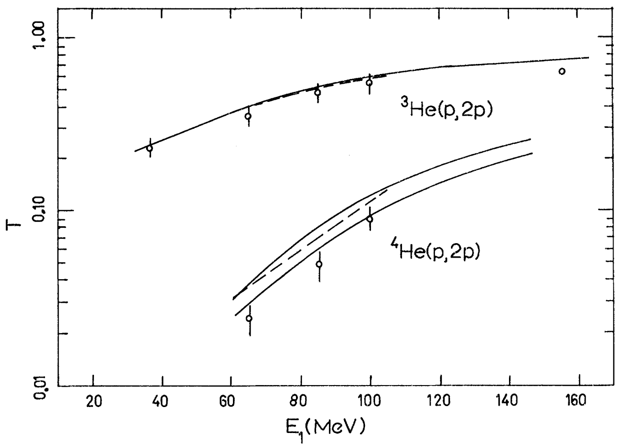

FIG. 3. Transmission coefficients from the attenuation model and from experiment (see Ref. 3) plotted against E_1, the incident proton energy, for the proton QFS reactions on ^{3}He and ^{4}He. Dashed lines, coefficients derived by Pugh *et al.* using the off-shell nucleon-nucleon scattering cross section of Ref. 10. The upper curve for $^4He(p,2p)^3H$ corresponds to the assumption $\sigma_T(p\text{-}^3H) = 2.5\sigma_T(p\text{-}n)$, while the lower one was computed using $\sigma_T(p\text{-}^3H) = 3.0\sigma_T(p\text{-}n)$.

$\times\sigma_T(p\text{-}n)$, respectively.

A look at our results indicates the extremely good fits of the attenuation model to the experimental data. The theoretical curves follow experiment closely for all three nuclei over the large range of incident proton energies considered. This consistency, over almost 2 orders of magnitude, is unlikely to be fortuitous and suggests physical reality underlying the semiclassical approach. Our success here leads us to conclude that all non-Faddeev analyses of coincidence experiments must include attenuation effects similar to those discussed in this paper. One also notes from Fig. 3 that the inclusion of off-shell effects in ^{4}He is consistent with our result for the more reasonable approximation to $\sigma_T(p\text{-}^3H)$, namely $\sigma_T(p\text{-}^3H) = 2.5\sigma_T(p\text{-}n)$. The off-shell curves in Fig. 3 are the results obtained by Pugh *et al.* using the computer code of Redish, Stephenson, and Lerner[10] and the Reid soft-core potential.

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Separability of the Neutrino Equations in a Kerr Background

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The first-order neutrino equations are shown to be separable in the Kerr-metric background. These equations are then used to show that the neutrino field is not superradiant, in contrast to the scalar, electromagnetic, and gravitational fields.

Recently Teukolsky[1] has shown that the electromagnetic and gravitational perturbations on a Kerr background are separable. Carter[2] had shown that the scalar equations are separable. The only other field with a consistent minimal-coupling generalization to a curved background is that described by the neutrino equations, which we have found also to be separable in Boyer-Lindquist[3] coordinates. Although the separability of the neutrino equations was derived by use of the Newman-Penrose[4] spin-coefficient formalism,[5] we present the results in the more traditional Dirac form.

The Kerr metric in Boyer-Lindquist coordinates is given by

$$ds^2 = -\frac{\Sigma\, dr^2}{\Delta} - \Sigma\, d\theta^2 + \frac{\Delta}{\Sigma}(dt - a\sin^2\theta\, d\varphi)^2 - \frac{\sin^2\theta}{\Sigma}[-a\,dt + (r^2+a^2)d\varphi]^2, \tag{1}$$

where

$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta.$$

A suitable choice of γ matrices obeying $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g_{\rm Kerr}{}^{\mu\nu}$ are given by

$$\gamma^t = \frac{r^2+a^2}{(\Sigma\Delta)^{1/2}}\gamma^0 + \frac{a\sin\theta}{\Sigma^{1/2}}\gamma^2,$$

$$\gamma^\varphi = \frac{a}{(\Sigma\Delta)^{1/2}}\gamma^0 + \frac{1}{\Sigma^{1/2}\sin\theta}\gamma^2, \tag{2}$$

$$\gamma^r = \left(\frac{\Delta}{\Sigma}\right)^{1/2}\gamma^3, \quad \gamma^\theta = \frac{1}{\Sigma^{1/2}}\gamma^1,$$

where γ^i are the ordinary flat-space Dirac matrices in the Bjorken-Drell[6] representation. The neutrino equation is now written as

$$\gamma^\mu(\partial/\partial x^\mu - \Gamma_\mu)\psi = 0, \tag{3}$$

where the Γ_μ are a set of spin-affine connections uniquely determined by the equations

$$\Gamma_\mu\gamma^\nu - \gamma^\nu\Gamma_\mu = \frac{\partial\gamma^\nu}{\partial x^\mu} + \Gamma_{\alpha\mu}{}^\nu\gamma^\alpha, \tag{4}$$

$$\mathrm{tr}(\Gamma_\mu) = 0. \tag{5}$$

We examine the solutions with $(1-\gamma^5)\psi = 0$. Letting $\psi = \binom{+\eta}{+\eta}$,

$$\eta = \frac{e^{-i\omega t}e^{-im\varphi}}{[\Delta\sin^2\theta\,(r + ia\cos\theta)^2]^{1/4}}\begin{pmatrix} R_1(r)S_1(\theta) \\ R_2(r)S_2(\theta)\end{pmatrix}, \tag{6}$$

we find the following set of coupled first-order equations for $R_1(r)$, $R_2(r)$ and $S_1(\theta)$, $S_2(\theta)$:

$$\left(\frac{d}{dr} - i\omega\frac{r^2+a^2}{\Delta} - i\frac{ma}{\Delta}\right)R_1(r) = \frac{k}{\Delta^{1/2}}R_2(r),$$

$$\left(\frac{d}{dr} + i\omega\frac{r^2+r^2}{\Delta} + i\frac{ma}{\Delta}\right)R_2(r) = \frac{k}{\Delta^{1/2}}R_1(r); \tag{7a}$$

$$\left(\frac{d}{d\theta} + \omega a\sin\theta + \frac{m}{\sin\theta}\right)S_1(\theta) = kS_2(\theta),$$

$$\left(\frac{d}{d\theta} - \omega a\sin - \frac{m}{\sin\theta}\right)S_2(\theta) = -kS_1(\theta). \tag{7b}$$

Here k is the separation constant determined by Eqs. (7b) for $S_1(\theta)$, $S_2(\theta)$ regular at $\theta = 0, \pi$. In the limit as $a = 0$, Eqs. (7b) reduce to those given by Schrödinger[7] for the spherically symmetric case. In the same limit Eqs. (7a) reduce to those for the Schwarzschild metric.[8] Similarly, as $M = 0$, we obtain the separated equations for neutrinos in flat space in spheroidal coordinates. No separation of variables seems possible with the above choice of Dirac matrices for the Dirac equation with mass.

The equations with $(1+\gamma^5)\psi = 0$ are obtained from those above by recalling that if ψ obeys the neutrino equations with $(1+\gamma^5)\psi = 0$, then $\hat{\psi} = \gamma^2\psi^*$ obeys those with $(1-\gamma^5)\psi = 0$ (the asterisk denotes complex conjugation).

For waves which are ingoing at the Kerr horizon $[\Delta = 0,\ r_H = M + (M^2 - a^2)^{1/2}]$, we find

$$R_1(r) \approx O(\Delta^{1/2}),$$

$$R_2(r) \approx \exp[-i(\omega + m\omega_H)]r^* + O(\Delta), \tag{8}$$

where r^* is defined by $dr^*/dr = (r^2+a^2)/\Delta$ and $\omega_H = a/2Mr_H$.

Defining the conserved neutrino number current as $J_{(x)}{}^\mu = \bar{\psi}_{(x)}\gamma^\mu\psi(x)$ ($\bar{\psi}$ being the Dirac adjoint $\psi^\dagger\gamma^0$), we find that the number density is always positive:

$$\sqrt{-g}\,J^t = \sqrt{-g}\,\bar{\psi}\gamma^t\psi = \frac{2}{\Delta^{1/2}}\Big([|R_1(r)S_1(\theta)|^2 + |R_2(r)S_2(\theta)|^2]\frac{r^2+a^2}{(\Sigma\Delta)^{1/2}}$$

$$+ i[R_2^*(r)R_1(r) - R_1^*(r)R_2(r)]\,S_1(\theta)S_2(\theta)\frac{a\sin\theta}{\Sigma^{1/2}}\Big)$$

$$> \frac{2}{\Delta^{1/2}}\left((|R_1S_1|^2 + |R_2S_2|^2)\frac{r^2+a^2}{(\Sigma\Delta)^{1/2}} - 2|R_2S_1||R_1S_1|\frac{a\sin\theta}{\Sigma}\right)$$

$$> \frac{2}{\Delta^{1/2}}\left((|R_1S_1| - |R_2S_2|)^2\frac{r^2+a^2}{(\Sigma\Delta)^{1/2}}\right) > 0.$$

Also, the net number current flowing down the black hole is always positive:.

$$\frac{\partial N}{\partial t} = -\int d\theta\, d\varphi \sqrt{-g}\, J^r = -2\int d\theta\, d\varphi [|R_1(r)|^2 S_1{}^2(\theta) - |R_2(r)|^2 S_2{}^2(\theta)] = -4\pi[|R_1(r)|^2 - |R_2(r)|^2]. \quad (10)$$

[We have normalized the angular functions so that $\int S_1{}^2(\theta)\,d\theta = \int S_2{}^2(\theta)\,d\theta = 1$.] This expression can be seen by Eqs. (7a) to be independent of r. By Eqs. (8) we have that $\partial N/\partial t = 4\pi |R_2(r_H)|^2 > 0$.

Of special interest also are the net energy and angular momentum flux carried down the black hole by the neutrinos. In particular, Bekenstein[9] has shown that if the ratio of energy to angular momentum carried down the black hole by a wave is given by ω/m, and if the conditions of Hawking's area theorem[10] are satisfied (namely, the positive-energy condition and the impossibility of naked singularities imply that the area of a black hole must increase), then superradiant scattering must occur; i.e., if $(\omega + m\omega_H) < 0$, the wave must extract energy from the black hole.

For neutrinos, the stress-energy tensor is given by[8]

$$T_{\mu\nu}{}^{(x)} = \frac{i}{8\pi}\bar{\psi}(x)\left[\gamma\mu\left(\frac{\partial}{\partial x^\nu} - \Gamma_\nu\right) + \gamma\nu\left(\frac{\partial}{\partial x^\mu} - \Gamma_\mu\right)\right]\psi(x) + \text{c.c.}$$

The conserved energy current is given by $T_t{}^\mu(x)$ and the conserved angular momentum current by $T_\varphi{}^\mu(x)$. Evaluating the energy and angular momentum currents down the black hole, we find

$$\partial E/\partial t = \int \sqrt{-g}\, T_t{}^r\, d\theta\, d\varphi = \omega \partial N/\partial t,$$

$$\partial L/\partial t = \int \sqrt{-g}\, T_\varphi{}^r\, d\theta\, d\varphi = m\partial N/\partial t.$$

The ratio of energy to angular momentum is ω/m, but we find no superradiant effect. For $\omega > 0$, there is always a net energy flux down the black hole, independent of the value of m.

This absence of superradiance is related to the fact[11] that the energy-momentum tensor $T_{\mu\nu}$ does not satisfy the positive-energy criterion inside the ergosphere (i.e., $T_{\mu\nu}t^\mu t^\nu < 0$ for $t_\mu t^\mu > 0$). Were the neutrino field a classical field, this result would be in immediate violation of the assumptions of both the area theorems and also singularity theorems of Hawking, Penrose, and Geroch.[12] The quantum-mechanical nature of fields make such conclusions unclear, and this question is being studied in detail elsewhere.

In addition, the intrinsic parity nonconservation of neutrinos (i.e., neutrinos have only one helicity) will lead to vacuum polarization effects about a spinning object (naively, the gravitational spin-spin interaction between neutrinos and a spinning object leads to vacuum polarization effects). This will also be discussed in detail elsewhere.

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