through almost a full 180°, sending the region of 90° spin flop towards the middle of the crystal where it stops after meeting the diverging state coming from the last plane. The energy will approach twice that given by (9). In a real crystal the nature of the surfaces and of the interior will determine the precise location of the spin-flop region, and may also raise the critical field above H_1.

As $b \to a$, according to (8) all $\sin^2\alpha_{2l} \to 1$, and hence the SSF region expands across the entire crystal and a first-order phase transition takes place into the SF phase. The condition $a^2 = b^2$ is

$$\xi^2 = \xi_3{}^2 \equiv 2\zeta - \zeta^2, \qquad (10)$$

which marks the well-known AF-SF free energy boundary,[5] $H_3 \equiv \xi_3 H_E$. Above H_3, Eqs. (8) and (9) no longer apply, since the integration constant in (7) ceases to be zero.

Finally, we introduce a $\cos^4\alpha$ anisotropy, characterized by $\zeta' \equiv H_A'/H_E$, and also a Dzialoshinski-Moriya[11] interaction, characterized by $\mu \equiv H_D/H_E$. After relating the canting angle to α, it is found eventually that Eq. (6) and all the consequences still hold, with, to first order in ζ and ζ',

$$a^2 = 2\zeta + 4\zeta' - \xi^2 - \mu^2, \quad b^2 = 2\zeta'.$$

We note that the fields $H_3{}^2 = 2H_E(H_A + H_A') - H_D{}^2$ and $H_4{}^2 = 2H_E(H_A + 2H_A') - H_D{}^2$ are separated to first order in ζ', as has been pointed out by Jacobs *et al.*[7]

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N-Body Forces in High-Density Nuclear Matter

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The contribution of n-body forces to nuclear matter energy is summed in closed form as a function of the nuclear density. It is seen that they provide an increasing amount of binding as the density increases beyond ordinary nuclear density. The presence of correlations suppresses this effect, but does not destroy it fully. The physics behind these phenomena is discussed.

We summarize here results obtained by us for the contribution of the *sum* of pionic many-body forces (i.e., intrinsic three-body forces + four-body forces + . . .) to the binding energy of nuclear matter in the presence of internucleon correlations. These higher many-body forces are certainly present in any real nuclear system, and have not yet been investigated, let alone summed. In addition, there are many interesting physical effects which may be expected to be produced by the sum of these forces, lending additional motivation for this work. The three-body force, on which a great deal of work has been done,[1-3] arises from the scattering of the virtual pion emitted by nucleon 1 with nucleon 2, before it is absorbed by nucleon 3. Similarly, higher many-body forces arise when the virtual pion scatters successively off many nucleons before being absorbed. Thus, the summation of the effect of all these forces is equivalent to the modification of the pion propagator (or, alternately, its effective mass) as it travels through nuclear matter. Now, if the "static" πN amplitude is dominantly attractive, as is the case on the average, then the pion feels an average negative potential energy in nuclear matter, and hence its effective mass is diminished and its "range" increased. This phenomenon will clearly increase with increasing nuclear density.

Further, since the binding of nuclear matter comes primarily from the tail of the one-pion exchange potential, increasing the range of this tail (reducing μ_π^{eff}) should lead to an increase in binding as ρ increases. Since nucleons can be packed to about 20 times the normal nuclear density before the hard cores touch one another, and since such high densities may well exist in neutron stars, these high-density effects are clearly of interest.

The fact that the nucleons with which the virtual pion scatters are not free, but are correlated with each other because of their strong short-range two-pody potential, should tend to diminish this effect. This is because hard-core correlations create wounds in the nucleon wave function,

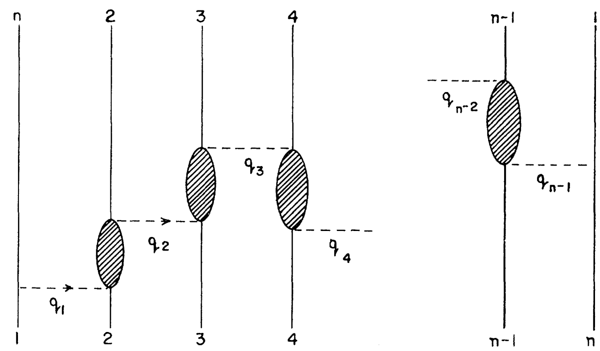

FIG. 1. Typical diagram giving rise to an n-body force. The first and last nucleons exchange places in the final state. We sum all such diagrams over n, the number of nucleons in the chain.

and consequently the extent of attraction felt by the virtual pion in nuclear matter should be reduced by these wounds.

It will be seen below that our formalism, which sums the many-body force contribution in a simple closed form, also brings out all these physical effects in a concrete way. This Letter is merely a summary of the results. Full derivations, more detailed discussions of the assumptions and results, and generalizations to include other form factors and partial waves, etc., will be published elsewhere.

The binding-energy contribution we are calculating is the sum of diagrams of the type shown in Fig. 1, summed over the index n. We take the initial wave function of the nucleons as

$$\psi_i^{(n)}(\vec{r}_1,\dots,\vec{r}_n)=\rho^{n/2}\exp[i(\vec{p}_1\cdot\vec{r}_1+\dots+\vec{p}_n\cdot\vec{r}_n)]\psi(r_{12})\psi(r_{23})\cdots\psi(r_{n-1,n}), \tag{1}$$

where ρ is the nucleon density. The $\psi(r_{ij})$ are correlation functions of the form $1-\eta(r_{ij})$, where $\eta(r_{ij})$ is the well-known two-body defect function. Note that the above wave function does not include correlations between all the $\frac{1}{2}n(n-1)$ pairs, but only between neighbors on the chain. However, this makes the calculation tractable, and contains the major physical effects of correlations. In any case, even with two-body forces, only hypernetted chains can be summed at best.[4]

We take the final wave function as

$$\psi_f^{(n)}(\vec{r}_1,\dots,\vec{r}_n)=\rho^{n/2}\exp[i(\vec{p}_n\cdot\vec{r}_1+\vec{p}_2\cdot\vec{r}_2+\dots+\vec{p}_1\cdot\vec{r}_n)]\psi(r_{12})\cdots\psi(r_{n-1,n}), \tag{2}$$

where, as with usual three-body force work, the end nucleons exchange places. (The direct-term contribution vanishes on spin averaging). The n-body potential corresponding to the multiple scattering of the virtual pion, as depicted in Fig. 1, is given by

$$W_n(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_n)=(2\pi)^{-3n+3}\int\exp[-i(\vec{q}_1\cdot\vec{r}_{12}+\vec{q}_2\cdot\vec{r}_{23}+\dots+\vec{q}_{n-1}\cdot\vec{r}_{n-1,n})]$$
$$\times M_n(\vec{q}_1,\dots,\vec{q}_{n-1})\,d^3q_1\cdots d^3q_n, \tag{3}$$

where

$$M_n(\vec{q}_1,\dots,\vec{q}_n)=\frac{f^2}{\mu^2}\vec{\sigma}^{(1)}\cdot\vec{q}_1\tau_\alpha^{(1)}\frac{1}{-(q_1^2+\mu_\pi^2)}T(\vec{q}_1\vec{q}_2)\frac{1}{-(q_2^2+\mu_\pi^2)}T(\vec{q}_2\vec{q}_3)\cdots\frac{1}{-(q_{n-1}^2+\mu_\pi^2)}\tau_\alpha^{(n)}\vec{\sigma}^{(n)}\cdot\vec{q}_{n-1}. \tag{4}$$

Note that q^2 always refers to the square of the three-vector $\vec{q}$. $T(\vec{q}_1\vec{q}_2)$ is the static-limit πN invariant amplitude at zero pion energy, and averaged over the nucleon spin and isospin. Then

$$\sum_n \langle\psi_f^{(n)}|W_n|\psi_i^{(n)}\rangle = N\rho^2\frac{f^2}{\mu^2}\int\int d^3q_1\, d^3q_{n-1}\,\vec{\sigma}^{(1)}\cdot\vec{q}_1\tau_\alpha^{(1)}\frac{f(\vec{q}_1,\vec{k}_0)}{q_1^2+\mu_\pi^2}F(\vec{q}_1\vec{q}_{n-1})\frac{f(\vec{q}_{n-1},\vec{k}_0)}{q_{n-1}^2+\mu_\pi^2}\tau_\alpha^{(n)}\vec{\sigma}^{(n)}\cdot\vec{q}_{n-1}, \tag{5}$$

with $F(\vec{k}\vec{k}')$ obeying the integral equation

$$F(\vec{k}\vec{k}') = T(\vec{k}\vec{k}') - \rho\int d^3k''\, T(\vec{k}\vec{k}'')f(\vec{k}'',\vec{k}_0)(k''^2+\mu_\pi^2)^{-1}F(\vec{k}''\vec{k}'), \tag{6a}$$

$$\begin{aligned} f(\vec{k}'',\vec{k}_0) &\equiv (2\pi)^{-3}\int\psi^2(r)\exp[i(\vec{k}_0-\vec{k}'')\cdot\vec{r}]\,d^3r \\ &= \delta(\vec{k}''-\vec{k}_0) - \int d^3r\,(2\pi)^{-3}[2\eta(r)-\eta^2(r)]\exp[i(\vec{k}_0-\vec{k}'')\cdot\vec{r}]. \end{aligned} \tag{6b}$$

For simplicity, let us average the integral in (6b) over the directions of $\vec{k}_0$,[5] since we are interested in no preferred direction of $\vec{p}_1 - \vec{p}_n = \vec{k}_0$. Then

$$f(\vec{k}'',\vec{k}_0) \equiv \delta(\vec{k}''-\vec{k}_0) - \tilde{f}_{k_0}(k''). \tag{6c}$$

The integral equation (6a) is easily solved if we take a factorized form for each partial wave of $T(\vec{k}\vec{k}')$, so that

$$T(\vec{k}\vec{k}') = \sum_l T_l(\vec{k}\vec{k}') = \sum_{lm}\alpha_l a_l(k)a_l(k')Y_{lm}(\hat{k})Y_{lm}^*(\hat{k}'), \tag{7}$$

where α_l contains the sign of the partial-wave amplitude, so that $a_l(k)$ is always taken as positive. Then the solution of (6a) is (upon averaging over the directions of $\vec{k}_0$)

$$F(\vec{k}\vec{k}') = \sum_{lm}\lambda_l a_l(k)a_l(k')Y_{lm}(\hat{k})Y_{lm}^*(\hat{k}') = \sum_l(\lambda_l/\alpha_l)T_l(\vec{k}\vec{k}'), \tag{8}$$

with

$$\lambda_l = \alpha_l\beta_l\left(1 - \rho\alpha_l\beta_l\int a_l^2(k'')\frac{\tilde{f}_{k_0}(k'')}{k''^2+\mu_\pi^2}k''^2\,dk''\right)^{-1}, \tag{9}$$

and

$$\beta_l = 1 - \frac{\rho\alpha_l a_l^2(k_0)}{4\pi[k_0^2+\mu^2+\rho T(\vec{k}_0\vec{k}_0)]}. \tag{10}$$

When $F(\vec{k}\vec{k}')$ from (8) is substituted into (5), the desired energy contribution is obtained in terms of the πN partial-wave amplitudes. All this is for a given $k_0 = |\vec{p}_1 - \vec{p}_n|$, which can then be integrated over, in a manner identical to the familiar three-body force case.

Let us now interpret this result. If there were no correlations $[\tilde{f}_{k_0}(k'')=0]$, then $\lambda_l = \alpha_l\beta_l$. In Eq. (10), β_l can be seen to have an effective pion propagator with $(\mu_\pi^2)^{\text{eff}} = \mu_\pi^2 + \rho T(\vec{k}_0\vec{k}_0)$, which is less than μ_π^2 if $T(\vec{k}_0\vec{k}_0)$ is negative (an attractive force between the π and the N). Such a modified propagator was obtained earlier by Brown and Green[3] as part of their three-body force evaluation. Their results, in fact, motivated our investigation. Note that pure three-body forces lead to such a modified propagator only in the low-density approximation, whereas when all higher many-body forces are added up, as we do, it is true for all ρ. We wish to take the ρ dependence of this propagator seriously for high ρ, and more importantly, to see how much of it survives despite correlations.

When correlations are included, through $\tilde{f}_{k_0}(k'')$, then the full expression [Eq. (9)] for λ_l must be used. It can be seen that the denominator in Eq. (9) (the integral in it is positive) will push the pole in ρ farther away as compared to Eq. (10). In other words, the correlations tend to suppress the lowering of the pion effective mass. To illustrate this further consider for simplicity a pure p-wave attractive πN amplitude. Most three-body force calculations use such an amplitude, given in a static-model dispersion-relation calculation by[6]

$$T(\vec{k}\vec{k}') = \sum_m\alpha_1 kk'Y_{1m}(\hat{k})Y_{1m}^*(\hat{k}')H(k)H(k'), \tag{11}$$

with $H(k)$ representing a form factor to account for extrapolation off the pion mass shell. The spin and isospin of the nucleon have been averaged over in (11). Then,

$$\lambda_1 = \alpha_1\left[1 + \rho\alpha_1\left(\frac{k_0^2H^2(k_0)}{4\pi(k_0^2+\mu_\pi^2)+2\rho\alpha_1k_0^2H^2(k_0)} - \int\frac{k''^4\tilde{f}_{k_0}(k'')H^2(k'')}{k''^2+\mu_\pi^2}dk''\right)\right]^{-1} \tag{12}$$

and

$$F(\vec{k}\vec{k}')=\lambda_1(\rho)\alpha_1^{-1}T(\vec{k}\vec{k}'). \qquad (13)$$

Now, if in the energy expression in Eq. (5), $F(\vec{k}\vec{k}')$ were replaced by $T(\vec{k}\vec{k}')$, one would have the familiar three-body force contribution. Equation (13) then tells us that the sum of the many-body force contributions is just a scaling factor $\lambda_1(\rho)/\alpha_1$ times the three-body force contribution. This affords a tremendous calculational simplification, in that existing numerical results[2] for three-body force contributions to the energy can be simply multiplied by $\lambda_1(\rho)/\alpha_1$ to predict the magnitude and density dependence of the sum of all n-body force contributions.

Secondly, looking at Eq. (12), one can clearly see the opposing effects of the attractive πN amplitude and the correlations, respectively, in the first and second terms inside the large parentheses. On putting in the numerical values, one finds that the first term dominates, i.e., that while correlations reduce the extent to which the pion propagator is modified in nuclear matter, they do not extinguish the effect. As a result, the expression inside the large parentheses is positive, leading to a pole in $\lambda_1(\rho)$, evaluated for an average value of k_0, given by

$$\bar{k}_0=(1.2)^{1/2}(\tfrac{3}{2}\pi^2\rho)^{1/3},$$

as a function of ρ/ρ_0, where ρ_0 is the normal nuclear matter density (corresponding to $k_F=1.36$ fm^{-1}). In evaluating Eq. (12), we have used[7]

$$\alpha_1=-5.82\mu_\pi^{-3},\quad H^2(k)=(\eta^2-\mu_\pi^2)/(k^2-\eta^2).$$

The $f_{k_0}(k'')$ is obtained using Eqs. (6b) and (6c), with a correlation function $\eta(r)$ as parametrized by Day.[8]

Since the sum of n-body force contributions to the energy is $\lambda_1(\rho)/\alpha_1$ times the three-body force contribution, and the latter is about -1 MeV, one can see from Fig. 2 that the n-body forces will contribute a rapidly increasing amount of binding as ρ increases. However, at $\rho=\rho_0$, $\lambda_1(\rho)/\alpha_1$ is about 1.1, confirming the popular belief that three-body and higher forces are not large at nuclear density and agreeing with a detailed calculation of the three- and four-body forces.[9]

The singularity in $\lambda_1(\rho)$ at $\rho=4.25\rho_0$ probably will not lead to a singularity in the binding energy of nuclear matter. We have not considered all possible n-body cluster contributions to the energy. For example, those from repulsive two-body forces, which will become increasingly important at high densities, may well mask the singularity from the many-body forces.

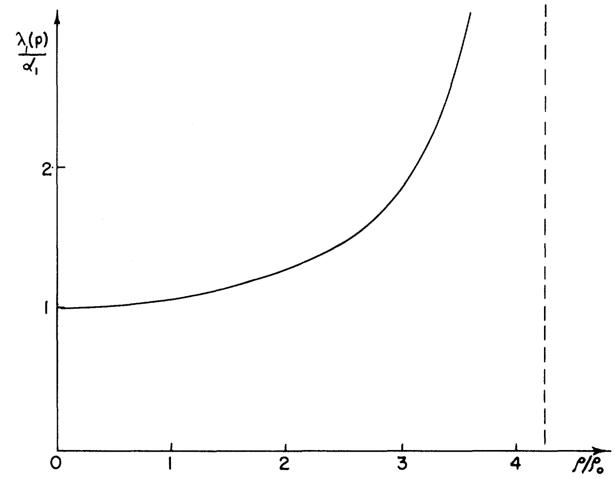

FIG. 2. The behavior of $\lambda_1(\rho)/\alpha_1$, the ratio of the sum of all many-body forces to the three-body force, as a function of ρ/ρ_0, where ρ_0 is the normal nuclear matter density.

The interplay between attractive and repulsive two-body forces leads to the usual minimum in the energy of nuclear matter at nuclear densities. It is conceivable that at higher densities, where we have shown that the attractive many-body forces become important, there is another region of stability. To investigate this possibility one must make a careful analysis of n-body energies due to two-body forces at high densities.[10]

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Gamma-Ray Spectroscopy in $A \cong 40$ Nuclei via Heavy-Ion–Induced Reactions: High-Spin States in ^{41}K and ^{41}Ca†

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Compound-nuclear reactions induced by bombardment of ^{24}Mg, ^{26}Mg, and ^{27}Al targets with 20–60-MeV ^{16}O, ^{18}O, and ^{19}F beams have been used to study γ transitions between high-spin states in nuclei in the mass region $32 \leq A \leq 44$. Results for ^{41}K and ^{41}Ca are presented to exemplify the method and results. The γ-ray decay of states with probable spins up to $\frac{19}{2}$ are observed in both nuclei.

The (heavy ion, xn, $\gamma\gamma$...) reaction has proven to be a powerful tool for studying nuclear spectroscopy in the mass region $A \gtrsim 100$[1,2] because of (a) large cross section, typically several hundred millibarns, (b) the transfer of large linear and angular momenta, (c) high alignment of the γ-emitting states, (d) strong energy dependence of the reaction cross section on bombarding energy and the concomitant ability to isolate quite well a particular final nucleus (value of x) by the choice of beam energy.

Investigations[3,4] of such reactions for lighter nuclei ($A < 80$) show the power of this method for this mass region, where all of the advantages mentioned above pertain, except (d). The lower Coulomb barrier allows proton and α-particle emission to compete with neutron emission. We then have reactions of the type (heavy ion, xn, yp, $z\alpha$, $\gamma\gamma$...) with x, y, z small integers. Typically ten nuclei are formed with comparable cross sections at any bombarding energy, and the major experimental problem is one of assigning the various γ-ray lines to particular nucleus. However, once the prejudice against studying several nuclei at once is overcome, it can be recognized that this method is a very efficient as well as a powerful spectroscopic tool.

A study of the nuclei with $32 \leq A \leq 44$ has been undertaken using reactions initiated by bombarding ^{24}Mg, ^{26}Mg, and ^{27}Al targets with ^{16}O, ^{18}O, and ^{19}F beams of 20–60 MeV from an MP tandem accelerator at Brookhaven National Laboratory. This provides nine reactions with compound nuclei between ^{42}Ca and ^{46}Ti for which the following experiments have been performed or are in progress: (1) γ-ray thin-target excitation functions, (2) γ-ray angular distributions, (3) γ-γ coincidence measurements, (4) recoil-distance lifetime measurements.

Targets were 200–300-μg/cm^2 metallic films on thick W backings, except for recoil-distance lifetime measurements[5] where a thin Ni backing was used. Excitation functions were taken in 5-MeV steps from 20 to 60 MeV. Angular distributions were recorded at seven angles between 0° and 90°.

The γ-γ coincidence data were recorded in 4096×4096 matrices by two Ge(Li) detectors placed at 0° and 90° to the beam with conventional electronics and a time gate of 60 nsec. After subtraction of random events, a background determined by channels near to a given photopeak was subtracted.

The results reported here for ^{41}K and ^{41}Ca considerably extend the information on high-spin states in mass-41 nuclei. The cross sections for ^{41}K and ^{41}Ca were largest for the ^{18}O + ^{26}Mg reaction, and next strongest in the ^{16}O + ^{27}Al and ^{19}F + ^{24}Mg reactions. The reaction mechanism is envisaged as one in which the formation of a compound nucleus with very high angular momentum (at 40-MeV bombarding energy the grazing angular momentum is $\sim 25\hbar$) is followed by emission of light particles (n, p, α)—often with γ emission competing—until the bound levels of the final nucleus are reached. At bombarding energies of