‡Present address: P. DOR, Los Alamos Scientific Laboratory, Los Alamos, N. Mex. 87544.
[1]J. C. Aarons and I. H. Sloan, Nucl. Phys. A182, 369 (1972), and Phys. Rev. C 5, 582 (1972).
[2]S. C. Pieper and K. L. Kowalski, Phys. Rev. C 5, 306 (1972).
[3]S. C. Pieper, Nucl. Phys. A193, 519, 529 (1972), and Phys. Rev. C 6, 1157 (1972), and Phys. Rev. Lett. 27, 1738 (1971).
[4]P. Doleschall, Phys. Lett. 38B, 298 (1972), and Hungarian Academy of Sciences Report No. KFKI-72-36, 1972 (unpublished), and Phys. Lett. 40B, 443 (1972).
[5]Y. Avishai and A. S. Rinat (Reiner), Phys. Lett. 37B, 487 (1971).
[6]J. Krauss and K. L. Kowalski, Phys. Lett. 31B, 263 (1970), and Phys. Rev. C 2, 1319 (1970).
[7]W. M. Kloet and J. A. Tjon, Phys. Lett. 37B, 460 (1971), and to be published.
[8]R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. 140, B1291 (1965).
[9]H. P. Noyes, in *Proceedings of the International Conference on Polarized Targets and Ion Sources, Saclay, France, 1966* (La Documentation Française, Paris, France, 1967), p. 309. See also H. P. Noyes, Ref. 6, p. 2.
[10]J. Hufner and A. De-Shalit, Phys. Lett. 15, 52 (1965).
[11]L. M. Delves and P. Brown, Nucl. Phys. 11, 432 (1959); L. M. Delves, *ibid.* 33, 482 (1962).
[12]R. D. Purrington and J. L. Gammel, Phys. Rev. 168, 1174 (1968).
[13]I. H. Sloan, Phys. Rev. 165, 1587 (1968), and Phys. Lett. 25B, 84 (1967).
[14]S. W. Bunker, J. M. Cameron, R. F. Carlson, J. R. Richardson, P. Tomas, W. T. H. van Oers, and J. Verba, Nucl. Phys. A113, 461 (1968).
[15]J. Zamudio, B. E. Bonner, F. P. Brady, J. A. Jungerman, and J. Wang, in *Proceedings of the Conference on Nuclear Structure Study with Neutrons, Budapest, Hungary, 1972* (Central Research Institute for Physics, Budapest, Hungary, 1972).
[16]R. A. Eldred, private communication.
[17]T. A. Cahill and R. A. Eldred, *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, Wis., 1971), p. 465.
[18]H. E. Conzett, G. Igo, and W. J. Knox, Phys. Rev. Lett. 12, 222 (1964).
[19]D. Garreta, A. Papineau, J. Sura, and A. Tarrats, Phys. Rev. Lett. 21, 1393 (1968).
[20]J. C. Faivre, D. Garetta, J. Jungerman, A. Papineau, J. Sura, and A. Tarrats, Nucl. Phys. A127, 169 (1969).
[21]R. L. Walter and C. A. Kelsey, Nucl. Phys. 46, 55 (1963).
[22]J. J. Malanify, J. E. Simmons, R. B. Perkins, and R. L. Walter, Phys. Rev. 146, 632 (1966).

Nuclear Excitation Energy in Muon Capture

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(Received 26 June 1973)

Sum-rule techniques are used to evaluate total μ-capture rates. They turn out to be strongly dependent on the mean nuclear excitation energy, whose behavior along the nuclear table is discussed.

As is well known, total μ-capture rates $\Lambda_{\mu c}$ can be roughly thought of as proportional to $Z_{\text{eff}}^4 = RZ^4$, where Z is the number of protons and R a factor describing the overlap of muon and nuclear wave functions. However, as can be seen in Fig. 1, a plot of $Z_{\text{eff}}^4/\Lambda_{\mu c}$ exhibits a wide variation over the nuclear table. This effect has been attributed by Primakoff[1] to nuclear correlations. Anyway, results obtained by this method are critically dependent on the value assumed for a parameter δ_a, measuring nuclear correlations, which enters the rate in addition to an average neutrino momentum $\langle\nu_a\rangle$. The value of δ_a is obtained by a "best fit," about which individual nuclei may of course fluctuate. Later attempts along this line have failed to improve the situation substantially.[2]

The aim of this work is to show that the general features of the process can be explained by the variation along the nuclear table of the average momentum of the emitted neutrino, on which total capture rates are strongly dependent. The corresponding value of the excitation energy in daughter nuclei is in agreement with the hypoth-

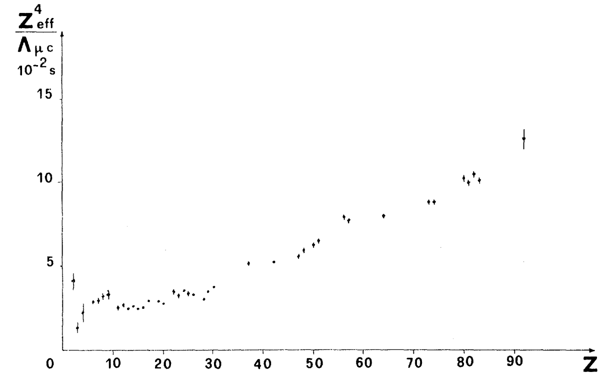

FIG. 1. $\Lambda_{\mu c}/Z_{eff}^4$ versus Z. Experimental values of $\Lambda_{\mu c}$ are taken from Ref. 11 and Z_{eff} from Ref. 8.

esis of Foldy and Walecka[3] and with shell-model calculations[4] on the relative contribution of different multipoles in the capture mechanism, and affords a deeper understanding of nuclear-structure effects in the process. We perform our calculations within the framework of the closure approximation, but use sum-rule techniques to overcome the uncertainties connected with the difficult evaluation of nuclear correlations.

Let us use the following simplified formula for the total capture rate:

$$\Lambda_{\mu c}=\frac{G_{\mu c}^{2}}{2\pi}\sum_{b}\nu_{ab}^{2}\int\frac{d\hat{\nu}}{4\pi}|\langle b|\sum_{i=1}^{A}\tau_i^{-}\exp(-i\vec{\nu}_{ab}\cdot\vec{x}_i)\varphi(x_i)|a\rangle|^2, \tag{1}$$

which can be easily obtained from the usual expression[5] through the following approximations: (a) neglecting relativistic effects (they give contributions that can increase the rate by ~10%[6]); (b) assuming $M_V^2=M_A^2=M_P^2$ (this actually overestimates M_A^2 and M_P^2 by 10–20%).[7] These two effects partially compensate, so that the overall agreement can be estimated to be better than 10%. The muon wave function $\varphi(x)$ has not been extracted from the matrix element; as we shall see this will affect the result in the high-A region.

The form factor appearing in (1), $G_{\mu c}^2=G_V^2+3G_A^2+G_P^2-2G_AG_P$, is only slightly dependent on the neutrino momentum and can be treated as a constant; evaluating it for an average value $\bar{\nu}=85$ MeV, with $g_A/g_V=-1.23$ and $G=1.023\times10^5M^{-2}$ (M is the nucleon mass), we obtain $G_{\mu c}^2=5.85\times10^{-10}M^{-4}$. On the contrary, the remaining part of (1) can be expressed as a rapidly varying function of the mean neutrino momentum.

In a transition from $|a\rangle$ to $|b\rangle$, momentum and energy conservation give

$$\nu_{ab}\simeq m_\mu-\epsilon_a-(E_b-E_a), \tag{2}$$

where $E_{b(a)}$ is the total energy of the corresponding nuclear state and ϵ_a is the binding energy of the muon in the K orbit of the mesonic atom. We are interested in the mean value of E_b-E_a that can be defined through the relation

$$\overline{E}=\frac{G_{\mu c}^{2}}{2\pi}\sum_{b}\frac{\bar{\nu}^2(E_b-E_a)}{\Lambda_{\mu c}}\int\frac{d\hat{\nu}}{4\pi}|\langle b|\sum_{i=1}^{A}\tau_i^{-}\exp(-i\vec{\nu}\cdot\vec{x}_i)\varphi(x_i)|a\rangle|^2. \tag{3}$$

Here we have replaced ν_{ab} with $\bar{\nu}\equiv m_\mu-\epsilon_a-\overline{E}$ in the numerator in order to make possible the calculation. In fact, using closure we get the following sum rule:

$$2\sum(E_b-E_a)|\langle b|\sum_{i=1}^{A}\tau_i^{-}\exp(-i\vec{\nu}\cdot\vec{x}_i)\varphi(x_i)|a\rangle|^2=-\langle a|[[H,\sum_{i=1}^{Z}\exp(-i\vec{\nu}\cdot\vec{x}_i)\varphi(x_i)],\sum_{j=1}^{Z}\exp(i\vec{\nu}\cdot\vec{x}_j)\varphi(x_j)]|a\rangle. \tag{4}$$

This is valid for a nuclear Hamiltonian $H=T+V^W$, i.e., kinetic energy and Wigner potential, and holds true to a good extent also in the presence of additional exchange potentials. Their influence will be discussed later. Therefore we obtain

$$\Lambda_{\mu c}\simeq-\frac{1}{2}\frac{G_{\mu c}^{2}}{2\pi}\frac{\nu^2}{\overline{E}'}\langle a|[[H,\sum_{i=1}^{Z}\exp(-i\vec{\nu}\cdot\vec{x}_i)\varphi(x_i)],\sum_{j=1}^{Z}\exp(i\vec{\nu}\cdot\vec{x}_j)]|a\rangle. \tag{5}$$

As we neglected the Coulomb potential in the Hamiltonian, the mean excitation energy $\overline{E}$ has been replaced by $\overline{E}'=\overline{E}+E_C$, where E_C is the Coulomb splitting between analog states in parent and daughter nuclei.

The commutator in (5) can be easily calculated if only the kinetic energy and Wigner potential are taken into account. The result is

$$\Lambda_{\mu c}=\frac{G_{\mu c}{}^2}{2\pi}\,\frac{1}{2M}\,\frac{\bar\nu^4}{\bar E'}\,\langle a|\sum_{i=1}^{Z}\left\{\varphi^2(x_i)+\frac{1}{\bar\nu^2}\,[\nabla_i\varphi(x_i)]^2\right\}|a\rangle. \tag{6}$$

Remembering that

$$\langle a|\sum_{i=1}^{Z}\varphi^2(x_i)|a\rangle=\frac{(m_\mu\alpha)^3}{\pi}Z_{\rm eff}{}^4,$$

and defining

$$1/\lambda^2=\langle a|\sum_{i=1}^{Z}[\nabla_i\varphi(x_i)]^2|a\rangle\langle a|\sum_{i=1}^{Z}\varphi^2(x_i)|a\rangle^{-1},$$

Eq. (6) becomes

$$\Lambda_{\mu c}(\bar\nu)=\frac{G_{\mu c}{}^2(m_\mu\alpha)^3}{(2\pi)^2M}\,\frac{\bar\nu^4}{(m_\mu-\epsilon_a+E_{\rm C}-\bar\nu)}\,Z_{\rm eff}{}^4\left(1+\frac{1}{\lambda^2\bar\nu^2}\right). \tag{7}$$

This equation gives the total capture rate as a function of the mean neutrino momentum $\bar\nu$ (only) or, equivalently, as a function of $\bar E'$.

The parameter $Z_{\rm eff}$ has been tabulated,[8] and λ can be calculated using an approximate expression for $\varphi(x)$.[9] The term containing λ would not have been present had we averaged $\varphi(x)$ over the nucleus as in the usual procedure; it turns out to be very small for light nuclei, but increases to 15% for heavy ones.

The contribution from exchange potentials, $V^{\rm ex}=\sum_{i<j}(V_{ij}{}^{\rm M}+V_{ij}{}^{\rm H}+V_{ij}{}^{\rm B})$, i.e., Majorana, Heisenberg, and Bartlett terms, is given by

$$\langle a|[[V^{\rm ex},\sum_{i=1}^{Z}\exp(-i\vec{\bar\nu}\cdot\vec{\rm x}_i)\varphi(x_i)],\sum_{j=1}^{Z}\exp(i\vec{\bar\nu}\cdot\vec{\rm x}_j)\varphi(x_j)]|a\rangle$$

$$\simeq|\varphi|_{\rm av}{}^2\langle a|\sum_{i=1}^{Z}\sum_{j=1}^{N}|\exp(-i\vec{\bar\nu}\cdot\vec{\rm x}_i)-\exp(-i\vec{\bar\nu}\cdot\vec{\rm x}_j)|^2(V_{ij}{}^{\rm M}+V_{ij}{}^{\rm H})|a\rangle. \tag{8}$$

Expanding the exponential we get

$$|\exp(-i\vec{\bar\nu}\cdot\vec{\rm x}_i)-\exp(-i\vec{\bar\nu}\cdot\vec{\rm x}_j)|^2=\bar\nu^2(x_i-x_j)^2-\tfrac{1}{12}\bar\nu^4(x_i-x_j)^4+\cdots.$$

The contribution of the $\bar\nu^2$ term has been evaluated by Bethe and Levinger[10] to be about 40% *of the dipole strength,* but the $\bar\nu^4$ term that here is still relevant reduces the contribution to about 30%. Higher-order terms are negligible. We stress the fact that exchange contributions depend essentially on $\langle|(x_i-x_j)^2V_{ij}{}^{\rm ex}|\rangle$, i.e., on the interaction range and not on the nuclear radius. Therefore, their importance relative to the dominant matrix element, which depends on the latter, is decreased in high nuclei.

Going back to Eq. (7), experimental values of $\Lambda_{\mu c}$[11] can be used to determine $\bar E'$; the results are shown in Fig. 2 for a wide range of nuclei. The general trend of the plot can be explained as follows:

(i) In light nuclei the value of $\bar E'$ agrees within experimental errors with the giant-dipole-resonance energy, according to the hypothesis that the capture process here occurs essentially through dipole excitation. The contribution of exchange forces would slightly raise $\bar E'$; as a matter of fact, multiplying the right-hand side of

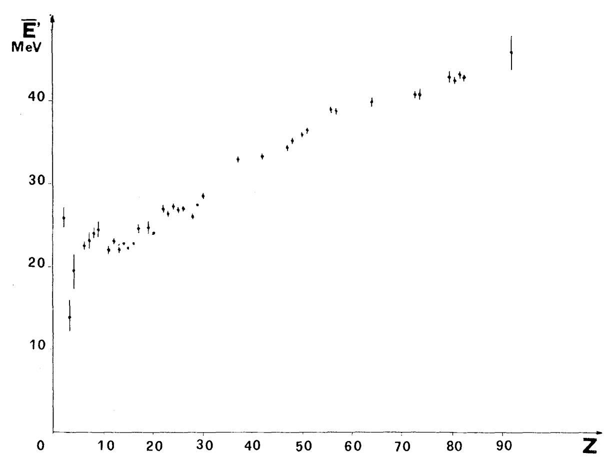

FIG. 2. $\bar E'$ obtained from Eq. (7) versus Z. The quoted errors are those coming from experimental uncertainty in $\Lambda_{\mu c}$.

Eq. (7) by a factor 1.3 would increase $\bar{E}'$ by ~3 MeV.

(ii) As Z increases, $\bar{E}'$ rises gradually because of a larger contribution from higher-order multipoles which become predominant in heavy nuclei. This can also be seen from the value of $\bar{\nu}R$, ~1.4 in ^{40}Ca and 2.5 in ^{208}Pb, and is confirmed by shell-model calculations.[4] Here exchange effects are not expected to be large because of the small relative contribution of the dipole term to the total transition strength. The smaller slope for very heavy nuclei can be probably explained by a hindrance of higher-order multipoles due to phase-space conditions.

Finally, we want to comment on the present approach. Since Eq. (7) has been obtained in a model-independent way and since the value of $\bar{E}'$ is not sensitively affected by exchange forces, we stress the importance of this parameter which appears to be the most natural one for μ capture.

[1]H. Primakoff, Rev. Mod. Phys. 31, 802 (1959).
[2]See, for example, J. S. Bell and J. Løvset, Nuovo Cimento 32, 433 (1964).
[3]L. L. Foldy and J. D. Walecka, Nuovo Cimento 34, 1026 (1964).
[4]J. Joseph, F. Ledoyen, and B. Goulard, Phys. Rev. C 6, 1742 (1972).
[5]See, for example, Ref. 2.
[6]See, for example, B. Goulard, G. Goulard, and H. Primakoff, Phys. Rev. 133, B186 (1964).
[7]F. Cannata, R. Leonardi, and M. Rosa-Clot, Phys. Lett. 32B, 6 (1970).
[8]K. W. Ford and J. G. Wills, Nucl. Phys. 35, 295 (1962).
[9]Y. N. Kim, *Mesic Atoms and Nuclear Structure* (North-Holland, Amsterdam, 1971), and references therein.
[10]H. A. Bethe and J. S. Levinger, Phys. Rev. 78, 115 (1950).
[11]M. Eckhause *et al*., Nucl. Phys. 81, 575 (1966).

Polarized-Proton Inelastic Scattering on ^{32}S and Possible Evidence for a Hexadecapole Phonon State

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Differential cross sections and analyzing powers for inelastic scattering of 24.5-MeV protons from ^{32}S have been measured, with special attention to the excitation of states which occur in the two-phonon region. The coupled-channel theory is used to interpret the excitation of the two-phonon states. An appreciably better fit to the data for the excitation of the second 2^+ state is obtained if a small admixture of the one-phonon state is assumed. However, a strong component of a 4^+ one-phonon state needs to be admixed to the 4^+ two-phonon state.

Various studies of s-d shell nuclei, theoretical as well as experimental, appear to indicate a transition with increasing A from pure rotational spectra to spectra of an anharmonic vibrator. The success of an interpretation in terms of the collective model is rather surprising in this region of light nuclei where one might expect individual-particle aspects to be dominant. The structure of sulfur remains a very puzzling one and we present here a tentative description of the first six states.

In the region of mass $A = 30-38$ where the application of the vibrational model has been previously suggested,[1,2] we have begun a study of ^{32}S and ^{34}S by inelastic scattering of polarized protons at 24.5 MeV. We have obtained good fits to the cross sections and analyzing-power measurements of the first 0^+, 2^+, and 3^- states of ^{32}S and ^{34}S by a coupled-channel analysis using the vibrational model.[3] This model predicts in its simple form a triplet of two-phonon states ($J = 0^+$, 2^+, 4^+) at around twice the energy of the one-phonon state. The states at 3.78 MeV (0^+), 4.29 MeV (2^+), and 4.46 MeV (4^+) have been tentatively identified by previous lifetime measurements[1] and (d, d') scattering[4] as the two-phonon states of ^{32}S.

In general, data for two-phonon states are rather sparse, since the cross sections are low and spacing of the states is small, requiring good energy resolution. However, these states represent a crucial test of the validity of the model.

The details of the experimental setup of the ex-