gradual motion of the plasma across the magnetic field. If the plasma surface has a convex curvature R and has thickness Δ, then the Rayleigh-Taylor instability develops with growth rate $\sim v_i(R\Delta)^{-1/2}$, where v_i is the ion thermal velocity. The growth of this instability is probably not important over the short time scales of interest (~ 50 nsec). The electric field at the plasma surface vanishes because of the space charge of the escaping ions. Thus a Rayleigh-Taylor instability at the plasma surface arising from the applied electric field is not expected to occur.

Within the electron sheath, two-stream microinstabilities between the counter-streaming electrons and between electrons and ions are of course possible. The effect of these is to cause scatter in the ion beam. This effect could be lessened by reducing the electron sheath thickness (decreasing $\mathcal{E}$) or by use of a geometrical configuration in which the ion beam is extracted without passing through the electron sheath. If both the electron sheath and anode plasma are reasonably stable then the cathode-anode spacing may be made very small.

As an illustration of the above formulas, consider a diode with voltage $V_0 = 2$ MV across a gap $d = 0.2$ cm, with a magnetic field $B_y = 70$ kG. From Eq. (7b) we find $\mathcal{E} \approx 0.1$ (which is small compared with unity as required for the theory), $x_* \approx 0.015$ cm, and $V_* \approx 100$ kV. From Eq. (8) the expected current density of ions, assumed to be deuterons, is $J_i \approx 3$ kA/cm^2. Thus with an anode area of ~ 35 cm^2 the total ion current is of the order of 10^5 A. A requirement on the anode plasma is that the random thermal flux of the ions, $\frac{1}{4}nv_i$, be much larger than the flux of escaping ions. That is, we need $\frac{1}{4}nv_i Ze > J_i$. For the above parameters this would require a deuteron plasma with, for example, $n \sim 10^{16}$ cm^{-3} and $T_i \sim 100$ eV. A powerful laser is clearly needed to furnish such a plasma.

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Controlled Excitation of Ion Acoustic Waves by Ion Sheet Beams

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When ion sheet beams modulated at a prescribed frequency are injected into plasmas, ion acoustic waves are found to be excited nearly perpendicularly to the ion sheet beams. The amplitude and frequency of the ion acoustic waves can be controlled by changing the modulation amplitude and frequency of the ion beams. The exciting mechanism is qualitatively explained by a fluid model.

There have been several methods for excitation of ion acoustic waves, i.e., coil excitation,[1] mesh excitation,[2,3] and large double-plasma-type excitation.[4] By using nonlinear effects of plasmas, the excitation of the higher harmonics of the ion acoustic waves is also reported,[5] which is useful for frequency conversion. Coil excitation has merit in that it does not disturb the plasma, as the coil can be set outside the plasma. However, it is thought to involve a weak coupling with plasmas. Mesh excitation is good for a localized excitation, but it has the disadvantage that insertion of the metal mesh creates direct disturbance in the plasma and it is not useful for high-temperature plasmas. Furthermore, the mesh excitation has been shown to excite a pseudowave easily[6] via the sheath around the mesh. The double-plasma device investigated by Taylor, Mackenzie, and Ikezi is good in exciting efficiency, but is not useful for localized excitation and high-temperature plasma. In this Letter, we report experimental and theoretical investigations of a new method for the controlled excitation of ion acoustic waves by using modulated ion sheet beams.[7]

Experiments were performed in a vacuum chamber, shown in Fig. 1(a), which is 160 cm long and 32 cm in diameter. Argon gas is used at a pressure of $P \approx 6.0\times10^{-4}$ Torr. Four oxide-coated cathodes (K) are set at different positions near the wall of the chamber (A) in order to obtain a uniform plasma. The wall of the chamber is used as an anode. The diffused plasma is used for the experiments. Langumuir probes are used to measure the electron and ion densities and the temperature of the electron gas. The typical plasma parameters are density $N_0 = 10^8 - 10^9$ cm^{-3} and electron temperature $T_e = 1-2$ eV. The plasma for the ion sheet beams is produced in the region A_b-K_b (A_b, the anode, K_b, the cathode). The plasma potential in the region A_b-K_b can be independent of that of the plasma produced by A-K. The application of a positive potential (V_{pb}) to the anode A_b relative to the other anode A results in the injection of ion beams into the plasmas. The ion beams are injected into the plasma through a slit 5.5 cm in length and 0.5 cm in width. By modulation of the applied potential (V_{pb}), ion charges perturbed with a prescribed frequency can be injected into the plasmas.

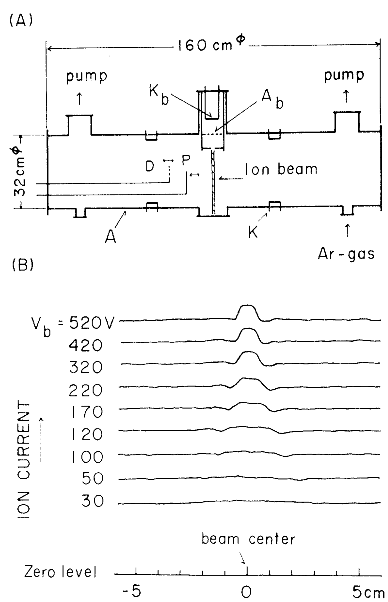

FIG. 1. (a) Simplified experimental setup. Anodes, A and A_b: cathodes, K and K_b; detector, D; probe, P. For ion wave excitation, the ion beams are modulated. (b) Ion current across the ion beams measured with a cylindrical probe, which is 5 cm away from the slit. Zero level is for the ion beam energy V_b=30 V. The level for a larger velocity is shifted successively.

Figure 1(b) shows the typical ion currents when a cylindrical probe is moved across the ion beams. The symbol V_b shows the ion beam velocity which is measured with a Faraday cup facing perpendicular to the ion beams. The ion beam velocity V_b is larger than the applied potential V_{pb} by 20 V for our experimental conditions. The ion currents show clearly the distribution of the ion beams in the z direction, where coordinates x, y, z are chosen such that x is perpendicular to the figure plane in Fig. 1(a), y is the direction along the ion beams, and z is perpendicular to the ion sheet beams. When the velocity of the ion beam is low, the beam diverges strongly in the z direction. However, the distribution does not greatly change for a beam energy greater than 300 eV. In the figure, the zero level is chosen for V_{pb}= 30 V. For a larger beam velocity, the zero level is shifted successively. As the ion current is measured with a cylindrical probe, twice the ion current of the plasma component is measured in comparison with that of the ion-beam component. So as to simplify the mechanism, the ion beam velocity for the wave excitation is chosen larger than that produced by V_b = 300 eV. Because the phase velocity of ion acoustic waves treated in the Letter corresponds to ion energies of nearly 1 eV, the use of ion beams faster than 300 eV results in a constant phase for the y (ion beams) direction, that is, the transit time of ion beams across the plasma can be neglected.

Figure 2(a) shows typical wave patterns, detected by a lock-in amplifier, of ion acoustic waves which are obtained perpendicular to the ion sheet beams. The frequency of the excited wave follows the modulation frequency of the ion beams. The phase of the excited wave for the y (ion beam) direction is nearly constant, that is, the wave can be said to propagate nearly perpendicular to the ion sheet beams. The observed wavelength and the relative amplitude are shown in Fig. 2(b). The wavelength is inversely propor-

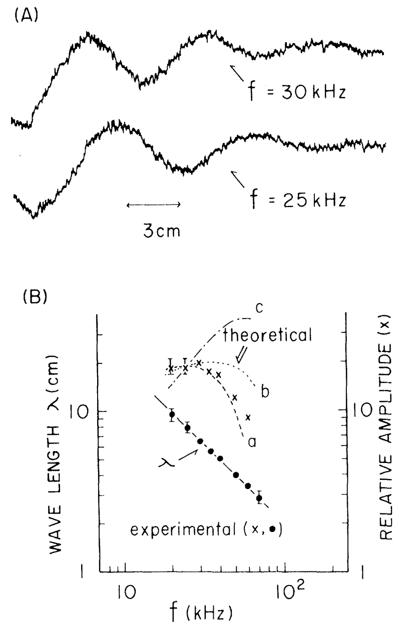

FIG. 2. (a) Typical wave patterns, detected by a lock-in amplifier, of ion waves which propagate nearly perpendicular to the ion sheet beams. V_b=420 eV. V_f=4 V peak to peak. (b) Wavelength and wave amplitude versus wave frequency. The theoretical curves of the wave amplitude are obtained (*a*) from the relation $\varphi(z) \propto \omega S_b \sin(kb)/k^{3/2}$, $b=1.5$ cm; (*b*) from $\omega S_b \sin(kb)/k^{3/2}$, $b=1.0$ cm; (*c*) from $\omega^2 N_b \sin(kb)/k^{3/2}$, $b=1.0$ cm in Eq. (8); all are normalized to the experimental results for f=25 kHz.

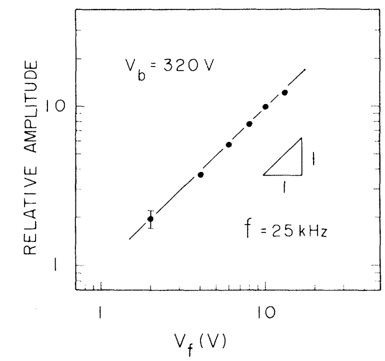

FIG. 3. Wave amplitude versus modulation voltage (V_f, peak-to-peak value). The ion beam velocity and the frequency are fixed.

tional to the exciting frequency, which is in accord with the property of ion acoustic waves, i.e., $\omega/k = f\lambda = (\kappa T_e/m_i)^{1/2} = \mathrm{const}$. The phase velocity $\sim 2\times10^5$ cm/sec is nearly in accord with the value calculated from the ion acoustic velocity $\omega/k = (\kappa T_e/m_i)^{1/2}$. The wave amplitude is plotted in terms of the value at the beam position, which is estimated using extrapolation that takes into account the damping length of the ion acoustic wave. The theoretical curve of the wave amplitude is discussed later. The excited wave amplitude is found to be proportional to the modulated amplitude of the ion beams as shown in Fig. 3, when the ion-beam velocity and the exciting frequency are fixed.

Now, we investigate theoretically the mechanism of the wave excitation by ion sheet beams. We simplify the model to the two-dimensional case working in the x-z plane, as the ion beam is long compared to the distance of observation. From the fundamental equations for ion waves (the continuity equations, the momentum equations, the linearized Boltzmann relation for the electrons, and Poisson's equation), one can obtain the following wave equation for ion waves:

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{pi}^2\right)\nabla^2\varphi-\frac{\omega_{pi}^2}{C_p^2}\frac{\partial^2}{\partial t^2}\varphi = -4\pi e\left(\frac{\partial}{\partial t}S_b+\frac{\partial^2}{\partial t^2}N_b\right), \tag{1}$$

where φ, $\omega_{pi}^2 = 4\pi N_0 e^2/m_i$, and $C_p^2 = \kappa T_e/m_i$ correspond to the wave potential, the ion plasma frequency, and the ion sound velocity, respectively. The effect of the injected beams is contained in two terms, viz., S_b representing a generating source in the ion continuity equation, and N_b for the number of injected charges in Poisson's equation. The term S_b is introduced here as a source term describing ionization due to the beam by a charge exchange or by other mechanisms. This term, which may explain the experimental results, is discussed later. In Eq. (1), the external sources are included in the right-hand side. If we assume the time dependence as $e^{j\omega t}$, Eq. (1) can be written as

$$\nabla^2\varphi + k^2\varphi = -\rho(\vec{r}), \tag{2}$$

$$k^2 \equiv \frac{\omega_{pi}^2}{C_p^2(-1+\omega_{pi}^2/\omega^2)}, \tag{3}$$

$$\rho(\vec{r}) \equiv \frac{4\pi e(j\omega S_b - \omega^2 N_b)}{-\omega^2+\omega_{pi}^2}. \tag{4}$$

The two-dimensional solution of Eq. (2) is obtained as

$$\varphi(\vec{r}) = (4j)^{-1}\int_{-a}^{a}dx\int_{-b}^{b}dz\,\rho(r')H_0^{(2)}(kr'), \tag{5}$$

where $H_0^{(2)}(kr')$ is a Hankel function of the second kind, and the beam is assumed to be infinitely long in the y direction. In Eq. (5), r' is the distance from the observation point to the region of integration in the x-z plane where charged particles exist, and $\vec{r}$ denotes the coordinate of the observation point. If the charges are assumed to be distributed uniformly over the cross section with a charge density ρ, one can obtain the following relation from Eq. (5) for the far-field approximation:

$$\varphi(R, \theta) = \frac{(1-j)\rho e^{-jkR}}{(\pi k R)^{1/2}} \frac{\sin(ka\sin\theta)}{k\sin\theta} \frac{\sin(kb\cos\theta)}{k\cos\theta}, \quad (6)$$

where R is the distance from the origin (the center of the beam cross section) to the observation point. The angle θ is that between the z axis and the line from the origin to the observation point, and the wave patterns in Fig. 2 are observed in the direction $\theta=0$. For the experimental condition $\theta=0$, Eq. (6) reduces to

$$\varphi(z) = \frac{(1-j)\rho a e^{-jkz}}{(\pi k z)^{1/2}} \frac{\sin(kb)}{k}. \quad (7)$$

By Eq. (7), the ion waves are found to be excited in the z direction by an injection of modulated ion beams. As $\rho = 4\pi e(j\omega S_b - \omega^2 N_b)/\omega_{pi}^2$ for $\omega^2 \ll \omega_{pi}^2$, $\varphi(z)$ becomes

$$\varphi(z) \propto (j\omega S_b - \omega^2 N_b)\sin(kb)/k^{3/2}. \quad (8)$$

Because the values of S_b and N_b are thought to be proportional to the modulated voltage of the ion beams, the experimental result of Fig. 3 that the wave amplitude is proportional to the modulation voltage of the ion beams is in accord with the theoretical result of Eq. (8). The frequency dependence expressed by $\omega S_b \sin(kb)/k^{3/2}$ is more in accord with the wave amplitude of the experimental results [Fig. 2(b)] than that contained in $\omega^2 N_b \sin(kb)/k^{3/2}$. For $2b=3.0$ cm, which is roughly twice the distance between the two valleys of the ion current in Fig. 1(b), the wave amplitude of the theoretical result is roughly in accord with the experimental result as shown in Fig. 2(b). The term $\sin(kb)$ becomes important as the charge width b cannot be neglected in comparison with the wavelength. However, the selection of $b=1.5$ cm which is larger than the width of the ion beams remains a problem. Further investigations on the amplitude of the excited waves are in progress.

In conclusion, ion acoustic waves are found to be excited and to propagate nearly perpendicular to ion sheet beams when modulated ion sheet beams are injected into plasmas. The excited wave can be controlled in frequency and amplitude by changing the modulation frequency and amplitude of the ion sheet beams. The amplitude of the excited wave is found to depend on the wavelength and the width of the ion sheet beams. The excitation mechanism is qualitatively explained with a fluid model, particularly by introducing the effect of the ion beams into the generating term in an ion-continuity equation. This type of excitation by modulated ion sheet beams has several merits: Localized wave excitation and excitation in high-temperature plasmas are possible, and the excitation efficiency may be good because the charged particles as a source couple directly with the ambient plasma.

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