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Experimental Demonstration of Backbending Behavior from a Band Crossing in ^{154}Gd†

T. L. Khoo, F. M. Bernthal, J. S. Boyno, and R. A. Warner
Cyclotron Laboratory, Departments of Physics and Chemistry, Michigan State University, East Lansing, Michigan 48824
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The members of the ground and β bands in ^{154}Gd have been identified to spin 18. Plots of $2\mathcal{J}/\hbar^2$ versus ω^2 yield backbending curves for both the β and yrast states. It is shown that the backbending in the yrast sequence results from a band crossing and it is speculated that the twin backbending may arise from the intersection of the ground and β bands by a third "intersecting band."

At high spins the rotational bands of a number of nuclei exhibit a phenomenon commonly referred to as backbending—a name derived from the characteristic S-shaped plots of the moment of inertia, $2\mathcal{J}/\hbar^2$, versus the square of the nuclear rotational frequency, $(\hbar\omega)^2$.[1-3] This anomalous behavior has been attributed to the Mottelson-Valatin[4] effect, which is a phase transition from the normal superfluid state to one in which the nucleon pairs have been coherently broken by the Coriolis force. It is also possible to explain the backbending effect in terms of a band which intersects the ground band to become the yrast band.[5,6] The question then reduces to one of the intrinsic character of the intersecting band: It might be the Mottelson-Valatin[4]-type unpaired band, the decoupled band of Stephens and Simon,[5] or perhaps even a quasiparticle or vibrational band. However, there has been no experimental evidence that explicitly demonstrates the band-crossing feature. We shall show here a case in which backbending in the yrast band can be directly attributed to band crossing. It would, however, be premature to generalize from this particular case that all other cases of backbending can be similarly explained.

The levels of ^{154}Gd have been studied by means of the reaction ^{154}Sm$(\alpha, 4n)^{154}$Gd using α beams from the Michigan State University sector-focused cyclotron. Measurements of the angular distributions of the γ rays and γ-γ coincidence experiments were performed with 48-MeV α beams, while excitation function measurements were made with beams of 41, 45, 48, and 50 MeV. Supplementary singles data from the reaction ^{152}Sm$(\alpha, 2n)^{154}$Gd with 24-MeV α's were also accumulated.

Two rotational bands—ostensibly the ground and β bands—have been identified and are shown in Fig. 1. The spin assignments were based on the angular-distribution and excitation-function data in the usual manner and also on the relative intensities of the γ rays in the $(\alpha, 4n)$ and $(\alpha, 2n)$ reactions. The assignments for the levels up to spin 16 agree with those previously reported.[7,8]. Each of the two spin-18 members, which have not been observed before, decays to both bands and there is an ambiguity regarding their assignment to the respective bands; we have chosen to group together in Fig. 1 the yrast and the yrare levels. [An yrare level is defined here as the first excited state above the lowest (yrast) state of the same spin.]

The interband-to-intraband $B(E2)$ ratio, $B(E2, I'\rightarrow I-2)/B(E2, I'\rightarrow (I-2)')$, given in Table I and in parentheses in Fig. 1, increases dramatically from $<10^{-3}$ to 1.4 as I' increases from 12 to 18; in addition $B(E2, 18\rightarrow 16')/B(E2, 18\rightarrow 16) = B(E2, 18'\rightarrow 16)/B(E2, 18'\rightarrow 16')$, within experimental errors. These features can be explained by con-

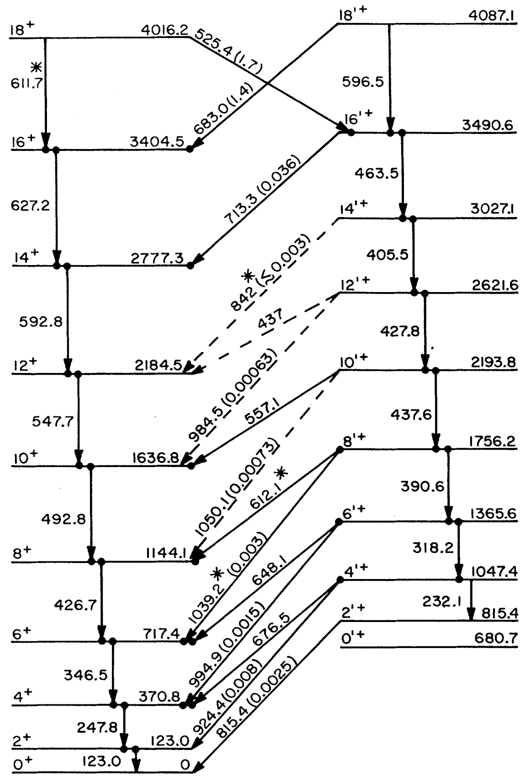

FIG. 1. Partial level scheme for ^{154}Gd, showing the yrast and yrare bands. The numbers in parentheses give the interband-to-intraband $B(E2)$ ratios, $B(E2,I' \to I-2)/B(E2,I' \to (I-2)')$ or $B(E2,I \to (I-2)')/B(E2,I \to I-2)$. The transitions indicated by asterisks are members of unresolved multiplets. The 612-keV line was separated into the $18^+ \to 16^+$ and $8^{+\prime} \to 8^+$ components based on the branching ratio, $I(8^{+\prime} \to 8^+)/I(8^{+\prime} \to 6^{+\prime})$, determined from the $(\alpha, 2n)$ reaction in which the $18^+ \to 16^+$ component was practically absent.

sidering an admixture between the ground and β bands. The wave functions for the states with spin I may be written as

$$|I\rangle = a_I|g\rangle + b_I|\beta\rangle, \quad |I'\rangle = b_I|g\rangle - a_I|\beta\rangle, \tag{1}$$

where the prime denotes the higher-lying state, and $|g\rangle$ and $|\beta\rangle$ are the unperturbed ground and β bands. a_I and b_I are the appropriate amplitudes given by

$$a_I b_I = H_{\beta g}/\Delta E \text{ and } a_I^2 + b_I^2 = 1, \tag{2}$$

where $H_{\beta g}$ is the interaction matrix element and ΔE is the observed separation between the two states with spin I. The $E2$ transition matrix elements connecting states differing in spin by 2 are then

$$\langle (I-2)' | Q | I'\rangle \approx (a_{I-2}a_I + b_{I-2}b_I)Q_0, \tag{3}$$

and

$$\langle (I-2)|Q|I'\rangle \approx (b_{I-2}a_I - a_{I-2}b_I)Q_0, \tag{4}$$

where it has been assumed that $\langle g|Q|g\rangle = \langle\beta|Q|\beta\rangle = Q_0$. The term $\langle g|Q|\beta\rangle$ has been neglected since $\langle g|Q|\beta\rangle \ll Q_0$. Using Eqs. (1)-(4), the $B(E2)$ branching ratios have been calculated using three interactions of the form (a) $H_{\beta g} = 0.078I(I+1)$, (b) $H_{\beta g} = 1.39[I(I+1)]^{1/2}$, and (c) $H_{\beta g} = 24.5$ keV (constant). In each case, the strength of the interaction has been adjusted to fit the $B(E2)$ ratios from the $I = 18$ members. As shown in Table I, the calculated $B(E2)$ ratios agree satisfactorily with the observed values for all three interactions over the limited range of spin values considered. The conclusions to be presented are thus relatively independent of the choice of interaction.

TABLE I. Interband-to-intraband $B(E2)$ ratios from two-band-mixing calculations.

I	a_I^2 [a]	b_I^2 [a]	$H_{\beta g}$ (keV)	$B(E2,I' \to I-2)/B(E2,I' \to (I-2)')$ Experiment	(a)[b]	(b)[b]	(c)[b]
18	0.83	0.17	26.7	1.66(35)[c]	1.52	1.50	1.51
18′	0.17	0.83	26.7	1.35(40)	1.52	1.50	1.51
16′	0.93	0.07	21.2	$3.6(3)\times10^{-2}$	3.8×10^{-2}	4.1×10^{-2}	4.3×10^{-2}
14′	0.996	0.004	16.4	$<3\times10^{-3}$	1.4×10^{-3}	1.7×10^{-3}	1.8×10^{-3}
12′	0.999	0.001	12.2	$6.3(1.4)\times10^{-4}$	1.6×10^{-4}	1.8×10^{-4}	1.5×10^{-4}
10′	1.0	0.0002	8.6	$7.3(7)\times10^{-4}$	3.9×10^{-5}	4.8×10^{-5}	1.6×10^{-5}

[a] a_I^2 and b_I^2 are the squares of the amplitudes of the β band and ground bands, respectively, in the mixed states; $H_{\beta g}$ is the magnitude of the interaction matrix element. The values given were derived using interaction (a).

[b] The three forms (a), (b), and (c) of $H_{\beta g}$ used in calculating the $B(E2)$ ratios are defined in the text.

[c] This value is for the ratio $B(E2, 18 \to 16')/B(E2, 18 \to 16)$.

The squares of the amplitudes and the interaction strength obtained using interaction (a) are also shown in Table I. [Similar values were obtained with interactions (b) and (c).] Note that the $I=18'$ (yrare) level contains more of the ground than of the β band, in contrast to the yrare levels of lower spin; conversely, the $I=18$ (yrast) level has a predominant β structure. A band crossing has therefore occurred between $I=16$ and 18. (A similar case of band crossing has also been observed[9] in ^{156}Dy.) Unless such a crossing occurs, the $B(E2)$ values of the transitions from each of the spin-18 members cannot have the comparable magnitudes observed. In general, for band mixing without band crossing, $B(E2, I' \to I-2)/B(E2, I' \to (I-2)') \ll 1$, provided the degree of mixing is similar (but not necessarily small) in the states with spins I and $I-2$. On the other hand, if the bands cross between spins I and $I-2$, the ratio $B(E2, I' \to I-2)/B(E2, I' \to (I-2)')$ is ∞ with no mixing, rapidly decreases to 1 with small admixtures ($\approx 15\%$), and becomes 0 in the limit of complete mixing. These results follow from Eqs. (3) and (4), with $a_I^2 \geqslant b_I^2$ when there is no band crossing, $a_I^2 \leqslant b_I^2$ when bandcrossing occurs, and $a_{I-2}^2 \geqslant b_{I-2}^2$ in both cases.

A conventional plot[10] of $2\mathcal{J}/\hbar^2$ versus $(\hbar\omega)^2$ for the yrast and yrare levels (closed symbols) is shown in Fig. 2(a). The yrare band shows the backbending feature which has also been observed by Ward *et al.*[7] In addition, the yrast band also exhibits the same phenomenon at spin 18. This is the first instance in which backbending occurs for more than one band in the same nucleus.[11] The open symbols in Fig. 2(a) represent the alternative band assignment of the spin-18 levels, i.e., according to the predominant intrinsic structure (ground or β) as opposed to the relative excitation (yrast or yrare). Clearly backbending does not occur in the predominantly ground band; it is a feature of the yrast sequence. Furthermore, from the previous discussion it is clear that backbending in the yrast sequence in ^{154}Gd arises from an intersection of the ground and β bands. This is the first experimental demonstration that backbending can result from a band crossing. In this regard, it should be noted that in previously reported cases, the anomalous behavior of backbending was a characteristic of the yrast levels, no distinction having been made between the yrast and the ground bands.

At the moment it is not clear whether backbending in other reported cases can be directly at-

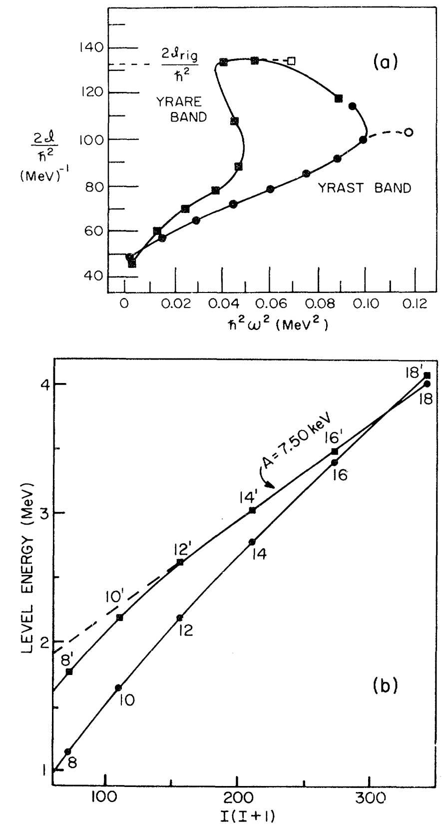

FIG. 2. (a) Plots of the moment of inertia against the nuclear rotational frequency for both the yrast and yrare levels (closed symbols). The open symbols represent the alternative band assignments for the spin-18 levels. The points were obtained in the manner described in Ref. 10. (b) Plot of the level energy against $I(I+1)$. The circles and squares denote the yrast and yrare levels, respectively; the lines connect states which are predominantly either ground or β band. The dashed portion, which is an extrapolation of the straight line through the points for $I=12'$, $14'$, $16'$, and 18 and which extends to $E=1.45$ MeV at $I=0$, illustrates the possible existence of the postulated "intersecting band."

tributed to a band crossing. Forking, which is the divergence of the yrast sequence into two branches and which may be interpreted as a band crossing in which only three of the four branches from the crossing point are seen, has been observed[12] in ^{100,102}Pd and also in other cases studied in this laboratory,[13] viz., ^{184,186}Os and ^{182}W. Thus, ^{154}Gd does not appear to be an isolated instance, although in this case the unusually large population of the yrare band has enabled us to trace the two bands which intersect. This is clearly illustrated in Fig. 2(b), where the level energies E have been plotted against $I(I+1)$. Notice that a straight line can be drawn through the points for $I=12'$, $14'$, $16'$, and 18. Thus, after the backbending region the β band behaves like a perfect rotor, with a constant moment of inertia, $2\mathcal{I}/\hbar^2 = 134\ \mathrm{MeV}^{-1}$, equal to that of the rigid-rotor value.

For $I \geq 12$, the "perfect rotor" behavior of the β band, the rigid moment of inertia, and the small matrix elements between its members and those of the ground band at these high spin values all lead one to question whether it still is the β band as such. In fact the twin backbending may be tentatively interpreted in terms of a third band which intersects the β band to become the yrare sequence for $I \geq 12$, and further intersects the ground band to form the yrast levels for $I \geq 18$. However, we have not yet found any direct evidence for the crossing of the β band and the postulated "intersecting band," nor is its intrinsic character understood. It would be difficult to identify it with the decoupled band of Stephens and Simon[5] since the level energies of such a band would not be expected to fit the straight line of Fig. 2(b). On the other hand, the observed rigid moment of inertia tempts one to speculate that it arises from an unpaired Mottelson-Valatin[4]-type state. Clearly further experiments are required to establish first the existence, and then the nature, of the postulated "intersecting band." There is also a need for a more critical examination of the level structure around the backbending region of yrast states than has heretofore been performed, in order to ascertain whether all such anomalous behavior results from a band crossing.

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Differential Cross Sections in $\pi^- p \to K^0\Lambda^0$ and $\pi^- p \to K^0\Sigma^0$ from 3 to 6 GeV/c*

C. E. W. Ward, I. Ambats, A. Lesnik,† W. T. Meyer, D. R. Rust,‡ and D. D. Yovanovitch§
Argonne National Laboratory, Argonne, Illinois 60439
(Received 10 August 1973)

We have measured the differential cross sections for the associated-production reactions $\pi^- p \to K^0\Lambda^0$ and $\pi^- p \to K^0\Sigma^0$ at 3, 4, 5, and 6 GeV/c, with a total of over 40 000 events. We find that both reactions have exponential forward peaks for $-t \lesssim 0.4\ (\mathrm{GeV}/c)^2$, with no indication of forward-direction flattening or turnover; the slopes of the forward peaks show little if any variation with momentum; and the two cross sections are equal within experimental error from $-t = 1.2\ (\mathrm{GeV}/c)^2$ out to at least $-t = 2.0\ (\mathrm{GeV}/c)^2$.

Knowledge of the associated-production reactions $\pi^- p \to K^0\Lambda^0$ and $\pi^- p \to K^0\Sigma^0$ at high energies is important for the amplitude analysis of hypercharge-exchange processes. Such an analysis requires high-quality data, and in this paper we present differential cross section measurements of these processes made at 3, 4, 5, and 6 GeV/c, with a total data sample of over 40 000 events.

This experiment was performed at the Argonne National Laboratory zero-gradient synchrotron accelerator, using the Argonne effective-mass spectrometer. The technique of the experiment