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Nonlinear Absorption of Radiation by Optical Mixing in a Plasma

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Parametric excitation of longitudinal plasma oscillations can occur when a plasma is simultaneously irradiated by two laser beams whose frequencies differ by $(2/n)\omega_{pe}$, $n=1, 2, 3, \ldots$, where ω_{pe} is the electron plasma frequency. We examine (i) the effect of a frequency mismatch on the excitation of the principal mode, (ii) the mode coupling in an inhomogeneous plasma, and (iii) the radiation absorption efficiencies.

Recent experimental[1] and theoretical results[2-4] indicate that enhanced absorption of radiation by a plasma can occur through the excitation of a large-amplitude longitudinal wave at the beat frequency of two laser beams. We investigate this excitation for a homogeneous, cold plasma, showing, in a weak-field approximation, that the parametric resonances fall into the regions of instability of the solutions of a nonhomogeneous Mathieu equation. The time dependence of the solutions is a function of the mismatch, $\delta\omega$, between the beat frequency, $\Delta\omega$, and the natural plasma frequency ω_{pe}. By considering $\delta\omega$ to be spatially dependent, we generalize our solution to the case of an inhomogeneous plasma. This treatment is presented as an alternative to that of Rosenbluth and Liu,[3] who solve the longitudinal wave equation directly, and to that of Kaufman and Cohen,[4] who formulate the interaction in terms of the local longitudinal dielectric constant with vanishingly small damping. The present method enables us to calculate the steady-state spatial distribution of power transfer as a function of the dissipation rate. In particular, in the presence of collisions, transfer occurs over a region defined by $\delta\omega(x)\simeq\pm\nu/2$, while when convection predominates, the zone of transfer is $\delta\omega(x)\simeq\pm\Delta\omega v_e^2/2v_p^2$; ν is the collision frequency, v_e the electron thermal velocity, and v_p the plasma-wave phase velocity. We then calculate the transfer efficiencies for parallel and antiparallel laser beam.

Consider two electromagnetic waves:

$$\vec{E}_i'(\vec{r},t)=\vec{E}_i\sin(\omega_i t-\vec{k}_i\cdot\vec{r}),\quad i=1,2, \tag{1}$$

linearly polarized in the positive y direction, propagating through a cold homogeneous plasma along the x direction, at frequencies ω_i much above the plasma frequency ω_{pe}. The equation of motion of an electron fluid element in the composite field $\vec{E}'=\vec{E}_1'+\vec{E}_2'$ is

$$\ddot{\vec{A}}+\nu\dot{\vec{A}}=(e/m)(\vec{E}'+\dot{\vec{A}}\times\vec{B}'/c+\vec{E}_s), \tag{2}$$

where $\vec{A}=\vec{r}-\vec{r}_0$ is the displacement of the fluid element from its equilibrium position $\vec{r}_0$, $\vec{E}_s$ is the self-consistent electrostatic field, and ν is the longitudinal damping rate. Assuming that $\alpha_i=(eE/mc\omega_i)\ll 1$, $A_x|\vec{k}_1-\vec{k}_2|\ll 1$, and $\nu\ll\omega_i$, and making use of $\omega_{pe}\ll\omega_i$, we find, for the longitudinal displacement A_x at the beat frequency $\Delta\omega$,

$$d^2A_x/d\tau^2+(\nu/\Delta\omega)dA_x/d\tau+[\delta-\epsilon\cos(\tau-kx_0)]A_x=-(\epsilon/k)\sin(\tau-kx_0). \tag{3}$$

Here, $\tau=\Delta\omega t$ and $\delta=(\omega_{pe}/\Delta\omega)^2$; ϵ and k depend on the mutual orientation of the wave vectors, $\vec{k}_i$. For parallel beams

$$k_p=k_2-k_1,\quad \epsilon_p=\tfrac{1}{2}\alpha_1\alpha_2, \tag{4}$$

and for antiparallel beams

$$k_a=k_2+k_1,\quad \epsilon_a=\tfrac{1}{2}\alpha_1\alpha_2(\omega_1+\omega_2)^2/\Delta\omega^2. \tag{5}$$

For small ϵ the regions of instability of the Mathieu equation (3) lie in the vicinity of the points $\delta = j^2/4$ in the (δ, ϵ) plane.[5] The frequency-matching conditions are, therefore, $\omega_{pe} = (j/2)\Delta\omega$, $j = 1, 2, 3, \ldots$. For $\epsilon < 1$ the growth rate of an unstable mode is of the order of ϵ^j. The right-hand side of Eq. (3) does not affect the stability regions, but we see that at the principal resonance, $j = 2$, we may expect parametric amplification along with the normal (exponential) instability. This amplification, as shown by Rosenbluth and Liu,[3] gives better coupling than the first-order instability at $\Delta\omega = 2\omega_{pe}$. We will therefore limit our analysis to first-order effects at the principal resonance, $\Delta\omega = \omega_{pe}$. Since for CO_2 laser beams of the same intensity, $\epsilon_a = 1.3 \times 10^{-14}\,\bar{S}$, where the laser beam Poynting flux $\bar{S}$ is in W cm^{-2}, we may, for intensities of interest, carry out an asymptotic analysis.[6] Writing

$$A_x(t, x_0) = a(t)\sin[kx_0 + \varphi(t) - \Delta\omega t], \tag{6}$$

we obtain to first order in ϵ for the slowly varying amplitude $a(t)$ and the phase $\varphi(t)$

$$\dot{a} + \nu a/2 = v_0 \sin\varphi, \tag{7}$$

$$a\dot{\varphi} = v_0 \cos\varphi - a\delta\omega, \tag{8}$$

where $v_0 = (\epsilon/2k)\Delta\omega$ and $\delta\omega = \omega_{pe} - \Delta\omega$. In order to clarify the behavior of the solution, we solve the system of Eqs. (7) and (8) in two limiting cases.

(A) When $\nu \to 0$, we obtain

$$a(t) = (2v_0/\delta\omega)\sin[(\delta\omega/2)t], \tag{9a}$$

$$\varphi(t) = \pi/2 - t\delta\omega/2. \tag{9b}$$

At perfect matching, $\delta\omega = 0$, we have $a = v_0 t$, in agreement with Rosenbluth and Liu.[3] When, however, $\delta\omega \neq 0$, the plasma wave is merely amplitude modulated.

(B) When $\nu \neq 0$, the system (7) and (8) admits of a steady-state solution

$$a_s = (\nu^2/4 + \delta\omega^2)^{-1/2}, \tag{10a}$$

$$\tan\varphi_s = \nu/2\delta\omega. \tag{10b}$$

The nonzero, constant phase shift φ_s is responsible for the irreversible transfer of energy from the transverse waves to the plasma, in contrast to the first case where the apparently irreversible transfer at $\delta\omega = 0$ results only in storing energy in the plasma wave.

We will now deal with the effect of plasma inhomogeneity, using the above results as a local approximation. This is possible when the wave number k is much larger than the reciprocal of the density-gradient scale length. Let us first calculate, in steady state, the average power $\bar{p}(x)$ transferred per unit volume located around a specific point in space given by $\delta\omega(x)$. We have

$$\bar{p}(x) = n(x)(1/T)\int_0^T \dot{A}_x F\,dt, \tag{11}$$

where T is a beat period or integral multiple thereof, $n(x)$ is the local electron density, A_x is given by (6), and the driving force $F = m\Delta\omega^2(\epsilon/k) \times \sin(\Delta\omega t - kx_0)$. With $\omega_1 + \omega_2 = 2\omega_L$ and $\bar{S}_i = cE_i^2/8\pi$, we obtain

$$\bar{p}(x) = \beta I(x, \nu)\bar{S}_1(x)\bar{S}_2(x), \tag{12}$$

where the coupling constant β for parallel and antiparallel beams is, respectively,

$$\beta_p = 2\pi^2\left(\frac{e}{mc^2}\right)^2\left(\frac{\Delta\omega}{\omega_L}\right)^4\frac{1}{\Delta\omega}, \tag{13}$$

$$\beta_a = 8\pi^2\left(\frac{e}{mc^2}\right)^2\left(\frac{\Delta\omega}{\omega_L}\right)^2\frac{1}{\Delta\omega}, \tag{14}$$

and the function

$$I(x, \nu) = \left(\frac{\omega_{pe}}{\Delta\omega}\right)^2\frac{1}{4\pi}\,\frac{\Delta\omega\,\nu}{\nu^2/4 + \delta\omega^2} \tag{15}$$

describes the spatial distribution of energy transfer. We may now define the width d of the interaction region as the full width at half-maximum amplitude of the distribution (15), i.e., the boundaries, $x_\pm$, of the region satisfy the equation $\delta\omega(x_\pm) = \pm\nu/2$. For example, in the case of a linear density distribution $(\omega_{pe}/\Delta\omega)^2 = 1 + x/l$, we have $\delta\omega = \Delta\omega x/2l$, so that $d = 2\nu l/\Delta\omega$. In the limit of vanishingly small dissipation $(\nu \to 0)$, we have

$$I(x, \nu \to 0) = \tfrac{1}{2}\delta(\mu(x)), \tag{16}$$

with $\mu = \delta\omega/\Delta\omega$, reflecting the boundedness for all times of the plasma wave amplitude except at $\delta\omega = 0$. One expects, therefore, that (16) must follow directly from (9) and (11) in the limit $T \to \infty$, as can indeed be demonstrated.

The function $I(x, \nu)$ in its form (15) directly represents the effect of collisions if ν is identified as the collision frequency. In contrast, let us now consider the collisionless case in which all power transferred to the plasma wave is convected away by the latter to be dissipated outside the region of transfer. The relevant damping rate ν_{conv} to be utilized in (15) is the inverse of the time when balance is attained between the power flowing into the growing, resonant mode and the outgoing flux associated with its propagation into the region of lower density. Assuming that the plasma wave is resonantly generated over a spatial region of the order of its inverse wave number, $1/k$, we obtain $\nu_{\text{conv}}=kv_g=\Delta\omega v_e^2/v_p^2$, where v_g, v_p, and v_e are the wave group, wave phase, and electron thermal velocities, respectively. The width d_{conv} of the interaction region, for a linear density distribution, is $d_{\text{conv}}=2lv_e^2/v_p^2$. We note that Perkins and Flick[7] found an identical interaction region for the excitation of the oscillating two-stream instability on a density gradient.

We may now proceed with the calculation of the absorption efficiency η, defined as the fraction of transverse flux transferred to the plasma waves:

$$\eta = 1-(\bar{S}_1^{\text{out}}+\bar{S}_2^{\text{out}})/(\bar{S}_1^{\text{in}}+\bar{S}_2^{\text{in}}), \tag{17}$$

where $\bar{S}_i^{\text{in}}$ and $\bar{S}_i^{\text{out}}$ are the incident and exciting transverse fluxes, respectively. The values of $\bar{S}_i^{\text{out}}$ are obtained by integrating the equation of transverse flux conservation

$$d\bar{S}_1/dx \pm d\bar{S}_2/dx = -\bar{p}(x) \tag{18}$$

using the condition of photon number conservation,

$$\bar{S}_1/\omega_1 \pm \bar{S}_2/\omega_2 = \text{const.} \tag{19}$$

The signs + and − refer to parallel and antiparallel beams, respectively. Let us assume $\omega_1>\omega_2$. The variables are separable, giving the following:

(i) For parallel beams,

$$\bar{S}_1^{\text{out}}=\sigma S/(\sigma+\Omega y), \tag{20}$$

$$\bar{S}_2^{\text{out}}=yS/(\sigma+\Omega y), \tag{21}$$

where $S=\bar{S}_1^{\text{in}}+\Omega\bar{S}_2^{\text{in}}$, $\sigma=\bar{S}_1^{\text{in}}/\bar{S}_2^{\text{in}}$, $\Omega=\omega_1/\omega_2$, $y=\exp[(\sigma+\Omega)q_p]$, and $q_p=\beta_p\bar{S}_2^{\text{in}}L/(\Omega-1)$, with L defined by (25).

(ii) For antiparallel beams, with $\vec{S}_1$ propagating in the positive x direction we have

$$\bar{S}_1^{\text{out}}=\bar{S}_1^{\text{in}}(\Omega+b_r)/\sigma, \tag{22}$$

$$\bar{S}_2^{\text{out}}=\bar{S}_2^{\text{in}}(\sigma-b_r)/\Omega, \tag{23}$$

where b_r is the nonzero root of the equation

$$(\sigma-b)(\Omega+b)=\Omega\sigma\exp(bq_a), \tag{24}$$

with $q_a=\beta_a\bar{S}_2^{\text{in}}L/(\Omega-1)$. The quantity

$$L(\nu)=\int_{(x)}I(x,\nu)\,dx \tag{25}$$

has the dimensions of length, depends in general on the dissipation rate, and is characteristic of the density distribution at $\delta\omega(x)=0$. Two special cases are of particular interest: (a) For approximately linear distributions, $\omega_{pe}^2(\mu\sim 0)=\Delta\omega^2(1+x/l)$, L is equal to the density-gradient scale length l, and is independent of the dissipation rate. (b) For a resonance at the maximum density, $\omega_{pe}^2(\mu\sim 0)=\Delta\omega^2(1-x^2/l^2)$, we obtain, in contrast to the latter case, $L\simeq l(\Delta\omega/\nu)^{1/2}$ (equal to lv_p/v_e when convection predominates).

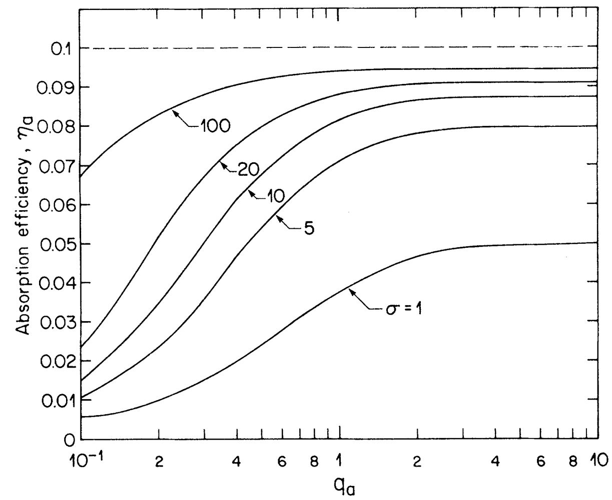

FIG. 1. The absorption efficiency η_a for the mixing of two antiparallel CO_2 laser beams of incident intensities $\bar{S}_1^{\text{in}}$ and $\bar{S}_2^{\text{in}}$ at wavelengths $\lambda_1=9.6\ \mu\text{m}$ and $\lambda_2=10.6\ \mu\text{m}$, respectively, as a function of $q_a=13.6\times10^{-13}\bar{S}_2^{\text{in}}L$. L is the length defined by (25) and $\sigma=\bar{S}_1^{\text{in}}/\bar{S}_2^{\text{in}}$. The dashed line corresponds to the maximum power extraction, $\Delta\omega/\omega_1$.

The above results are no longer valid when the inverse of the damping rate, $t_d=1/\nu$, is larger than the time t_{nd} at which the amplitude (proportional to v_0t) of the growing, resonant mode is limited by some nondissipative mechanism.[8] When t_d exceeds t_{nd}, the dissipation rate is reduced by a factor $\simeq(t_{nd}/t_d)^2$. Also, the singularity of $I(x,\nu)$ at $\delta\omega=0$ is now removed, so that $L\to 0$ (and, therefore, $\eta\to 0$) as $\nu\to 0$.

The transfer (or absorption) efficiencies η can be represented model free as functions of the parameters σ and q. As a function of σ, η increases to the asymptotic value $\Delta\omega/\omega_1$, whereas, as a function of q, η saturates below $\Delta\omega/\omega_1$, at a value dependent on σ. Because of better coupling ($\beta_a\gg\beta_p$), the antiparallel case is always more efficient (for the same S_{10}, S_{20}, and L, $\eta_a>\eta_p$). In Fig. 1, η_a is plotted as function of q_a for various values of the parameter σ. In the case of two CO_2 laser beams at 9.6 and 10.6 μm, respectively, we have $\Omega-1=0.1$ and $\beta_a=13.6\times10^{-14}\ \text{W}^{-1}\ \text{cm}$. Take then, for example, $L=2$ cm and $\sigma\simeq 20$. If we require 9% efficiency, then

$q_a \simeq 1.5$ and it follows that the required intensities would be $\bar{S}_2^{in} = 5.5 \times 10^{11}$ W cm^{-2} and $\bar{S}_1^{in} = 1.1 \times 10^{13}$ W cm^{-2}. Such power densities are presently available.[9,10]

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Experimental Investigation of Plasma Heating by a High-Frequency Electric Field near the Electron Cyclotron Resonance in the FM-1 Spherator*

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Plasma heating due to a high-frequency electric field near the electron cyclotron resonance is investigated in a toroidal plasma confinement device, the FM-1 spherator. It is observed that electrons and ions are heated "anomalously" when the incident high-frequency field exceeds a threshold value. Above the same threshold the parametric decay instability of upper and lower hybrid waves takes place. We investigate the effect of the decay instability and plasma heating on the confinement time.

Plasma heating by high-power, high-frequency electric fields near the electron cyclotron frequency have been investigated in a number of experiments.[1-4] Recently, similar experiments have been also performed in toroidal devices such as the TM-3[5] and the Lawrence Livermore Laboratory Levitron.[6]

In this Letter we report results of detailed experimental studies of plasma heating and confinement properties when a high-frequency electric field with high power is applied near the electron cyclotron frequency (i.e., the upper hybrid frequency) in a toroidal device, the FM-1 spherator. The main subjects of the present investigation are as follows: (a) Anomalous ion heating is studied by measuring the ion temperature with the Doppler-broadening method. Above a threshold of incident rf power an anomalously fast heating of the main body of ions is observed. (b) This threshold is shown to correspond to excitation of the parametric decay instability of upper and lower hybrid waves.[7-9] The presence of such an instability is observed experimentally, and it is proposed that parametrically excited lower hybrid waves are responsible for the observed ion heating. (c) The effect of decay instability and plasma heating upon particle confinement is also investigated. (d) Since the experiment can be carried out with much lower neutral pressure than in linear devices, it is possible to calculate accurately the distribution of input energy in the plasma. To the best of our knowledge this is the first time that the foregoing points (a) and (c) have been demonstrated experimentally.[10]

The FM-1 spherator has a superconducing ring levitated magnetically to produce closed magnetic surfaces for confining the plasma.[11,12] The superconducting ring is excited with $I_p = 275$ kAt, and the ratio of the ring current I_p to the toroidal field current I_T is 0.94. In this magnetic field configuration the fluctuation level is minimum.[11] The average magnetic field is about 2 to 4 kG. Plasmas are produced by using 10.5-GHz microwave power with a filling helium gas of 4×10^{-7}