

Fiz. Tekh. Poluprov. 2, 113 (1968) [Sov. Phys. Semicond. 2, 93 (1968)]; V. A. Chaldyshev and V. N. Pokrovskii, *Izv. Viz. Fiz. SSSR* 2, 173 (1960), and 5, 103 (1963).

²J. L. Shay, E. Buehler, and J. H. Wernick, *Phys. Rev. B* 3, 2004 (1971).

³N. A. Goryunova, S. S. Grigor'yan, and L. B. Zlatkin, *Izv. Akad. Nauk SSSR, Neorg. Mater.* 2, 2125 (1966).

⁴J. L. Shay, R. F. Leheny, E. Buehler, and J. H. Wernick, *J. Lumin.* 1, 851 (1970); R. E. Nahory, J. Shah, R. C. C. Leite, E. Buehler, and J. H. Wernick, *Phys. Rev. B* 1, 4677 (1970).

⁵J. L. Shay, in *Proceedings of the Eleventh International Conference on the Physics of Semiconductors, Warsaw, 1972*, edited by The Polish Academy of Sci-

ences (PWN—Polish Scientific Publishers, Warsaw, 1972).

⁶E. Buehler and J. H. Wernick, *J. Cryst. Growth* 8, 324 (1971).

⁷B. Ray, A. J. Payne, and G. J. Burrell, *Phys. Status Solidi (b)* 35, 197 (1969).

⁸M. Cardona, *Modulation Spectroscopy* (Academic, New York, 1969).

⁹P. J. Dean and D. J. Thomas, *Phys. Rev.* 150, 690 (1966).

¹⁰A. Shileika, in *Proceedings of the First International Conference on Modulation Spectroscopy, Tucson, Arizona, 23–26 November 1972* (to be published).

¹¹J. L. Shay, *Phys. Rev. Lett.* 29, 1008 (1972).

¹²J. A. Van Vechten, *Phys. Rev.* 187, 1007 (1969).

Investigation of the Spin Dependence in the Deuteron-Nucleus Interaction

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Measurements of the cross section, the vector analyzing power, and the three tensor analyzing powers have been made for deuteron elastic scattering from ^{90}Zr at 5.5 MeV. The measurements are explained by an optical-model potential whose spin-dependent parts are calculated from the nucleon-nucleus potential. The predicted spin-orbit potential, which is considerably different from the potentials used in previous phenomenological analyses, also gives excellent agreement with the vector analyzing power at 9 MeV.

The optical model has been used to describe deuteron elastic scattering from a wide range of target nuclei. Watanabe¹ has suggested that the deuteron-nucleus potential might be calculated from the empirically known nucleon-nucleus optical potentials. In this so-called folding model, the deuteron potential is taken to be the sum of the neutron-nucleus and proton-nucleus potentials, averaged over the internal motion of the deuteron.

The purpose of the present Letter is to investigate whether the folding model correctly predicts the spin-dependent parts of the deuteron optical-model potential. Phenomenological spin-dependent potentials have shown some resemblance to the folding-model predictions,^{2,3} but no systematic investigation of the accuracy of the spin-dependent potentials has been made.

When the nucleon-nucleus potentials are made up of central and spin-orbit terms, the deuteron potential will likewise contain a central and a spin-orbit term.¹ In addition, the folding-model potential will contain a tensor potential of the

form

$$V_{\vec{r}}(\vec{r}) = F(r)T_r \quad (1)$$

provided that the effects of the deuteron D state are included.⁴ In Eq. (1), r is the deuteron-nucleus separation and the tensor operator, T_r , is defined by

$$T_r = (\vec{s} \cdot \hat{r})^2 - \frac{2}{3}, \quad (2)$$

where \vec{s} is the deuteron spin.

The spin-dependent potentials predicted⁵ from the folding model for 5.5-MeV deuterons incident on ^{90}Zr are shown in Fig. 1. These potentials were calculated using formulas given by Raynal.⁶ The tensor potential is complex because the central nucleon-nucleus potentials are complex. The spin-orbit potential results from the neutron and proton spin-orbit potentials and is thus purely real.

The spin-orbit and tensor potentials have only a small effect on the elastic scattering cross section, but do affect the analyzing powers⁷ (i.e., cross section for polarized incident deuterons).

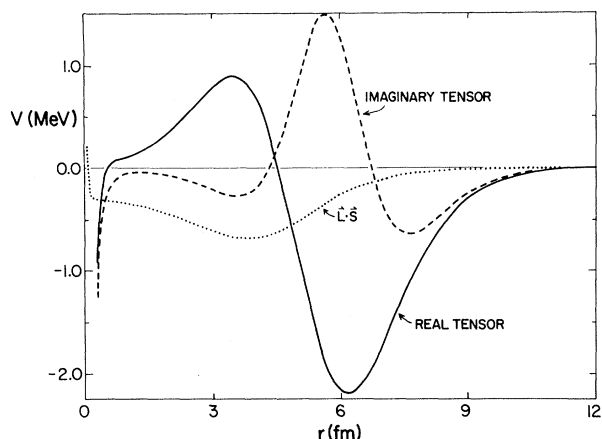


FIG. 1. Spin-dependent parts of the folding-model potential for ^{90}Zr at 5.5 MeV as a function of deuteron-nucleus separation. The real (imaginary) part of $F(r)$ is shown by the solid (dashed) curve. The spin-orbit potential is obtained by multiplying the dotted curve by $\vec{L} \cdot \vec{s}$.

When the spin-dependent potentials are small enough to be treated as perturbations, the spin-orbit potential produces a nonzero vector analyzing power (iT_{11}), while the tensor potential primarily affects the tensor analyzing powers (T_{20} , T_{21} , T_{22}).⁸

There have previously been several optical-model studies in which acceptable fits to vector analyzing-power data were obtained by using phenomenological spin-orbit potentials.⁹ It has been stated² that the phenomenological spin-orbit strength is consistent with that expected from the folding model. However, a simple comparison of the spin-orbit strengths for deuterons and nucleons is not meaningful because the radial dependence of the deuteron spin-orbit term employed in the phenomenological analyses is in fact not consistent with that predicted from the folding model. Optical-model studies involving tensor analyzing powers have not been very successful and, as a result, there is little phenomenological information on the tensor interaction in deuteron scattering from intermediate-weight nuclei.

In the present experiment the analyzing powers and differential cross section were measured for deuteron elastic scattering from ^{90}Zr at 5.5 MeV. This energy is below the Coulomb barrier and thus the analyzing powers are small. The experimental techniques which were used have been described in detail elsewhere.^{10,11} The results¹² are shown in Fig. 2.

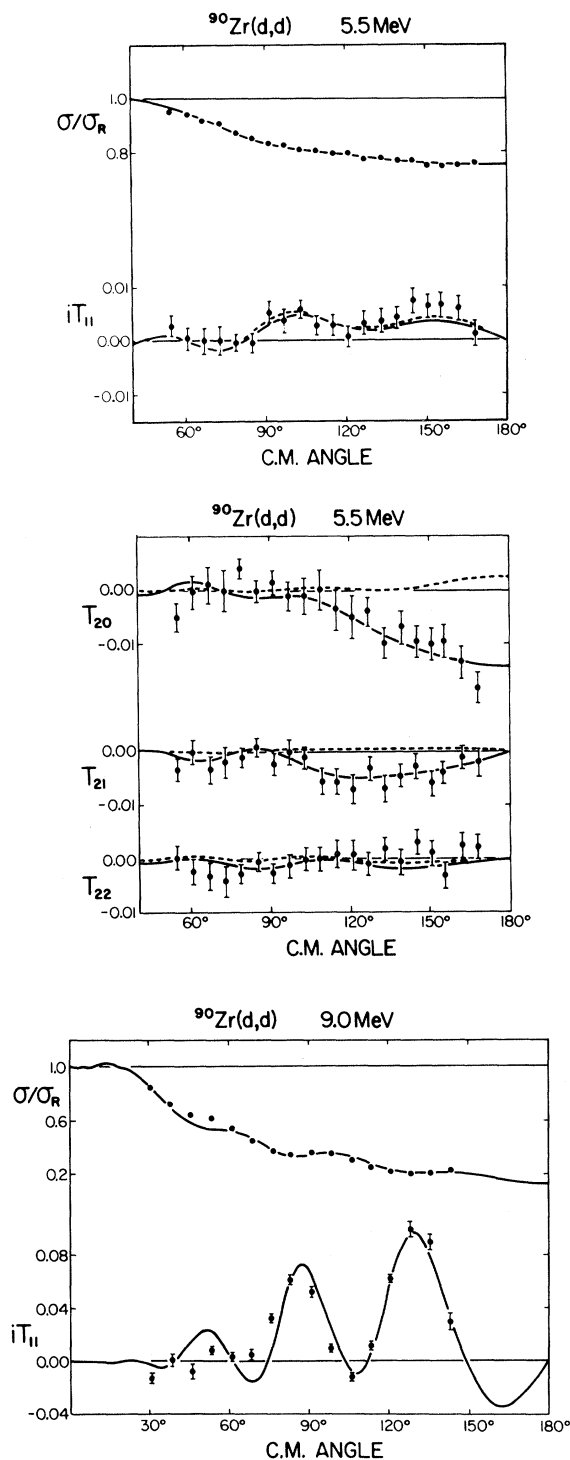


FIG. 2. Angular distributions of the cross section and analyzing powers for deuteron elastic scattering from ^{90}Zr at 5.5 and 9.0 MeV. Measurements at 9.0 MeV are from Ref. 18. Cross sections are plotted as ratios to the Rutherford cross section and have relative errors of 2% at 5.5 MeV and 5% at 9.0 MeV. The calculated curves are described in the text. Where not shown, the dashed curves coincide with the solid curves.

TABLE I. Parameters describing the central parts of the optical-model potentials. The notation is that of Ref. 13.

	V_R (MeV)	r_R (fm)	a_R (fm)	W_{SF} (MeV)	r_I (fm)	a_I (fm)
5.5 MeV	94.80	1.200	0.580	6.54	1.774	0.716
9.0 MeV	89.41	1.200	0.550	5.28	1.919	0.749

We wish to compare the measurements with the predictions of the folding model. The central parts of the optical-model potential were parametrized in the usual way (see Ref. 13), and consisted of a real Woods-Saxon well, a surface-peaked imaginary potential, and the Coulomb potential of a uniformly charged sphere of radius $1.3A^{1/3}$ fm. The folding-model spin-orbit potential was included, and the parameters describing the central potentials were determined by fitting the cross section and vector analyzing power. No tensor potential was used during this part of the analysis.

The final parameters are listed in Table I, and the fits are shown by the dashed curves in Fig. 2. The agreement with the vector analyzing-power data indicates that the folding model provides an adequate description of the spin-orbit potential, but the calculated tensor analyzing powers are much smaller in magnitude than the measurements. This provides evidence for the existence of a tensor interaction, as can be shown by using arguments based on perturbation theory. Since the vector analyzing power is very small in magnitude, the spin-orbit potential can be treated as a perturbation. If no tensor potential is present, the tensor analyzing powers are second order in the strength of the spin-orbit interaction, while the vector analyzing power is first order.⁸ As a result, the calculated tensor analyzing powers must be substantially smaller in magnitude than the vector analyzing power.¹⁴ Since the measured tensor analyzing powers are as large in magnitude as the vector analyzing power, tensor forces must be present in the deuteron-nucleus interaction.

To test the accuracy of the tensor part of the folding-model potential, the tensor potential¹⁵ was included while the central and spin-orbit parts of the optical-model potential remained unchanged. The resulting analyzing powers are given by the solid curves in Fig. 2. The agreement with the measured tensor analyzing powers is entirely satisfactory. The present study is

the first case in which angular distributions of all five elastic-scattering observables have been well reproduced by an optical-model calculation.¹⁶ This is especially significant when one considers that the spin-dependent potentials contain no adjustable parameters, and that agreement with the tensor analyzing powers was obtained without readjustment of the parameters describing the central potentials. We conclude that the folding model provides a good description of the optical-model tensor potential.¹⁷ However, the test of the folding model is not very stringent since the analyzing powers are small and show little structure.

As a further test of the folding-model spin-orbit potential, we have analyzed previously reported cross-section and vector analyzing-power data for deuteron scattering from ⁹⁰Zr at 9.0 MeV.¹⁸ The parameters describing the central parts of the optical-model potential were determined as in the analysis described above. The final parameters are listed in Table I, and the calculated curves are shown in the bottom part of Fig. 2. The fits are a significant improvement over those obtained in Ref. 18. The spin-orbit potential used in Ref. 18 is of the Thomas form with $V_{s.o.} = 7$ MeV, $r_{s.o.} = 0.75$ fm, and $a_{s.o.} = 0.50$ fm, and is typical of the phenomenological spin-orbit potentials found in the literature.⁹ This potential is much deeper (2.1 MeV) and is more sharply peaked than the folding-model spin-orbit potential shown in Fig. 1, which can be approximated by a Thomas potential with $V_{s.o.} = 5.63$ MeV, $r_{s.o.} = 0.98$ fm, and $a_{s.o.} = 1.00$ fm. It is interesting that the present analysis, which used a fixed spin-orbit potential, resulted in an improved fit over that of Ref. 18 in which the spin-orbit potential was varied. Apparently the previous analysis did not explore parameter space in sufficient detail, because of the large number of parameters used.

We have shown that the folding model provides a description of the spin-dependent parts of the deuteron optical-model potential which is suffi-

ciently accurate to describe the analyzing powers for sub-Coulomb elastic scattering. In addition we have found that the predicted spin-orbit potential is consistent with the measured vector analyzing power at higher energies. Additional optical-model studies at higher energies (including tensor analyzing-power data) must be made before stronger conclusions can be drawn. These studies may eventually show the necessity for modifying the folding-model spin-dependent potentials. Should that be the case, the model would still provide a physically reasonable starting point for further optical-model analysis.

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¹S. Watanabe, Nucl. Phys. **8**, 484 (1958).

²W. Haeberli, Rev. Brasil. Fis. **2**, 187 (1972).

³J. Raynal, Phys. Lett. **29B**, 93 (1969).

⁴G. R. Satchler, Nucl. Phys. **21**, 116 (1960).

⁵The deuteron wave function used in this calculation was taken from T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962). The neutron and proton optical-model potentials were obtained from F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. **182**, 1190 (1969).

⁶J. Raynal, Ph.D. thesis, Commissariat à l'Énergie Atomique, Centre d'Études Nucléaires de Saclay, Report No. CEA-R 2511, 1965 (Argonne National Laboratory Report No. ANL-TRANS-258) (unpublished).

⁷S. E. Darden, in *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, Wis., 1970), p. 39.

⁸R. C. Johnson, in *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, Wis., 1970), p. 143.

⁹See, for example, P. Schwandt and W. Haeberli, Nucl. Phys. **A123**, 401 (1969); J. A. R. Griffith, M. Irshad, O. Karban, and S. Roman, Nucl. Phys. **A146**, 193 (1970), and **A166**, 675(E) (1971).

¹⁰H. S. Liers, R. D. Rathmell, S. E. Vigdor, and W. Haeberli, Phys. Rev. Lett. **26**, 261 (1971).

¹¹N. Rohrig and W. Haeberli, to be published.

¹²The analyzing powers are defined according to the Madison Convention, as found in *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, Wis., 1970).

¹³Becchetti and Greenlees, Ref. 5.

¹⁴This argument does not hold at higher energies for which the vector analyzing power is large in magnitude. See, for example, J. Raynal, Phys. Lett. **7**, 281 (1963).

¹⁵The off-diagonal parts of the T_T potential were treated properly in the calculation. Tensor potentials of the T_L type (see Ref. 4) were not used. The tensor potential which arises from the interaction of the deuteron quadrupole moment with the Coulomb field of the nucleus was also included in the calculation. This potential is discussed in Ref. 2.

¹⁶P. W. Keaton and D. D. Armstrong, Bull. Amer. Phys. Soc. **17**, 686 (1972).

¹⁷Further calculations have shown that the overall quality of the fits does not change significantly when the imaginary tensor term is omitted in the calculation. Thus the present work is unable to establish the existence of the imaginary tensor potential.

¹⁸J. M. Lohr and W. Haeberli, Bull. Amer. Phys. Soc. **17**, 563 (1972), and to be published.

Analysis of the Anomaly in the Reaction $^{88}\text{Sr}(d, p_0)^{89}\text{Sr}$ Using Polarized Deuterons at the Threshold of the Neutron Analog Channel*

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The theory of Lane for the anomaly in the excitation function of the (d, p_0) reaction near the threshold of the analog (d, n) channel has been applied to polarized-beam measurements. The anomaly effects, observed at several angles in both cross section and analyzing power in the reaction $^{88}\text{Sr}(d, p_0)^{89}\text{Sr}$, are reproduced by a distorted-wave Born-approximation calculation, if the radial matrix elements for the proton exit channel with $l=1$, $j=\frac{3}{2}$ are modified by a resonance term.

Anomalous effects in excitation curves of the (d, p_0) reaction near the threshold of the (d, n) analog channel have been observed with several spin- 0^+ targets in the $A=90$ mass region^{1,2} and pronounced effects have been found for the reac-

tion $^{88}\text{Sr}(d, p_0)^{89}\text{Sr}(\frac{5}{2}^+)$.³ The anomaly is assumed to originate from the isospin coupling of the $3p$ neutron single-particle resonance to the proton analog channel.^{1,4} For $A \approx 90$ nuclei the $3p$ resonance is near the neutron emission threshold.