Test of the Lane Model for Nucleon-Nucleus Optical Potentials*

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Isospin-dependent potentials, determined from observed quasielastic (p,n) angular distributions, and a good set of proton optical potentials, are used to determine a set of neutron optical potentials. The elastic scattering predictions from these neutron potentials agree with existing neutron scattering data.

The Lane model[1] of the nucleon-nucleus optical potential postulates a simple dependence of the potential upon isospin:

$$U = U_0 + (4/A)(t \cdot T)U_1, \quad (1)$$

where t is the projectile isospin and T the nuclear isospin. Such a model predicts direct relationships between the potentials for proton elastic scattering, neutron elastic scattering, and the interaction which produces quasielastic (p,n) transitions. These relations are obtained from matrix elements of expression (1):

$$U_p = U_0 - U_1 (N-Z)A^{-1}, \quad (2)$$

$$U_n = U_0 + U_1(N-Z)A^{-1}, \quad (3)$$

$$U_{p,n} = 2U_1[(N-Z)/A]^{1/2}. \quad (4)$$

In general U_0 and U_1 are complex and may have differing radial shapes and energy dependences.[2]

If the Lane model is applicable, an analysis of (p,n) quasielastic scattering data can be used to determine U_1. Once U_1 is known, separate determinations of U_p and U_n by elastic scattering measurements would be redundant. Since both proton elastic and (p,n) quasielastic scattering are more easily measured than neutron elastic scattering, it would be desirable to utilize the Lane model for the purpose of obtaining the neutron-nucleus optical potential without a separate neutron scattering measurement.

In order to determine the validity of the Lane model we studied cases in which proton elastic scattering data, (p,n) quasielastic data, and neutron elastic scattering data existed. The four nuclei chosen for detailed analysis were ^{56}Fe, ^{58}Ni, ^{115}In, and ^{120}Sn. Elastic proton scattering data near $E_p = 22$ MeV existed for all except ^{115}In. Quasielastic (p,n) data for these four nuclides were taken from the University of Colorado (p,n) reaction survey[3] at $E_p = 23$ MeV. Because of the energy dependence of the nucleon optical potential, neutron elastic scattering data for this study should be taken at a neutron energy corresponding to the outgoing neutron energy in the (p,n) quasielastic studies. Available neutron data did exist near appropriate energies for the above four cases.

The general procedure we followed was to make self-consistent distorted-wave Born-approximation (DWBA) calculations of the (p,n) quasielastic angular distributions. The code DWUCK[4] was used for these calculations. The proton parameters were *fixed* to the "best-fit" values of Becchetti and Greenlees (BG).[5] The form factor was given by the difference in proton and neutron optical potentials via the expressions (2), (3), and (4). The form factor was varied (and the neutron potential appropriately modified) until a good fit to the (p,n) data was obtained. The angular distribution predicted for neutron elastic scattering by the final neutron potential so obtained was then compared with existing neutron data with the results shown in Fig. 1.

The dashed lines in Fig. 1 indicate the (p,n) quasielastic prediction obtained with our starting parameters which were (a) the "best-fit" BG proton parameters; (b) a form factor given by the (BG) isospin-dependent term which has volume real and surface imaginary components; (c) a self-consistent neutron potential obtained by simply changing the sign of the isospin-dependent terms in the BG proton potential (and dropping the Coulomb correction which appears in the proton potential). Use of the neutron potential given by BG (which is not self-consistent with the BG proton potential) made very little difference in our DWBA calculations. The energy dependence given by BG was taken into account.

After two or three iterations our search procedure found a slightly modified form of U_1 which gave an improved fit to the (p,n) data as shown by the solid line in Fig. 1. The main effect of our search procedure was to extend the surface-peaked imaginary term of U_1 to a larger radius

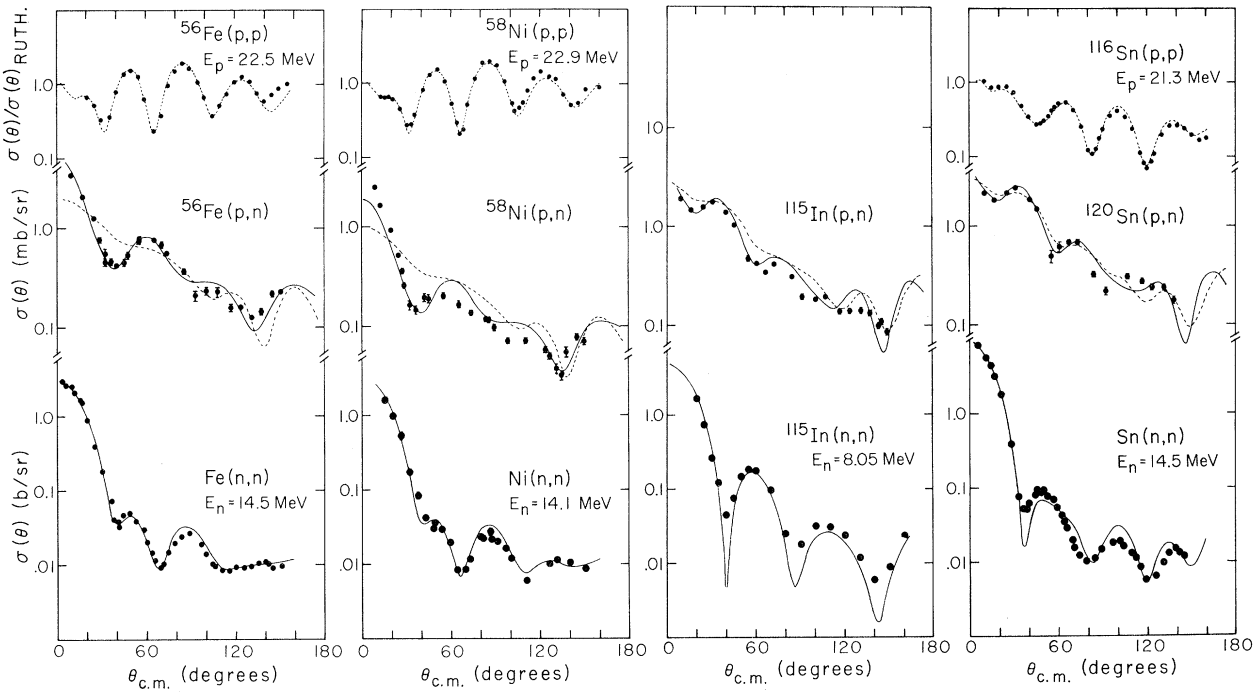

FIG. 1. Comparison of experimental cross-section data with Lane-model predictions (solid curves) for nucleon elastic and quasielastic scattering. For comparison, (p,p) and (p,n) predictions based on the Becchetti-Greenlees (Ref. 5) "best-fit" parameters are shown as a dashed curves. Isospin-dependent potentials are found which give improved fits to the (p,n) data for each nucleus. These potentials and the Becchetti-Greenlees proton potentials are then used to derive neutron potentials in the spirit of the Lane model. The neutron data are from Refs. 7–9 and the proton data from Refs. 10–12. Since the neutron scattering data for Fe, Ni, and Sn were obtained with natural isotopic mixtures, the neutron potential was suitably averaged for comparison with the data.

by increasing the diffuseness parameter as indicated in Table I. Table II gives the resulting neutron parameters.

The neutron parameters so obtained give fits to the neutron elastic scattering angular distributions which are apparently as good as existing parameters, such as those of BG.[5] The curves shown for neutron scattering in Fig. 1 do not include any corrections for compound-elastic scattering. We have made Hauser-Feshbach calculations which indicate that compound-elastic scattering is negligible for these cases. Further, the compound-elastic scattering for 14-MeV neutrons on Ni can be estimated by an analysis of the evaporationlike spectrum of inelastically scattered neutrons.[6] Such an estimate yields a value of less than $\frac{1}{2}$ mb/sr, which is negligible for our purposes. The neutron reaction cross sections, σ_R, predicted by our parameters are somewhat closer to experiment than those given by the BG neutron parameters. The BG predictions for σ_R are between 10% and 20% too large compared to data for Fe, Ni, and Sn. The potentials we obtain predict reaction cross sections between 2% and 5% too large.

In order to help determine whether the apparent success of the Lane model was either fortuitous or essentially independent of small changes in U_1, we made the following two calculations. First, for the case of ^{115}In, we used the U_1 given by BG for their best-fit proton potential to pro-

TABLE I. Charge-exchange potentials found to give an improved description of the (p,n) analog data compared to corresponding potentials from Ref. 5. The form of the potential and all parameters not listed are the same as the "best-fit" proton potentials of Ref. 5.

	V_1 (MeV)	W_1 (MeV)	a_I (fm)	r_I (fm)
^{56}Fe	17.8	8.9	0.90	1.29
^{58}Ni	15.6	7.8	0.90	1.29
^{115}In	18.8	9.4	0.83	1.29
^{120}Sn	21.2	10.6	0.83	1.29
BG	24.0	12.0	$0.545+0.35(N-Z)A^{-1}$	1.29

TABLE II. The neutron optical parameters derived using the Lane model compared with the corresponding parameters of Ref. 5.[a] All parameters not listed are the same as the "best-fit" parameters of Ref. 5. Suitable averages were made for the natural isotopic mixtures in Fe, Ni, and Sn.

	V_R (MeV)	W_V (MeV)	W_{SF} (MeV)	R_I (fm)	a_I (fm)	W_{SF}' (MeV)	r_I' (fm)	a_I' (fm)
Fe (14.5 MeV)								
BG	50.01	1.59	8.55	1.26	0.58	...[b]		
Lane model	48.55	0.49	9.00	1.32	0.56	−1.24[c]	1.32	0.90
Ni (14.1 MeV)								
BG	50.68	1.50	8.92	1.26	0.58	...[b]		
Lane model	49.04	0.40	8.83	1.32	0.54	−0.72[c]	1.32	0.90
In (8.05 MeV)								
BG	50.18	0.17	9.21	1.26	0.58	...[b]		
Lane model	49.30	0.0	11.56	1.32	0.62	−2.80[c]	1.32	0.83
Sn (14.5 MeV)								
BG	47.88	1.59	7.48	1.26	0.58	...[b]		
Lane model	46.66	0.49	10.06	1.32	0.62	−3.35[c]	1.32	0.83

[a]The parameters given in the above table refer to an optical-model potential of the form

$$-V_R f(r,R_R,a_R) - iW_V f(r,R_I,a_I) + iW_{SF} 4a_I (d/dr)[f(r,R_I,a_I)] + iW_{SF}' 4a_I' \times (d/dr)[f(r,R_I',a_I')] + V_{s.o.}(\hbar/m_\pi c)^2 r^{-1} (d/dr)[f(r,R_{s.o.},a_{s.o.})]\,\vec{\sigma}\cdot\vec{l},$$

where $f(r,R,a) = [1+\exp(r-RA^{1/3})/a]^{-1}$.

[b]Since the imaginary part of U_1 found in the present work had a geometry different from the imaginary part of the proton optical potentials, it was necessary to add an additional term to the above expression. This term is absent in the BG potentials.

[c]This term is represents the slight modification of the proton potential required by the Lane model. A single suitably averaged derivative Woods-Saxon form can be found to replace W_{SF} and W_{SF}' which gives essentially identical results.

duce a Lane-model consistent neutron potential. The neutron scattering it predicts agrees with the data in the forward peak but is low on the second maximum by a factor of 0.8 and lower still on the third maximum (near 110 deg) by a factor of 0.6. A second interesting test was made for the ^{56}Fe data. We began with the proton potential of Perey[7] which differs from that of BG but still gives an excellent fit to the data. We then used our U_1 to obtain a "Perey" neutron potential from the Lane model. The result was again an excellent fit to the neutron elastic scattering data.

In summary, we have used only the BG proton optical potential and a fit to (p,n) quasielastic scattering data to determine the full isospin-dependent nucleon optical model for four nuclei for which appropriate neutron elastic scattering data existed. The agreement obtained with data lends strong support to the Lane model for nucleon optical potentials. An attractive application of these results is the possibility of obtaining neutron optical-model parameters from proton elastic scattering and (p,n) quasielastic measurements alone. A complete description of the optical-model analysis of the Colorado (p,n) survey data is presently being completed and includes a smoothed-parameter optical model for all nuclei with $A \geq 40$.

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Population of 0^+ Excited States in ^{238}Pu and ^{240}Pu by Single-Neutron Transfer Reactions*

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Studies of the single-particle transfer reactions ^{239}Pu$(d,p)^{240}$Pu and ^{239}Pu$(d,t)^{238}$Pu indicate that—contrary to theoretical expectations—the ratios of the cross sections for population of the $K^\pi = 0^+$ excited states to those for population of the ground states are of the same order of magnitude as the ratios for the equivalent (p,t) reactions.

The 0^+ excited states in deformed nuclei have lately attracted much experimental interest, and they have been observed in rare-earth[1-3] and actinide nuclei.[4,5] Throughout the latter region of the nuclide chart, the (p,t) reaction has been found to excite the 0^+ states with uniform and fairly high cross sections.

There has also been a great deal of theoretical discussion[6-8] of the character of these 0_1^+ states. Griffin, Jackson, and Volkov[6] proposed a model in which the pairings between oblate and prolate levels were unequal, and van Rij and Kahana[7] believed that this model could explain the high ratio of the cross sections for (p,t) reactions to the $K^\pi = 0^+$ excited states to the cross sections for reactions to the ground state. As pointed out in Ref. 7, one prediction of this model is that the excited $K^\pi = 0^+$ states should be weakly excited in (d,p) and (d,t) reactions. Our purpose in this paper is to test this prediction. It should be noted that Chasman,[8] who based his calculations on experimentally determined single-particle energies, could not agree with their calculated energies for the excited $K^\pi = 0^+$ states, and found that states of the predicted character would be at much higher excitation.

Reference 7 also argued that the experiments of Bjørnholm, Dubois, and Elbek,[9] who studied single-particle transfer reactions to states in ^{234}U, gave evidence for weak single-particle excitation of excited $K^\pi = 0^+$ states. However, since the configuration of the target nucleus prevented excitation of low-spin states in these experiments, this evidence was not definitive.

To learn more about the structure of these states and about their possible two-quasiparticle components, we have studied their population by use of single-neutron transfer reactions on ^{239}Pu, which is a more suitable target because its low spin ($I = \frac{1}{2}$) and its $\frac{1}{2}^+[631]$ configuration would lead to higher cross sections for populating states with I = 0, 2, and 4. Also, the reaction[5] ^{242}Pu$(p,t)^{240}$Pu and the reaction[10] ^{240}Pu$(p,t)^{238}$Pu had been studied, and several $K^\pi = 0^+$ excited states in each final nucleus had been identified through studies either of these reactions or of radioactive decay.

The experimental results are shown in Figs. 1 and 2. The ^{239}Pu$(d,p)^{240}$Pu spectrum from the present work and the ^{242}Pu$(p,t)^{240}$Pu spectrum given by Maher *et al.*[5] are plotted on the same energy scale in Fig. 1. Our ^{239}Pu$(d,t)^{238}$Pu spectrum and the ^{240}Pu$(p,t)^{238}$Pu spectrum given by Schiffer *et al.*[10] are similarly plotted in Fig. 2. All four spectra were taken with an Enge split-pole spectrograph at the Argonne FN tandem Van de Graaff. A 12-MeV deuteron beam was used for the present work, and data for each target were taken at two or three angles and were analyzed by use of the computer program AUTOFIT.

The cross sections and energies of the rotational states observed in the different $K^\pi = 0^+$ bands in ^{240}Pu and ^{238}Pu are listed in Tables I and II, respectively. The energies and cross sec-