[5]Brackets on indices indicate here "physical components," i.e., components on orthonormal basis vectors.

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New Interpretation of the Euclidean-Markov Field in the Framework of Physical Minkowski Space-Time

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The Euclidean-Markov field, extensively used in constructive quantum field theory, is shown to be the same as the lowest energy generalized stochastic process associated with classical field theory through the procedure of Nelson's stochastic mechanics. As a result, the underlying four-dimensional manifold, on which the Markov field is defined, can be considered as the physical space-time.

The constructive quantum field-theory program,[1-3] advanced by Glimm and Jaffe and their followers, has been enriched by new powerful methods in the last two years. These methods[4-7] rely on ideas from Euclidean field theory and use probabilistic techniques and concepts. A central role is played by the Euclidean-Markov field $\varphi(f)$, introduced already in the pioneering studies of Schwinger,[8] Nakano,[9] and Symanzik,[10] which became a very powerful and effective tool after the discovery by Nelson that its Markov property is crucial for the reconstruction of the physical theory from the Euclidean theory and for the study of the Euclidean theory itself.

At present the situation seems to be the following: A large class of boson quantum field theories,[11] satisfying Wightman axioms,[12] have a mathematical image living in Euclidean space-time (time imaginary) and described by generalized stochastic processes.[13] Through a reconstruction theorem,[5] the physical theory can be derived from the Euclidean one.

To study and construct the Euclidean image for interacting fields is easier because powerful probabilistic techniques are available and because there is a deep analogy[14,15] between the Euclidean field theory and classical statistical mechanics, which makes it possible to use the rigorous methods of modern statistical mechanics[16] for the study of quantum field theory. Euclidean methods have been especially useful for the study of the infinite-volume limit of two-dimensional boson theories, with important results by Glimm and Spencer,[17] Nelson,[18] Simon and Griffiths,[19,20] Simon,[21,22] and Albeverio and Høegh-Krohn.[23,24]

The purpose of this note is to report about progress in the physical interpretation of the Euclidean-Markov theory. These results are a byproduct of our study of a generalization of Nelson's stochastic mechanics[25,26] to systems with an infinite number of degrees of freedom, on which we will report elsewhere.[27] Our main result is the following: *The Euclidean-Markov field coincides with the lowest energy generalized stochastic process associated with classical field theory through the procedure of Nelson's stochastic mechanics.* In the rest of this note we give a proof of this statement in the case of free fields and mention how to extend it to the interacting case. We also deal with the relativistic properties of a transformation of the Euclidean-Markov field.

From a conceptual point of view the most important consequence of our proposal of interpretation is that *the underlying four-dimensional manifold on which the Markov field is defined can be considered as the physical space-time.* The fourth coordinate is the real physical time and not the analytically continued imaginary time, as in the current interpretation of Euclidean field theory.

Nelson's stochastic mechanics can be considered as a method of quantization based on the theory of Markov processes. Here we sketch the main ideas in the simple case of a one-dimensional system with Lagrangian

$$L(q,\dot{q}) = \tfrac{1}{2}m\dot{q}^2 - V(q).$$

As a basic assumption, the classical configuration variable $q(t)$ must be interpreted as a Markov random process satisfying the stochastic differential equation

$$dq(t) = b(q(t),t)\,dt + dw(t),$$

where $b(x,t)$ is a velocity field to be determined and w is a Brownian motion with expectations

$$E(dw(t)) = 0, \quad E([dw(t)]^2) = \hbar\,dt/m.$$

It is also assumed that $dw(t)$ does not depend on $q(s)$ for $s \leq t$.

The dynamics of the process are described by the Newton equation $ma = -\partial V/\partial q$, where a is the mean acceleration of the process, defined by $a = (DD^* + D^*D)q(t)/2$. Here, D and D^* are the mean forward and backward derivatives defined respectively by

$$DF(q(t),t) = \lim_{h\to 0+} h^{-1}E_t[F(q(t+h),t+h) - F(q(t),t)],$$

$$D^*F(q(t),t) = \lim_{h\to 0+} h^{-1}E_t[F(q(t),t) - F(q(t-h),t-h)],$$

where E_t is the conditional expectation with respect to $q(t)$.

The process $q(t)$ is completely described by the probability density $\rho(x,t)$ and the current velocity $v(x,t)$ defined through

$$E(F[q(t),t]) = \int F(x,t)\rho(x,t)\,dx,$$

$$v(q(t),t) = (D + D^*)q(t)/2.$$

These must satisfy the basic equations

$$\partial\rho/\partial t = -\partial(\rho v)/\partial x,$$

$$m\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial t}\right) = -\frac{\partial V}{\partial x} + \frac{\hbar^2}{2m}\frac{\partial}{\partial x}\left(\rho^{-1/2}\frac{\partial^2}{\partial x^2}\rho^{1/2}\right),$$

which are consequences of the stochastic and dynamical assumptions made on $q(t)$. It also turns out that

$$b(x,t) = v(x,t) + (\hbar/2m)\partial \ln\rho/\partial x.$$

The connection with usual quantum mechanics is obtained by introducing the wave function

$$\psi(x,t) = \rho^{1/2}(x,t)\exp[iS(x,t)/\hbar],$$

where S is such that

$$v(x,t) = \partial S(x,t)/\partial x.$$

Then we can check that when ρ and v evolve according to the basic equations, ψ evolves according to the Schrödinger equation and *vice versa*. In particular, if we know the wave function, we can recover the associated stochastic process.

As an example we can consider the harmonic oscillator

$$V(x) = m\omega^2x^2/2,$$

with normalized ground-state wave function

$$\psi(x,t) = (m\omega/\hbar\pi)^{1/4}\exp(-m\omega x^2/2\hbar - i\omega t/2).$$

The associated stochastic process $q(t)$ has

$$\rho(x,t) = |\psi(x,t)|^2 = (m\omega/\hbar\pi)^{1/2}\exp(-m\omega x^2/\hbar),$$

$$v(x,t) = 0.$$

The basic stochastic differential equation is

$$dq(t) = -\omega q(t)\,dt + dw(t).$$

In this way we recognize that the process $q(t)$ is a Gaussian stochastic process characterized by the expectations $E(q(t)) = 0$ and $E(q(t)q(t')) = (\hbar/2m\omega)\exp(-\omega|t-t'|)$, and therefore is the Markov process of the Euclidean (i.e., imaginary-time) image of the harmonic oscillator.[28] Therefore we have our first result: *The Euclidean-Markov field associated with the harmonic oscillator coincides with the process associated with the ground state through Nelson's stochastic mechanics.* We would like to stress the difference in the two interpretations of the same process: In the first interpretation the parameter t is imaginary time, in the second one t is real (physical) time.

Consider now the Klein-Gordon equation for a real classical field $\varphi(x)$: $(\Box + X^2)\varphi = 0$ where $\Box = \partial^2/\partial t^2 - \Delta$, and X is a parameter with dimensions L^{-1} (we put $c = 1$, but let $\hbar$ retain its dimensions). The Hamiltonian is given by

$$H = \tfrac{1}{2}\int[(\partial\varphi/\partial t)^2 + (\nabla\varphi)^2 + X^2\varphi^2]\,d^3X.$$

Our objective is to find the stochastic process associated with this classical system by generalizing Nelson's stochastic mechanics to systems with an infinite number of degrees of freedom. In this simple case we discretize the system by enclosing the field in a cubic box of side L, imposing periodic conditions at the boundary. The system is then reduced to a collection of independent harmonic oscillators, with canonical coordinates $\Phi_k(t)$ and Hamiltonians

$$H_k = \tfrac{1}{2}[\dot{\Phi}_k^{\,2} + (k^2 + X^2)\Phi_k^{\,2}],$$

where $k \equiv k_1, k_2, k_3$ and $k_i = 2\pi n_i/L$, n_i are relative integers. We then consider the ground state for each oscillator and the associated stochastic process, as in the first step (now $m = 1$ and $\omega^2 = k^2 + X^2$). Through straightforward calculations, passing to the limit $L \to \infty$, we obtain that the ground-state stochastic process associated with the classical Klein-Gordon equation is a Gaussian generalized process $\varphi(x)$ with zero mean and covariance given by

$$E(\varphi(x)\varphi(y)) = \frac{\hbar}{(2\pi)^4}\int \frac{e^{-ik\cdot(x-y)}}{k_0^2+\vec{k}^2+X^2}d^4k$$

$$\equiv S(x-y),$$

where x, y, k are the vectors in Minkowski space-time and $k\cdot(x-y) = k_0(x_0-y_0) - \vec{k}\cdot(\vec{x}-\vec{y})$ is the Lorentz-invariant scalar product. Since the change of variable $k_0 \to -k_0$ does not modify the covariance of S, we recognize immediately that the process we have constructed is nothing but the free Euclidean-Markov process,[15] with mass X in a system in which $\hbar = 1$.

We stress the important fact that the label x of the process is the physical space-time. In this way our main statement is proved for the free-field case.

For the interacting case it is convenient to consider firstly the anharmonic oscillator [the $P(\varphi)_1$ theory], whose quantum-mechanical properties are well known.[29] Starting from the quantum theory we can construct the stochastic process associated with the ground state according to stochastic mechanics and compare it with the Markov process of the Euclidean theory [Euclidean $P(\varphi)_1$ theory], which is also well known (see Sect. II of Ref. 16). These considerations extend also to the $P(\varphi)_2$ (space-cutoff) theory as will be reported elsewhere.[27] Our last comment is about the relativistic transformation properties of the smeared field $\varphi(f) = \int \varphi(x)f(x)\,d^4x$, with covariance following from $S(x-y)$. Space-time translations can be trivially realized. To represent the Lorentz transformations $x \to x' = \Lambda x$, where x and x' are coordinates of the same world point in two frames Ω and Ω', it is necessary to impose the following transformation for the test function f: $f \to f'$, with $\tilde{f}(k)(k_0^2+\vec{k}^2+X^2)^{-1/2} = \tilde{f}'(k')(k_0'^2+\vec{k}'^2+X^2)^{-1/2}$, where $\tilde{f}$ are Fourier transforms and $k' = \Lambda k$. Then the field φ, as seen in Ω, and the field φ', as seen in Ω', have the same covariance (this statement is the content of relativistic invariance under Lorentz transformations) if they are related by $\varphi'(f') = \varphi(f)$.[30] The following remark, which can also be extended to the interacting case, will help to clarify the Lorentz covariance. The white noise $A(x)$, $x \in \mathbb{R}^4$, defined as a generalized Gaussian random process with expectations $E(A(x)) = 0$ and $E(A(x)A(y)) = \delta(x-y)$, enjoys manifest Lorentz covariance, because its two-point function is a δ function. This Lorentz covariance can be immediately transferred to the Euclidean field in the form given above, because the Euclidean field can be expressed as a function of the white noise through the formula

$$\varphi(f) = A((-\Delta + X^2)^{-1/2}f),$$

where Δ is the four-dimensional Laplacian.

In conclusion it seems possible to give an interpretation of the Euclidean-Markov field in the framework of the physical space-time and to achieve a very simple merging of stochastic mechanics for systems with an infinite number of degrees of freedom with the Euclidean-Markov theory.

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Further Observation of the Decay $K_L^0 \to \mu^+\mu^-$*

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A new search for the decay $K_L^0 \to \mu^+\mu^-$ has been performed at the Brookhaven National Laboratory alternating-gradient synchrotron. We used the experimental technique employed by Carithers *et al.*, with some modifications to suppress the background further and eliminate possible systematic effects. The analysis of this experiment yields three events satisfying criteria for this decay mode.

The experimental observation by Carithers *et al.* of six events satisfying criteria for the decay $K_L^0 \to \mu^+\mu^-$ gives a value for the branching ratio $\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma(K_L^0 \to \text{all}) = 14^{+13}_{-7} \times 10^{-9}$ (90% confidence-level errors).[1] This is in contradiction with the result from Clark *et al.*, in which no $K_L^0 \to \mu^+\mu^-$ candidates were found, thus placing an upper limit for this branching ratio of 3.3×10^{-9} at the 90% confidence level.[2] In this Letter we report the final results of a new search for the decay $K_L^0 \to \mu^+\mu^-$.

Three modifications were made to the spectrometer used by the original Columbia/CERN/New York University experiment shown in Fig. 1 of Ref. 1 [which we will refer to as $K_{\mu2}$(I)]:

(1) A new high-resolution horizontal hodoscope has been added to the system in order to improve background rejection in the identification of muons. This array of 23 scintillator elements, each $2\frac{1}{2}$ in. wide and 84 in. long, is mounted behind a 2-in.-thick steel plate and placed downstream from the other muon detectors. The overall size of the hodoscope is sufficient to detect most $K_L^0 \to \mu^+\mu^-$ decays which penetrate the hadron absorber. The efficiency of each of the three muon hodoscope planes (horizontal, vertical, and new horizontal) was found to be 97% from a carefully selected sample of $K_{\mu3}$ decays.

(2) The spectrometer magnet was operated at 240 MeV/c transverse momentum ($P_\perp$). This was chosen to be different from $P_\perp = 210.6$ MeV/c used in the $K_{\mu2}$(I) experiment for the following reasons. (a) Increased bending power tends to compress the $\mu^+\mu^-$ invariant-mass spectrum due to the