tering contributions at large angles. However, from our calculations the double scattering will become more important for higher vibrational-level excitations or at lower electron impact energies. These and other results will be reported later in a detailed publication. The excellent result from the present calculation encourages further exploration of this simplified general approach for the molecular vibrational excitations.

It is a pleasure to acknowledge valuable discussions with Dr. S. Trajmar and Dr. D. G. Truhlar.

*Research supported in part by the National Aeronautics and Space Administration.

[1]S. Trajmar, D. G. Truhlar, J. K. Rice, and A. Kuppermann, J. Chem. Phys. 52, 4516 (1970).

[2]R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.*, (Interscience, New York, 1959), Vol. I, p. 315; V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

[3]This separation of the two regions is only meant for a physical description. In actual calculation, no critical impact parameter is needed to divide the two regions. Rather, the cross-section contributions from these two potentials are simply calculated separately and then summed.

[4]D. G. Truhlar and J. K. Rice, J. Chem. Phys. 52, 4480 (1970), and references therein. We have used the value of $1.8a_0$ for the cutoff parameter a_p, consistent with those obtained by others.

[5]W. Kolos and L. Wolniewicz, J. Chem. Phys. 46, 1426 (1967).

[6]S. C. Wang, Phys. Rev. 31, 579 (1928).

[7]V. Franco, Phys. Rev. Lett. 20, 709 (1968); H. Tai, R. H. Bassel, E. Gerjuoy, and V. Franco, Phys. Rev. A 1, 1819 (1970); F. W. Byron, Jr., Phys. Rev. A 4, 1907 (1971).

[8]P. M. Morse, Phys. Rev. 34, 57 (1929).

Radiative Capture and Bremsstrahlung of Bound Electrons Induced by Heavy Ions*

P. Kienle,† M. Kleber,† B. Povh,‡ R. M. Diamond, F. S. Stephens, E. Grosse,§ M. R. Maier,† and D. Proetel†
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 31 July 1973)

X rays emitted during the radiative capture of target electrons into the K shell of fast and highly stripped projectiles were observed. When using targets with strongly bound electrons, a high-energy x-ray tail was also found; this new process is a type of electron bremsstrahlung. The cross sections for both these processes can be explained quantitatively.

In the x-ray emission accompanying the passage of fast highly stripped heavy ions through gases, we have been able to distinguish three types of beam-associated radiation:

(1) Characteristic x rays emitted following electron transfer into the L, M, and outer shells of a stripped ion.

(2) A broad x-ray band from the radiative capture of bound electrons from the target into the K shell of the fast projectile. The energies correspond to the difference of the electron binding energies in the initial and final state plus the electron kinetic energy relative to the projectile, and reflect the momentum distribution of the bound electrons. This radiation has been recently identified by Schnopper *et al.*[1]

(3) A high-energy tail going up to an energy corresponding roughly to $\hbar v/a$, where v is the projectile velocity and a is equal to the Bohr radius. This new process we ascribe to bremsstrahlung from initially bound target electrons in the Coulomb field of the projectile.

The x-ray emission was studied when high-energy ^{40}Ar, ^{20}Ne, and ^{14}N beams passed through targets of gaseous molecules and simple noble gases up to argon. Beams of ^{40}Ar were obtained from the Lawrence Berkeley Laboratory (LBL) SuperHILAC (heavy-ion linear accelerator) at energies up to 288 MeV (7.2 MeV/nucleon) and then stripped in the 6-μm Al entrance window of the gas target cell to an equilibrium average state of about 17^+. The ^{20}Ne and ^{14}N were accelerated in the LBL 88-in. cyclotron to 140 MeV (7.0 MeV/nucleon) and to 160 MeV (11.4 MeV/nucleon) and 250 MeV (17.9 MeV/nucleon), respectively, and were essentially completely stripped upon entering the gas target. The x-ray spectrometer was a 5-mm-diam Si detector coupled to a low-noise pulsed-light feedback amplifier with a peaking time of 9.5 μsec and a fast pileup rejector. The energy resolution measured with Fe $K\alpha$ x rays was 165 eV full width at half-maximum. The instantaneous count rates were kept below 3000/sec to avoid spectral distortions due to pileup. The x rays at 90° to the beam were collimated to an angular divergence of less than 0.1 rad to keep the Doppler broadening of the x rays emitted by the fast ions smaller than the detector resolution. Using a ^{57}Co standard source, we could determine the detection efficiency at 6.4 and 14.4 keV absolutely. The background produced by the beam passing through the evacuated chamber was measured to be negligible in all cases.

Figure 1 shows sample x-ray spectra from the He and Ne gas targets. In the Ar run, besides the $K\alpha$ and $K\beta$ lines which have energies expected for transitions in Ar^{16+}, one sees at higher energy a broad line. Such lines (marked with arrows in Fig. 1) are beam associated, because their peak energies change only with the energy and the species of the projectile, but not appreciably with the target atom. In agreement with Ref. 1, we assign these to radiative capture of bound electrons from He or Ne into (mainly) K orbits of the highly stripped projectiles. If one considers the electrons quasifree, with an intrinsic momentum $\vec{p}_i$ in a potential U_i, moving with a momentum $\vec{k}$ towards the projectile considered at rest, one can intuitively derive the radiation frequency ω when the electron is captured in an orbit with a binding energy E_f:

$$(\vec{p}_i+\vec{k})^2/2m_e+U_i=\hbar\omega+\boldsymbol{E}_f \text{ or } \hbar\omega=(E_i-E_f)+k^2/2m_e+\vec{k}\cdot\vec{p}_i/m_e. \qquad (1)$$

This energy for the capture maximum is in fair agreement with the experimental results.

In addition, in the spectra of all targets with $Z>6$ we observed high-energy tails, such as those shown in Fig. 1. These radiation tails must have a different origin because their intensity relative to the capture peak increases with larger beam velocities. We assign this radiation component to bremsstrahlung from strongly bound electrons. Such electrons can take up more momentum during an encounter with the projectile (before they get knocked out of the target) than can loosely bound electrons. The cross section for bremsstrahlung radiated by an electron during a collision with a heavy particle of charge Z and velocity v is classically given by[2] (we use atomic units)

$$\frac{d^2\sigma_B}{d\Omega\, d\omega}\approx\frac{2}{c\pi\omega}\left(\frac{Z\sin\theta}{cv}\right)^2\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right). \qquad (2)$$

For a given frequency ω one has $\rho_{\max}\approx v/\omega$ because for impact parameters larger than $\rho_{\max}$ the collision time $\tau\approx\rho/v$ is too long to produce a significant radiation at that frequency. For bound electrons the quantum limit on the impact parameter is the limit down to which the electron can be localized. The upper limit to the bremsstrahlung spectrum $\omega_{\max}\approx v/\rho_{\min}\approx v/a$ increases with decreasing orbital radius a. Thus, strongly bound $1s$ electrons lead to high-frequency bremsstrahlung and we write $\omega_{\max}=\lambda v\langle 1/r\rangle_{1s}$, where λ is a factor of the order of 1.

Loosely bound electrons, however, contribute mainly to the production of high-energy x rays via radiative capture into inner atomic shells of the projectile. The radiative capture of completely free electrons is treated by Bethe and Salpeter.[3] Bound target electrons are characterized by their Fermi motion, i.e., by a distribution of velocities. If the velocity of the projectile is high compared to the orbital velocity of the target electrons, one can use unperturbed wave functions for the electron in target and projectile. We calculate the cross section for radiative capture within the impact parameter method.[4] For hydrogenlike wave functions, the result for the capture cross section due to the $\vec{A}$ field of the projectile is

$$\frac{d^2\sigma}{d\Omega\, d\omega}=N_iN_f\frac{256\omega\sin^2\theta}{(\pi v)^2c^3}\int d^3p\,\frac{\kappa_i^{\,5}\kappa_f^{\,5}(k+p_z)^2}{[(\vec{p}+\vec{k})^2+\kappa_f^{\,2}]^4(p^2+\kappa_i^{\,2})^4}\,\delta((\omega-k^2/2-E_i+E_f-p_zk)/k). \qquad (3)$$

N_i and N_f stand, respectively, for the number of electrons in the target shell with principal quantum number n_i and for the number of available electron states in the projectile shell with principal quantum number n_f. The momenta κ_i and κ_f depend on the screened nuclear charges Z_{sc} through $\kappa=Z_{sc}/n$. The δ function contains the dispersion relation of Eq. (1). In the region where the cross section is peaked ($p_z\approx 0$) the term $(\vec{p}+\vec{k})^2+\kappa_f^{\,2}$ varies slowly and can be taken out of the integral. The cross section then factorizes into two parts: one which contains the properties of the projectile, and the other which gives information on the target. The latter enters in the form of its Compton profile,[5] and it thus reflects the momentum distribution of the electrons in the target.

When, as in the lower plots in Fig. 1, $k \gg \kappa_f \gg \kappa_i$, then Eq. (3) predicts a ratio of radiation tail to capture peak which is independent of the bombarding energy. This does not correspond to the experiment, and therefore we conclude that for the highest projectile velocities the high-energy radiation tail is dominated by bremsstrahlung even in targets with loosely bound electrons. (Indeed, theoretically the cross section for bremsstrahlung drops more slowly with increasing energy than that for radiative capture.)

The theoretical curves for the He targets were calculated from Eq. (3) and then folded with the energy resolution of the detector. We summed up the contributions of all target electrons into states of the projectile with principal quantum number $n_f \leq 5$. The high-energy radiation peaks marked with arrows in Fig. 1 are due to K-shell capture and contributions from $n_f > 1$ are very small. These peaks are mainly caused by the loosely bound (outer) target electrons. We do not show the capture peaks into states with $n_f > 1$ because these peaks lie close to the bremsstrahlung limit $k^2/2m_e$ of loosely bound target electrons. For neon targets, because the inner electrons are so strongly bound that their upper bremsstrahlung energy exceeds the energy freed in the radiative K-shell capture of outer electrons, we added the cross sections for capture and bremsstrahlung. Equation (2) is only qualitative, and agreement with experiment is achieved if we

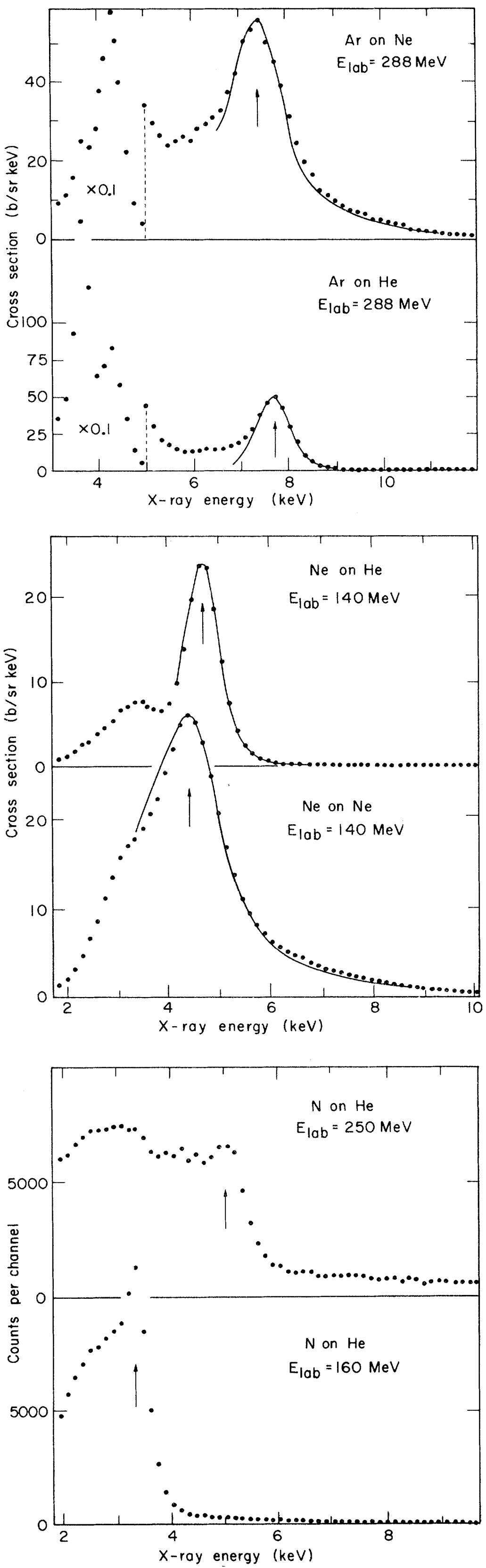

FIG. 1. Cross sections for x rays at $\theta = 90°$ in collisions between fast ions and simple gases. The experimental results are shown as points, and the solid curve is the theoretical shape. On the low-energy side of the peaks, absorption of the x rays in the windows and in the air path between the chamber and detector becomes important and leads to a cutoff in the spectra at ~ 3 keV. For the topmost curve, Ar on Ne, there was an additional 0.012-mm Al absorber to reduce the intensity of the Ar $K\alpha$ and $K\beta$ x rays at ~ 4 keV. The peaks corresponding to radiative capture into the K shell are marked by arrows, and the ordinate scale is appropriate to them. The importance of bremsstrahlung compared with radiative capture in the Ne targets is apparent in the plots for Ar and Ne projectiles. At high energies, as shown with the N projectiles, the amount of bremsstrahlung relative to the radiative capture peak increases and nearly obscures the latter.

weight the cross section for bremsstrahlung by a factor of 8 in the case of Ne on Ne and by a factor of 10 in the case of Ar on Ne. The large weighting factors indicate that the approximations involved in Eq. (2) may not be adequate. For λ we chose the reasonable values 2 and 3.5, respectively. Finally, we determined the screened nuclear charges from Slater's[6] prescription. At the maximum of the K-shell capture peak the calculated cross sections in b/(sr keV) are found to be 24 for 7.0-MeV/nucleon Ne^{10+} on He, and 53 for 7.21-MeV/nucleon Ar^{17+} on He. The experimental numbers are 24 and 50, respectively. For Ne as target the theoretical values are 73 (Ne^{10+}) and 155 (Ar^{17+}) compared to the experimental numbers 32 and 56. In both these cases, the ratio between bremsstrahlung of strongly bound electrons and capture process is 0.2 in the peak, and the high-energy radiation tails are mainly due to bremsstrahlung. The discrepancy between theory and experiment vanishes if one assumes a lower average charge of the projectiles in the case of the Ne target. The assumption is reasonable because the radiationless electron capture (which we do not consider here) increases rapidly with the nuclear charge of the target. The number 73 is reduced to 41 if one uses Ne^{9+} instead of Ne^{10+}. Note that both Ne^{8+} and Ar^{16+} do not contribute to the radiative capture into the K shell.

To compare the experimental and theoretical shapes we have normalized the cross sections to equal height at their maximum. The binding energies $-E_i$ are taken from experiment and the binding energies $-E_f$ in the highly ionized projectile are calculated with Slater's method. Equation (3) predicts maximal intensity at an energy slightly higher than that observed in experiment; in the figures we shifted the theoretical maximum down to the observed one. The shifts were 300 eV for Ne^{10+} on He and Ne^{9+} on Ne, 50 eV for Ar on He, and 400 eV for Ar on Ne. They are probably due to screening effects. [In this paper we also did not consider relativistic corrections which are of the order $(v/c)^2 \approx 1.5\%$.]

To summarize, we have established the main features which determine the spectral distribution of the capture radiation of weakly bound electrons by fast ($v/c \simeq 0.12$) heavy ions. The radiation frequency is connected by a simple dispersion relation [Eq. (1)] to the momentum component p_z of the captured electron. The cross section for a given frequency is proportional to the probability to find an electron with the corresponding momentum component p_z (Compton profile). In addition a new radiation process was found, a type of bremsstrahlung of strongly bound electrons. Bremsstrahlung becomes important in the x-ray spectrum at the highest beam velocities because its cross section drops more slowly with increasing beam energy than that for radiative capture. At lower beam velocities the capture radiation becomes more important, whereas at still lower velocities (which we did not study) one would expect that the radiative capture process should turn into transitions between quasimolecular states formed during a slow collision.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

†Permanent address: Technische Universität München, Munich, W. Germany.

‡Permanent address: Universität Heidelberg, Heidelberg, W. Germany.

§Permanent address: Max Planck Institut für Kernphysik, Heidelberg, W. Germany; supported by Deutsche Akademische Austausch Dienst.

[1]H. W. Schnopper, H. D. Betz, J. P. Delvaille, K. Kalata, A. R. Sohval, K. W. Jones, and H. E. Wegner, Phys. Rev. Lett. 29, 898 (1972).

[2]For discussion and references see J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), p. 509.

[3]H. A. Bethe and E. E. Salpeter, in *Encyclopedia of Physics*, edited by S. Flügge (Springer, Berlin, 1957), Vol. 35.

[4]G. F. Drukarev, Izv. Acad. Nauk SSSR, Ser. Fiz. 24, 979 (1960) [Bull. Acad. Sci. USSR, Phys. Ser. 24, 981 (1960)]; D. R. Bates and R. McCarroll, Advan. Phys. 11, 39 (1962).

[5]M. Cooper, Advan. Phys. 20, 453 (1971).

[6]J. C. Slater, Phys. Rev. 36, 57 (1930).