cross sections and analyzing powers reported here and in Ref. 3 for the 0^+, 2^+, 2^+, 4^+, and 3^- levels can be understood in terms of a coupled-channel vibrational-model calculation where a mixture of one- and two-phonon states is introduced.

In view of these findings, it would be very interesting to have (a) further microscopic calculations on 4^+ states in ^{32}S (e.g., by extending the work of Gunye[15]); (b) decay scheme and lifetime studies of 6.44-MeV state in ^{32}S to clarify its relationship to the 4.46-MeV state.

It is a pleasure to thank Dr. Kamitsubo, J. L. Escudié, J. Gosset, and B. Mayer for the analyses of the data and Dr. Delaroche for helping us with the use of the Jupiter-1 program. We are grateful to Professor Arima and Dr. Do Dang for stimulating discussions.

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Electromagnetic Radiation from an Unmoving Charge*

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Gravitational perturbations in the neighborhood of a point charge can induce electromagnetic radiation even if the symmetry of a configuration indicates that the charge is not moving. Wave equations for the electromagnetic radiation are given in the Newman-Penrose formalism, and the radiation for a simple physical system is calculated and discussed.

In classical nongravitational physics the generation of electromagnetic waves is always associated with the acceleration of charges. When gravitational fields are present, however, the space between an electric charge and an observer is distorted, so the simple classical criterion is insufficient. In gravitational physics the breakdown of this criterion, used with the equivalence principle, leads to some well-known paradoxes, in particular the question of how a freely falling charged particle can radiate.

In this paper we clarify the relationship between the generation of electromagnetic radiation and space-time oscillations.[1] Of particular interest are physical configurations for which it seems reasonable to say that no charge is moving. One such example is that of two uncharged bodies, of equal mass, on opposite sides of a charged particle, and executing circular orbits about the charge (see Fig. 1). We find that this configuration radiates electromagnetic waves and also has a magnetic moment. There is no classical analog of either phenomenon.

In the mathematical approach to such problems we assume that the stress-energy associated with charges and electromagnetic fields makes a neg-

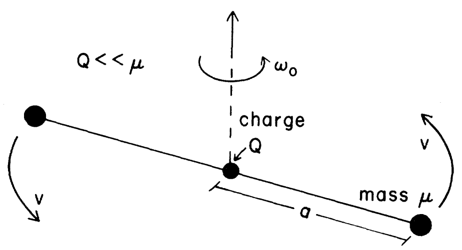

FIG. 1. A physical configuration in which an unmoving charge generates electromagnetic radiation.

ligible contribution to the curvature of space-time. The sources of space-time oscillations, though much more energetic than the electromagnetic perturbations, are taken to be small enough so that the gravitational oscillations can be considered as first-order perturbations. We give here equations in the Newman-Penrose (NP) formalism[2,3] for perturbations about both flat (Minkowski) and Schwarzschild space-time backgrounds.

The coordinate system and NP null tetrad $\vec{l}, \vec{n}, \vec{m}$ are chosen to be the "special system" of Ref. 2 ($\vec{l}, \vec{n}, \vec{m}$ are parallel propagated in the $\vec{l}$ direction). In the special system the peeling theorem,[2] of considerable importance in our analysis, imposes limits on the rate at which all outgoing NP fields fall off in radius at constant retarded time. The mathematical notation of this paper follows that of Ref. 3 very closely. Two notational conveniences of that reference which are used here deserve comment. (i) The indices of the projections of the Weyl tensor Ψ_i $(i=-2,-1,0,1,2)$ and of the electromagnetic field tensor Φ_i $(i=-1, 0, 1)$ denote the spin-weight of the fields. This notation differs from the usual NP notation of Ref. 2. (ii) All perturbation fields can be represented by their "despun" equivalents. The despun field is formed by operating on the field repeatedly with $-2^{-1/2}\eth$ or with $-2^{-1/2}\bar{\eth}$ until the spin weight is zero. Despun quantities are denoted here by carets. The great convenience of despun quantities is that their real parts correspond to even-parity fields, and their imaginary parts correspond to odd parity.[3] We use here units in which $c=G=1$, and the sign conventions of Newman and Penrose.[2]

For the unperturbed Schwarzschild (or flat with mass $M=0$) space-time geometry, we choose the null tetrad to be

$$\vec{l}=f^{-1}\vec{e}_t+\vec{e}_r, \quad \vec{n}=\tfrac{1}{2}\vec{e}_t-\tfrac{1}{2}f\vec{e}_r,$$
$$\vec{m}=2^{-1/2}r^{-1}(\vec{e}_\theta+i\vec{e}_\varphi/\sin\theta), \quad f\equiv 1-2M/r, \tag{1}$$

in terms of the usual Schwarzschild basis vectors. We consider two cases: (1) a point charge Q with negligible stress-energy in its field, at the coordinate origin, (2) a slightly charged Schwarzschild black hole—actually a Reissner-Nordström black hole with charge $Q \ll M$—at the origin. In either case the unperturbed electromagnetic field is described by

$$\Phi_0=-Q/2r^2, \quad \Phi_1=\Phi_{-1}=0. \tag{2}$$

When first-order perturbations of Maxwell's equation are calculated, perturbations in metric quantities multiplying Φ_0 must be kept. These perturbed equations (with no perturbation of the charge-current) can be combined to give a wave equation for Φ_{-1}. For l-pole radiation the wave equation in either a Schwarzschild or flat background is

$$W(\hat{\Phi}_{-1}r)\equiv 2\Delta r^2 D(\hat{\Phi}_{-1}r)+l(l+1)\hat{\Phi}_{-1}r$$
$$=Q[l(l+1)(r^{-1}\hat{U}_B-\hat{\mu}_B)+2\Delta\hat{\omega}_B{}^*]. \tag{3}$$

Here $\hat{U}_B, \hat{\omega}_B$ are (despun) perturbations of the NP metric functions U, ω, and $\hat{\mu}_B$ is the (despun) perturbation of the spin coefficient μ. The B subscript denoting perturbations will be dropped for simplicity in most of the expressions to follow.

The right-hand side of Eq. (3) can be eliminated in favor of stress-energy terms Φ_{ij} and Λ, directly describing the source of the gravitational perturbations.[2] This is done differently in the Schwarzschild and in the flat space-time case.

In the Schwarzschild background the Bianchi identies can be used to derive a wave equation for $r^2\Psi_{-1}$ which uses the same differential operator W as in Eq. (3) and which has the same metric perturbations on the right. When the two wave equations are combined, a wave equation results,

$$W(\hat{\theta})=(Q/9M)\{2l(l+1)[\Delta(r^3\hat{\Phi}_{11})-\tfrac{1}{2}r^4D(r^{-1}\hat{\Phi}_{22})+\tfrac{1}{2}r^2\hat{\Phi}_{21}-r^2\hat{\Phi}_{12}+r^2\Delta(r\hat{\Phi}_{11})]$$
$$+2\Delta(r^3\hat{\Phi}_{20})-2\Delta(r^4D\hat{\Phi}_{21})+4\Delta[r^4f^{-1}\Delta(f\hat{\Phi}_{10})]\}, \tag{4}$$

which relates stress-energy perturbations to a field variable

$$\theta\equiv r\Phi_{-1}-(Q/3M)r^2\Psi_{-1}, \tag{5}$$

carrying information about both gravitational and electromagnetic perturbations. To find the Φ_{-1} field it is necessary to solve for the Ψ_{-1} perturbation. From the perturbed Bianchi identies a wave equation relating the perturbation in Ψ_{-2} to the perturbing stress-energy is found to be[4]

$$r^{-4}f\Delta[r^4f^{-1}D(\hat{\Psi}_{-2}r)]+[\tfrac{1}{2}r^{-1}(l-1)(l+2)+3Mr^{-2}]\hat{\Psi}_{-2}$$
$$=-\tfrac{1}{2}(l-1)(l+2)[\tfrac{1}{2}l(l+1)r^{-1}\hat{\Phi}_{22}+fr^{-2}\Delta(r^2f^{-1}\hat{\Phi}_{21})+r^{-4}f\Delta(r^4f^{-1}\hat{\Phi}_{21})]-r^{-4}f\Delta[r^4f^{-1}\Delta(r\hat{\Phi}_{20})]. \quad (6)$$

One of the Bianchi identities,

$$D(\hat{\Psi}_{-2}r)=-\tfrac{1}{2}(l-1)(l+2)\hat{\Psi}_{-1}+3\hat{\lambda}M/r^2\ , \quad (7)$$

can then be used to find Ψ_{-1}. (Since by the peeling theorem for outgoing radiation we have $\lambda\sim r^{-1}$, $\Psi_{-1}\sim r^{-2}$, the dominant behavior of Ψ_{-1} can be computed without solving for λ explicitly.) For dipole fields the above scheme for finding Ψ_{-1} cannot be used [Eq. (7) is vacuous]. In this case, however, it can be shown that

$$\hat{\Psi}_{-1}=iKr^{-4}(1+M/r)\cos\theta, \quad (8)$$

where K is a real constant related to the angular momentum perturbation of the geometry.

In a flat background the Bianchi identities and equations for derivatives of the spin coefficients and of the tetrad perturbations can be used to cast Eq. (3) as a wave equation relating the Φ_{-1} field to sources. With the rather intricate source term simplified by the equations of motion, this wave equation is

$$W\{r\hat{\Phi}_{-1}+[2Q/(l-1)(l+2)][-\hat{\lambda}-r\hat{\Psi}_{-1}+\tfrac{1}{4}l(l+1)r\hat{\Psi}_0]\}=S_1+S_2+S_3, \quad (9)$$

$$S_1=W\{[2Q/(l-1)(l+2)][-r\hat{\Phi}_{21}+\tfrac{1}{4}l(l+1)r(\hat{\Phi}_{11}+\hat{\Lambda})]\}, \quad (10a)$$

$$S_2=-(Q/3)(2r^2D\hat{\Phi}_{[12]}+4r^2\Delta\hat{\Phi}_{[10]}+r\hat{\Phi}_{[02]}-3r\hat{\Phi}_{[10]}), \quad (10b)$$

$$S_3=-Q[r\hat{\Phi}_{(02)}+r\hat{\Phi}_{(01)}-4r\hat{\Phi}_{(12)}+\tfrac{1}{2}l(l+1)r(\hat{\Phi}_{11}+3\hat{\Lambda})]$$
$$+[Q/(l-1)(l+2)][-4r^2D\hat{\Phi}_{(12)}+l(l+1)r^2D(-\hat{\Phi}_{22}+\tfrac{1}{2}\hat{\Phi}_{11}+\tfrac{3}{2}\hat{\Lambda})], \quad (10c)$$

$$\hat{\Phi}_{[ij]}\equiv\tfrac{1}{2}(\hat{\Phi}_{ij}-\hat{\Phi}_{ji}),\ \ \hat{\Phi}_{(ij)}\equiv\tfrac{1}{2}(\hat{\Phi}_{ij}+\hat{\Phi}_{ji}). \quad (10d)$$

By the peeling theorem, the λ, Ψ_{-1}, and Ψ_0 terms in the wave operator W fall off faster than $r\Phi_{-1}$, so that they need not be computed to find the dominant behavior of Φ_{-1}. If the stress-energy source is of finite extent (e.g., a rotating dumbbell), the source terms in S_1 can be considered part of the variable on the left-hand side of the equations; these terms vanish at large r so they need not be taken into account in evaluating the radiation. Since $\hat{\Phi}_{ij}=(\hat{\Phi}_{ji})^*$ and Λ is real, the S_2 source term is pure imaginary and couples only to odd-parity (magnetic multipole) radiation, and the S_3 term is pure real and couples only to even-parity (electric multipole) radiation. For odd parity Eq. (9) can be put in relatively simple form which is also applicable to $l=1$ modes:

$$W[r\hat{\Phi}_{-1}+\tfrac{1}{2}Q(\hat{\tau}+\hat{\mu})]=S_2. \quad (11)$$

Again, by the peeling theorem, μ and τ can be ignored at large r.

In unperturbed Schwarzschild or flat ($M=0$) space-time, the relation of Φ_{-1} and electromagnetic fields is[5]

$$\Phi_{-1}=-2^{-3/2}f^{1/2}[E^{[\theta]}+B^{[\varphi]}-i(E^{[\varphi]}-B^{[\theta]})][E^{[\theta]}+B^{[\varphi]}-i(E^{[\varphi]}-B^{[\theta]})], \quad (12)$$

so that the power radiated per unit solid angle is

$$dP/d\Omega=(r^2/2\pi)|\Phi_{-1}|^2. \quad (13)$$

When space-time is perturbed the relation of the radiated power to the perturbations of the electromagnetic field tensor is obscured by coordinate gauge arbitrariness. It can be shown, however, that even when space-time perturbations are present, Eq. (13) remains true if Φ_{-1} is computed in the special system.

The equations given above then suffice to calculate electromagnetic radiation in a variety of interesting problems. The configuration in Fig. 1 is a simple example of a situation where, by symmetry, the central charge is not moved in a classical sense by the particles in orbit around it. For nonrelativistic motion of the particles the dominant radiation would be electric quadrupole. Because the odd-parity equation has a source term so much simpler we present here the result for the magnetic octupole, the dominant odd-parity mode for nonrelativistic motion. The radial stresses holding the particles in orbit contribute only to even-parity radiation (although the proper mass of the "ropes" holding the particles in orbit would also generate odd-parity radiation). From Eqs. (11) and (13) it is straightforward to show that the power radiated in the magnetic octupole mode is

$$P=(4096/2835)\mu^2Q^2\omega_0^{\,4}v^6. \tag{14}$$

It is interesting to compare this result with that for the classical magnetic octupole radiation that would result if the charge were carried on the orbiting particles. In that case the radiation is greater than that of Eq. (14) by a factor of order $(a/\mu)^2$. Since the orbital radius is assumed to be much larger than the Schwarzschild radius of the particles, the radiation from the unmoving-charge configuration is much smaller than the radiation from the same charge moving with the velocity of the uncharged particles. This same factor $(a/\mu)^2$ applies as well for the comparison of other multipoles.

In addition to the radiation fields there is also a stationary magnetic dipole associated with the configuration of Fig. 1. The magnetic moment is

$$m=4Q\mu v, \tag{15}$$

for nonrelativistic motion. The gyromagnetic ratio of this configuration is then

$$g=4a^{-1}\times(\text{combined mass of the uncharged particles}) \tag{16}$$

and depends on how compact the configuration is. The stationary magnetic dipole associated with the rotation of mass about an electric charge has been investigated by Cohen, Wald, and Tiomno[6] and others. Their calculations, involving rotating shells and very different mathematical techniques, give a gyromagnetic ratio in agreement with Eq. (16).

The wave equations in the Schwarzschild background predict qualitatively the same sort of radiation as the above for nonrelativistic motion, except that they allow consideration of electric dipole radiation, which is just the classically expected radiation due to the small motions of the charged black hole relative to the center of mass.

For relativistic particle motion the form of the wave equations as well as physical intuition strongly suggest that the radiation in the flat space-time case will be qualitatively similar to synchrotron radiation, and in the Schwarzschild case will be similar to geodesic synchrotron radiation[7]; radiation should be generated predominantly at frequencies much higher than orbital frequencies, and should be confined to small angles from the orbital plane. Any astrophysical significance of the radiation mechanism described here will probably require such a synchrotron mechanism to achieve observationally interesting frequencies.

Though radiation from an unmoving charge is classically unexpected, it is not at all surprising from the standpoint of general relativity. For the configuration in Fig. 1, for example, the uncharged particles can be thought of as changing the effective distance between the central charge and an observer, thereby inducing an oscillating field at the observer. For a less mystical explanation one can view the gravitational field of the uncharged particles as distorting the Coulomb electrical field of the central charge so that it is no longer spherically symmetric. It is the rotation of this distorted electric field which gives rise to the radiation.

The details of the calculations outlined here as well as a discussion of related matters will be presented elsewhere.

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*Work supported in part by the National Science Foundation under Contract No. GP38991.

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[1]The problem of the generation and propagation of electromagnetic radiation in curved space-time is, of course, not a new one. [See, e.g., P. C. Peters, Phys. Rev. D 7, 368 (1973).] Here, however, we are concerned with electromagnetic radiation directly attributable to time-dependent perturbations of the space-time geometry.

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New Interpretation of the Euclidean-Markov Field in the Framework of Physical Minkowski Space-Time

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The Euclidean-Markov field, extensively used in constructive quantum field theory, is shown to be the same as the lowest energy generalized stochastic process associated with classical field theory through the procedure of Nelson's stochastic mechanics. As a result, the underlying four-dimensional manifold, on which the Markov field is defined, can be considered as the physical space-time.

The constructive quantum field-theory program,[1-3] advanced by Glimm and Jaffe and their followers, has been enriched by new powerful methods in the last two years. These methods[4-7] rely on ideas from Euclidean field theory and use probabilistic techniques and concepts. A central role is played by the Euclidean-Markov field $\varphi(f)$, introduced already in the pioneering studies of Schwinger,[8] Nakano,[9] and Symanzik,[10] which became a very powerful and effective tool after the discovery by Nelson that its Markov property is crucial for the reconstruction of the physical theory from the Euclidean theory and for the study of the Euclidean theory itself.

At present the situation seems to be the following: A large class of boson quantum field theories,[11] satisfying Wightman axioms,[12] have a mathematical image living in Euclidean space-time (time imaginary) and described by generalized stochastic processes.[13] Through a reconstruction theorem,[5] the physical theory can be derived from the Euclidean one.

To study and construct the Euclidean image for interacting fields is easier because powerful probabilistic techniques are available and because there is a deep analogy[14,15] between the Euclidean field theory and classical statistical mechanics, which makes it possible to use the rigorous methods of modern statistical mechanics[16] for the study of quantum field theory. Euclidean methods have been especially useful for the study of the infinite-volume limit of two-dimensional boson theories, with important results by Glimm and Spencer,[17] Nelson,[18] Simon and Griffiths,[19,20] Simon,[21,22] and Albeverio and Høegh-Krohn.[23,24]

The purpose of this note is to report about progress in the physical interpretation of the Euclidean-Markov theory. These results are a byproduct of our study of a generalization of Nelson's stochastic mechanics[25,26] to systems with an infinite number of degrees of freedom, on which we will report elsewhere.[27] Our main result is the following: *The Euclidean-Markov field coincides with the lowest energy generalized stochastic process associated with classical field theory through the procedure of Nelson's stochastic mechanics.* In the rest of this note we give a proof of this statement in the case of free fields and mention how to extend it to the interacting case. We also deal with the relativistic properties of a transformation of the Euclidean-Markov field.

From a conceptual point of view the most important consequence of our proposal of interpretation is that *the underlying four-dimensional manifold on which the Markov field is defined can be considered as the physical space-time.* The fourth coordinate is the real physical time and not the analytically continued imaginary time, as in the current interpretation of Euclidean field theory.

Nelson's stochastic mechanics can be considered as a method of quantization based on the theory of Markov processes. Here we sketch the main ideas in the simple case of a one-dimensional system with Lagrangian

$$L(q,\dot{q}) = \tfrac{1}{2}m\dot{q}^2 - V(q).$$

As a basic assumption, the classical configuration variable $q(t)$ must be interpreted as a Mar-