B':C'D'}^{AB:CD})^2 \frac{f_{C'}{}^2}{(s'-m_{D'}{}^2)^2} K_{C'D'}{}^{A'B'}(s',t',m_{C'}{}^2=0). \tag{6}$$

In the SU(3) limit, $s=s'$ and $t=t'$. In broken SU(3) our limiting procedure yields some constraints among the (s,t) and (s',t').

The differential cross section (in the c.m. system) for the reaction $A+B\to C(\text{soft})+D$ is given by $d\sigma/d\Omega \propto (p_f/sp_i)K_{CD}{}^{AB}(s,t,0)$; thus Eq. (6) now gives the broken-SU(3) relation between the differential cross sections involving one *soft* pseudoscalar meson:

$$f_C{}^2\left(\frac{p_i}{p_f{}^3}\right)\frac{d\sigma}{d\Omega}^{AB\to CD}(s,t) = |f_{A'B':C'D'}^{AB:CD}|^2 f_{C'}{}^2\left(\frac{p_i{}'}{p_f{}'^3}\right)\frac{d\sigma}{d\Omega'}^{A'B'\to C'D'}(s',t'), \tag{7}$$

where we have used $2\sqrt{s}\,p_f=(s-m_D^2)$. Equations (6) and (7) are exact, in our framework, but are derived under the assumption that both C and C' are "soft." We find, however, that Eq. (7) indeed involves the $l=1$ effective angular-momentum barrier. Although the extrapolation from soft mesons to physical mesons is a complicated matter, we expect that the extrapolation preserves the effective angular-momentum barrier for physical processes. We can, in fact, demonstrate[7] this trend from the reactions involving both the pion and kaon in the final states which allow the explicit comparison between the soft-pion and -kaon limits. Therefore, we assume that our formula (7) is also satisfied reasonably well for physical reactions. The appearance of $l_{\text{eff}}=1$ for the total cross section in Eq. (7) suggest that the barrier effect for the partial wave may also take a form with $l=1$ in Eq. (2).

For the decay processes where C and C' are the pseudoscalar mesons, we obtain an analogous SU(3) relation for the width Γ by using the same procedure:

$$f_C{}^2(1/p_f{}^3)\Gamma(A\to B+C) = |f_{A';B'C'}^{A;BC}|^2 f_{C'}{}^2(1/p_f{}'^3)\Gamma(A'\to B'+C'). \tag{8}$$

Equation (8) is again exact when C and C' are "soft," but we assume that it is also valid for *physical* mesons. From the appearances of Eqs. (7) and (8), one may be tempted to think that only the angular momentum $l=1$ is actually contributing. However, the situation is vastly different. In our framework of (a), (b), and (c) there arises an SU(3)-breaking effect on the coupling constants involved. When the coupling constants are eliminated to obtain Eqs. (7) and (8), the net effect appears as the $l=1$ effective angular-momentum barrier. We can show this below in the soft-meson limit. As an illustration, consider the decay of the baryon, $A(J=L+\frac{1}{2}, P=\pm 1)\to B(\frac{1}{2}^+)+C(0^-)$, where L is a positive integer. With the effective-interaction Hamiltonian

$$H=g\bar{\psi}_{\mu_1\cdots\mu_L}(1 \text{ or } \gamma_5)\psi\partial_{\mu_1}\cdots\partial_{\mu_L}\varphi,$$

one can calculate the decay width using the Rarita-Schwinger wave function[8] of A,

$$\Gamma(J=L+\tfrac{1}{2}, p=\pm 1)\propto g^2[(A\pm B)^2-C^2]A^{-2}P_f{}^{2L+1}, \tag{9}$$

where the signs $\pm$ depend on the choice of P, and where A, B, and C denote the masses involved. Clearly, the usual barrier effect is present in Eq. (9). In the soft-meson limit ($C=C''=0$), Eqs. (8) and (9) give for the couplings g_{ABC} and $g_{A'B'C'}$

$$f_C g_{ABC}\frac{A\pm B}{A}\left(\frac{A^2-B^2}{A}\right)^{L-1} = f_{C'}|f_{A';B'C'}^{A;BC}|g_{A'B'C'}\frac{A'\pm B'}{A'}\left(\frac{A'^2-B'^2}{A'}\right)^{L-1}. \tag{10}$$

This turns out to be exactly the same as the one derived by Oneda and Matsuda[9] under the same assumption. Thus the true origin of the appearance of $l_{\text{eff}}=1$ is the *broken*-SU(3) coupling-constant relation, Eq. (10). Equation (10) gives a remarkable prediction[9,10] (consistent with experiment[10]) for the ratio of the $Y(1405)$ couplings, $R \equiv g_{YpK}/g_{Y\Sigma\pi}) = (Y-p)/(Y-\Sigma)$, with $f_\pi \approx f_K$.

We now compare our results with experiment. Consider the reactions appearing in Eq. (1). Equation (7) leads to

$$\tfrac{1}{3} f_\pi^2\left(\frac{p_i}{p_f^3}\right)\frac{d\sigma}{d\Omega}(\pi\Delta) = f_K^2\left(\frac{p_i'}{p_f'^3}\right)\frac{d\sigma}{d\Omega'}(KY^*) = f_\pi^2\left(\frac{p_i''}{p_f''^3}\right)\frac{d\sigma}{d\Omega''}(\pi Y^*) = f_K^2\left(\frac{p_i'''}{p_f'''^3}\right)\frac{d\sigma}{d\Omega'''}(K\Xi^*). \tag{11}$$

In the SU(3) limit, where $f_\pi = f_k$ and $p_f = \ldots = p_f'''$ if we take $p_i = \ldots = p_i'''$, Eq. (11), of course, reduces to Eq. (1). Equation (11) is almost the same as the one introduced *phenomenologically* by Trilling, if one choses to compare the above relations at the *same* scattering angle and the incident particle momentum in the lab frame. We now face the problem of how to compare the prediction, Eq. (11). In the SU(3) limit, $s' = s(p_i = p_i')$ and $t' = t(\theta = \theta')$, and SU(3) holds for each p_i, θ. In broken SU(3), mass splittings somewhat obscure the problem.[2] Trilling[1] and Rosenfeld[4] argued that the two SU(3)-related reactions should be compared at $p_i = p_i'$. For differential cross sections, one of the variables, θ, t, u, must also be chosen case by case, depending on the reactions. Although our limiting procedure imposes some constraints upon our variables, they are obtained only in the soft-meson limit and cannot be taken *too* seriously until we establish the reliable extrapolations. Within the errors of neglecting meson masses, we find that the constraints do not forbid[11] us to compare our relation (11) at the same p_i and θ for each reaction as in the case of exact SU(3). Then, Eq. (11) is also valid for the *total* cross section, replacing $d\sigma/d\Omega$ by σ.

Figure 1 shows the comparison of our prediction (with $f_K/f_\pi = 1$) with the experiments. The three curves for $\pi p \to \pi\Delta$, $\pi p \to KY^*$, and $Kp \to K\Xi^*$ are the predictions with the input reaction $Kp \to \pi Y^*$ given by the solid curve.[12] The agreement is good and resolves the discrepancies found by Ref. 2.

Figure 2 gives our prediction (with $f_K/f_\pi = 1$) on the Reaction (1b). The curve for $K^-p \to K^0\Xi^0$ is the prediction with the input reaction $K^-p \to \pi^+\Sigma^-$.

Since our result always involves the $l_{\rm eff} = 1$ angular-momentum barrier, it will resolve most of the similar difficulties encountered[1,2] when naive SU(3) is applied to the reactions *involving* pseudoscalar mesons.

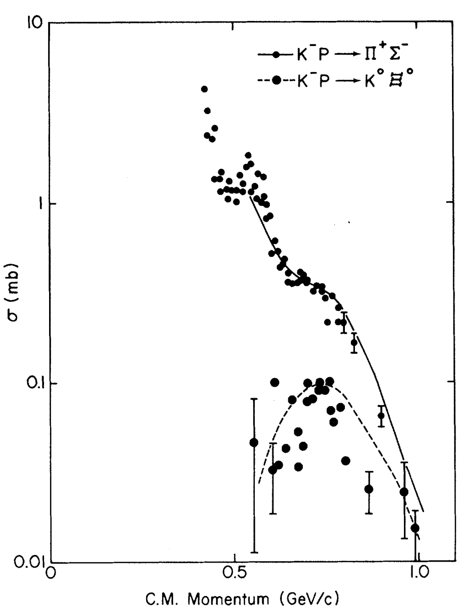

FIG. 2. Cross sections for the reactions $K^-p \to \pi^+\Sigma^-$ and $K^-p \to K^0\Xi^0$. The curve for $K^-p \to K^0\Xi^0$ is the prediction with the input reaction $K^-p \to \pi^+\Sigma^-$.

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[12]For Fig. 1, we used the data given in Refs. 1 and 2. To avoid confusion, in the figure we indicate the error bar only for $Kp\to\pi Y^*$.