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Generation of Intense Ion Beams in Pulsed Diodes*

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The generation of high-current (~ 10^5 A) pulsed ion beams with ion energy in the range 0.5–10 MeV appears to be possible by modifications of present electron-beam technology.

The success achieved in generating high-current pulsed relativistic electron beams[1] suggests the possibility of producing high-current (~ 10^5 A) pulsed ion beams in the 0.5–10 MeV range by modifications of this technology. Such ion beams would have a number of important applications: (a) Intense ion beams could be used to heat rapidly a plasma to fusion temperatures. The ion beam may be expected to interact directly with the plasma ions by collective processes provided the electron thermal velocity is greater than the ion beam velocity. (b) Ion beams of say D_2 with energy of the order of 1 MeV can produce fusion reactions directly when used to bombard a suitable target plasma. A net gain in energy is possible provided the electron temperature of the target is 5 keV.[2] Such electron temperatures could be produced by auxiliary means. (c) Intense ion beams could find use in nuclear studies which are beyond present capabilities, for producing intense neutron fluxes, isotope fluxes, and multiple nuclear reactions. It is clear that development of intense ion beams would provide a new technology which may have unforeseen uses.

Important problems for the efficient generation of intense ion beams are the suppression of the normally occuring electron current and the preparation of the anode surface so as to allow ions to be emitted readily. This Letter discusses conditions for the suppression of the electron current by application of a transverse magnetic field, requirements on an (externally produced) anode plasma in order for this plasma to serve as the ion source, and the expected space-charge–limited ion current. (A preliminary experiment[3] aimed at generating an ion beam in a pulsed diode has been done using a laser-heated metallic anode but without a transverse magnetic field. Preliminary experiments[4] have also been done on the suppression of the electron current with a magnetic field; however, a previous theoretical study[5] on the effect of a transverse magnetic field is inapplicable to the situation considered here because of the neglect of the electron space charge.)

By way of introduction it is noted that the high-current electron beams are produced basically by the application of a high-voltage (0.5–10 MeV) pulse (of typical length 50 nsec) to a diode consisting of two closely spaced metal surfaces. Electrons from the cathode surface are accelerated across the diode and extracted. The electron current across the diode is described approximately by the Child-Langmuir law[6] for space-charge–limited flow if account is taken of the fact that the effective cathode-anode gap decreases rapidly during a pulse as a result of the expansion of anode and cathode plasmas created by the beam emission.[7] We now discuss the ion current expected from a planar diode. Subsequently, we investigate the effect a transverse magnetic field has in preventing electrons from crossing the diode.

A planar diode is considered with constant separation d between the cathode and anode surfaces. We later point out some advantages of a different geometry. The x direction is taken to be normal to these surfaces, and it is assumed that the physical parameters are independent of y and z. The electric field is in the x direction, $\vec{E}=(E_x(x), 0, 0)$, and the corresponding potential $V(x)$ is takin to have $V(0)=0$ at the cathode and $V(d)=V_0$ at the anode, where V_0 is the voltage applied externally to the diode. With no magnetic field the self-consistent potential $V(x)$ within the diode is found in a straightforward way following the derivation of the Child-Langmuir law.[6] However, for the present situation we assume a plasma anode (created by external means) from which ions may be readily extracted. Thus the space charge within the diode arises from both electrons flowing from the cathode to the anode and ions flowing in the opposite direction. Assuming for simplicity nonrelativistic motion of both electrons and ions, one obtains

$$\frac{d^2V}{dx^2}=4\pi\left(\frac{m}{2e}\right)^{1/2}J_e[V^{-1/2}-r(V_0-V)^{-1/2}], \qquad (1)$$

where $r\equiv(M/Zm)^{1/2}J_i/J_e$, $-e$ and m (Ze and M) are the charge and mass of an electron (ion), and J_e and J_i are the electron and ion current densities, which are independent of x. Space-charge–limited emission at both the cathode and anode implies $(dV/dx)_{x=0}=0$ and $(dV/dx)_{x=0}=0$. Under these conditions Eq. (1) has a solution only for $r=1$,

$$J_i=(Zm/M)^{1/2}J_e, \qquad (2a)$$

$$J_e=(h^2/9\pi)(2e/m)^{1/2}V_0^{3/2}d^{-2}. \qquad (2b)$$

Equation (2b) is the standard Child-Langmuir current density except for the numerical factor

$$h=\tfrac{3}{4}\int_0^{\pi/2}d\theta\,\sin(2\theta)(\sin\theta+\cos\theta-1)^{-1/2},$$

which is of order unity. From Eq. (2a) it is clear[8] that almost all of the power supplied to the diode appears in the electrons; the ion power is only $(Zm/M)^{1/2}$ times the electron beam power.

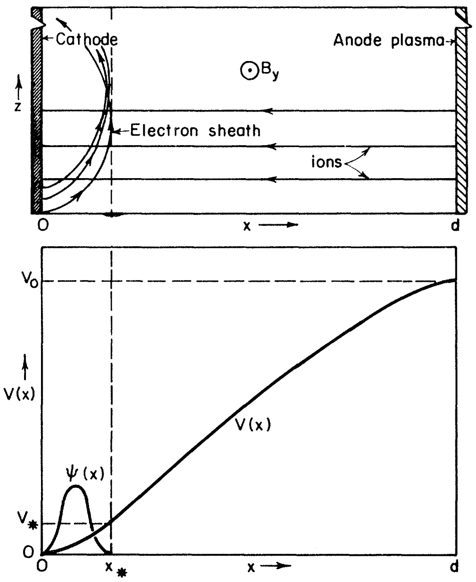

FIG. 1. Geometry of ion diode. $\psi(x)$ is enlarged relative to $V(x)$.

For the efficient generation of an ion beam in a diode device it is necessary to prevent the electron current from crossing the diode. This may be accomplished with a transverse magnetic field. We consider a uniform constant magnetic field $\vec{B}=(0, B_y, 0)$ which fills the diode gap. It is assumed that the magnetic field B_y is sufficiently strong to reverse the motion of the electrons acquired in the electric field within the diode gap but not so strong as to significantly influence the ion motion. Figure 1 is a sketch of the envisaged diode. We consider the limit of nonrelativistic electron motion and neglect the diamagnetic effect of the electron current in the z direction; the constraint imposed by these approximations is discussed later on. Utilizing the single-particle constants of the motion, namely the ion energy, the electron energy, and the electron canonical momentum $P_z=mv_z+eB_yx/c=0$, we obtain

$$d^2V/dx^2=4\pi(m/2e)^{1/2}J_e\{2U(x)[V-(m/2e)\omega_c^2x^2]^{-1/2}-r(V_0-V)^{-1/2}\}, \qquad (3)$$

where $\omega_c=eB_y/mc$ is the electron cyclotron frequency, J_e is the electron current density in the $+x$ direction (which is equal in magnitude to that in the $-x$ direction), and J_i is the ion current density. Both J_e and J_i are independent of x. In Eq. (3) the factor $U(x)$ is unity in the region accessible to electrons and zero otherwise.

The region accessible to electrons, the electron sheath, is determined by the condition

$$0 \leq \psi(x) \equiv V(x) - (m/2e)\omega_c^2 x^2. \quad (4)$$

This condition holds between $x=0$ and $x=x_*$.

In the region not accessible to electrons, $x_* < x \leq d$, the solution of Eq. (3) gives

$$V(x) = V_0 - (9\pi)^{2/3}(M/2Ze)^{1/3} J_i^{2/3}(d-x)^{4/3}, \quad (5a)$$

$$J_i = (1/9\pi)(2Ze/M)^{1/2}(V_0 - V_*)^{3/2}(d - x_*)^{-2}, \quad (5b)$$

[where $V_* = V(x_*)$] provided the ion emission at the anode surface is space-charge limited with $(dV/dx)_{x=0} = 0$. It remains to determine x_* and V_*.

Within the electron sheath, $0 \leq x \leq x_*$, we may neglect the x dependence of the ion velocity provided $V_* \ll V_0$. In this limit the contribution of the ion charge in Eq. (3) may be approximated by $-rV_0^{-1/2}$. For space-charge-limited emission of electrons from the cathode $[dV(0)/dx = 0]$, the first integral of Eq. (3) then gives

$$(d\psi/dx)^2 = 16\pi(2m/e)^{1/2} J_e \psi^{1/2} - (2m/e)(\omega_c^2 + \omega_p^2)\psi, \quad (6)$$

where $\omega_p \equiv (4\pi e J_i / m v_i)^{1/2}$ and $v_i = (2eZV_0/M)^{1/2}$. For sufficiently small x, it is seen that $\psi(x) \approx V(x) \propto J_e^{2/3} x^{4/3}$. For increasing x, $\psi(x)$ increases to a maximum and then decreases until $x = x_*$, where $\psi(x_*) = 0$. Evidently, $\psi(x)$ is symmetric about $x = x_*/2$.

At $x = x_*$, we have $\psi(x_*) = 0$ and $d\psi(x_*)/dx = 0$, which from Eq. (4) imply $V(x_*) = V_* = (m/2e)\omega_c^2 x_*^2$ and $dV(x_*)/dx = (m/e)\omega_c^2 x_*$. Matching these conditions to $V(x_*)$ and $dV(x_*)/dx$ obtained from Eq. (5a) gives

$$x_* = \tfrac{3}{2} d[1 - (1 - 8\mathcal{E}/q)^{1/2}], \quad V_* = \mathcal{E}^{-1}(x_*/d)^2 V_0, \quad (7a)$$

where

$$\mathcal{E} \equiv 2(eV_0/mc^2)(\omega_c^2 d^2/c^2)^{-1}, \quad (7b)$$

$\mathcal{E}^{1/2}$ is essentially the ratio of the electron gyroradius (corresponding to the energy it gains in the electric field) to the diode gap d. Evidently, for $\mathcal{E} < 1$ the magnetic field reverses the electron motion within the diode, i.e., $x_* < d$. However, our prior assumption in obtaining Eq. (6) requires $V_* \ll V_0$, which is satisfied with $\mathcal{E} \ll 1$. In this limit $x_* \approx (\frac{2}{3})\mathcal{E} d$ and $V_* \approx (\frac{4}{9})\mathcal{E} V_0$.

In the limit $\mathcal{E} \ll 1$ the ion current density is, from Eq. (5b),

$$J_i = (1/9\pi)(2Ze/M)^{1/2} V_0^{3/2} d^{-2}. \quad (8)$$

The electron current density obtained from Eq. (6) is

$$J_e = [(3/4\pi)\mathcal{E}^{-1/2}](1/9\pi)(2e/m)^{1/2} V_0^{3/2} d^{-2}. \quad (9)$$

One also finds $\omega_p^2/\omega_c^2 = \frac{2}{9}\mathcal{E} \ll 1$. From the approximate expressions for x_* and V_* given above, it is clear that the electron current density corresponds roughly to a diode (without a magnetic field) having a voltage V_* across an effective gap spacing of x_*.

The prior assumption that the electrons are nonrelativistic requires $eV_*/mc^2 \ll 1$, or $\frac{4}{9}(eV_0/mc^2\mathcal{E} \ll 1$. This condition can be satisfied even for $eV_0/mc^2 > 1$ by making the magnetic field sufficiently strong (so that $\mathcal{E}$ is sufficiently small). The neglect of the diamagnetic field ΔB_y arising from the electron current in the z direction requires $\Delta B_y/B_y \ll 1$. For $\mathcal{E} \ll 1$ an estimate is $\Delta B_y/B_y \sim (eV_0/mc^2)\mathcal{E}$. Thus the diamagnetic effect is negligible if the above condition for nonrelativistic motion holds. Our previous assumption that the ion motion is not influenced by the magnetic field may be expressed as $\mathcal{E} \gg Zm/M$.

It is noted that in a practical design of a diode it may be important to provide a closed path for the diamagnetic electron current; otherwise a space-charge gradient could build up and the associated electric field could allow transport of electrons across the diode gap. A cylindrical diode, with concentric cathode and anode surfaces and the magnetic field parallel to the axis of the cylinders, permits the diamagnetic current to flow in a closed path. However, in cases where it is impractical to provide a closed path for the diamagnetic current it would be necessary to rely on the high conductivity of the electrodes to short out the space-charge gradient.

The above discussion has assumed a plasma anode. A diamagnetic surface current in the plasma, $K_z = 4\pi p/B_y$ (where p is the plasma pressure), establishes an equilibrium, thus inhibiting plasma motion if $8\pi p/B_y^2 \ll 1$. The finite conductivity of the plasma does, however, permit a gradual motion of the plasma across the magnetic field. If the plasma surface has a convex curvature R and has thickness Δ, then the Rayleigh-Taylor instability develops with growth rate $\sim v_i(R\Delta)^{-1/2}$, where v_i is the ion thermal velocity. The growth of this instability is probably not important over the short time scales of interest (~50 nsec). The electric field at the plasma surface vanishes because of the space charge of the escaping ions. Thus a Rayleigh-Taylor instability at the plasma surface arising from the applied electric field is not expected to occur.

Within the electron sheath, two-stream microinstabilities between the counter-streaming electrons and between electrons and ions are of course possible. The effect of these is to cause scatter in the ion beam. This effect could be lessened by reducing the electron sheath thickness (decreasing $\mathcal{E}$) or by use of a geometrical configuration in which the ion beam is extracted without passing through the electron sheath. If both the electron sheath and anode plasma are reasonably stable then the cathode-anode spacing may be made very small.

As an illustration of the above formulas, consider a diode with voltage $V_0 = 2$ MV across a gap $d = 0.2$ cm, with a magnetic field $B_y = 70$ kG. From Eq. (7b) we find $\mathcal{E} \approx 0.1$ (which is small compared with unity as required for the theory), $x_* \approx 0.015$ cm, and $V_* \approx 100$ kV. From Eq. (8) the expected current density of ions, assumed to be deuterons, is $J_i \approx 3$ kA/cm^2. Thus with an anode area of ~35 cm^2 the total ion current is of the order of 10^5 A. A requirement on the anode plasma is that the random thermal flux of the ions, $\frac{1}{4}nv_i$, be much larger than the flux of escaping ions. That is, we need $\frac{1}{4}nv_i Ze > J_i$. For the above parameters this would require a deuteron plasma with, for example, $n \sim 10^{16}$ cm^{-3} and $T_i \sim 100$ eV. A powerful laser is clearly needed to furnish such a plasma.

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Controlled Excitation of Ion Acoustic Waves by Ion Sheet Beams

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When ion sheet beams modulated at a prescribed frequency are injected into plasmas, ion acoustic waves are found to be excited nearly perpendicularly to the ion sheet beams. The amplitude and frequency of the ion acoustic waves can be controlled by changing the modulation amplitude and frequency of the ion beams. The exciting mechanism is qualitatively explained by a fluid model.

There have been several methods for excitation of ion acoustic waves, i.e., coil excitation,[1] mesh excitation,[2,3] and large double-plasma-type excitation.[4] By using nonlinear effects of plasmas, the excitation of the higher harmonics of the ion acoustic waves is also reported,[5] which is useful for frequency conversion. Coil excitation has merit in that it does not disturb the plasma, as the coil can be set outside the plasma. However, it is thought to involve a weak coupling with plasmas. Mesh excitation is good for a localized excitation, but it has the disadvantage that insertion