with $G_{\mu\nu}(p_2, k_2, p_1, k_1)$ being the ordinary Green's function in momentum space,

$$G_{\mu\nu}(p_2, k_2; p_1, k_1) = \int d^4x_2\, d^4y_2\, d^4x_1\, d^4y_1 \exp[i(p_2\cdot x_2 - p_1\cdot x_1 + k_2\cdot y_2 - k_1\cdot y_1)] \times \langle 0 | T[\psi(x_2) J_\mu(y_2) J_\nu(y_1) \overline{\psi}(x_1)] | 0\rangle. \quad (20)$$

A more detailed exposition will derive the reduction formula in a class of gauges for a general scattering process and will include a discussion of the Gupta-Bleuler condition and of the choice of coherent states corresponding to a given experimental situation. Applications will be presented and in particular an expression for cross sections in terms of finite covariant on-mass-shell amplitudes, reduced S-matrix elements.

This work has profited from many enjoyable discussions with numerous colleagues. In particular I wish to thank N. Kroll, H. Stapp, A. Goldhaber, Y. Frishman, S. Brodsky, P. Tsai, N. Pananikolaou, A. Mueller, and A. Sirlin for their stimulating and informative criticism.

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[1]It is misleading that in the Yennie gauge the singularity is a pole. This is due to a cancelation between states of positive and negative metric, and the analog of Eq. (3) holds also in the Yennie gauge.

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[5]This reaction is chosen because it is simple and illustrates the reduction formula for both electrons and photons. An explicit calculation by the present method of the amplitude for Coulomb scattering was presented in Ref. 3.

Measurement of Cosmic-Ray Muon Charge Ratio at Sea Level Between Energies of 10 and 1500 GeV*

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We have measured the cosmic-ray muon charge ratio, μ^+/μ^-, from 10 GeV to 1.5 TeV with a sea-level magnetic spectrometer at eight different zenith angles between 0° and 90°. The data are consistent with a constant charge ratio between 50 GeV and 1.5 TeV, in agreement with the predictions of the scaling hypothesis. Assuming the charge ratio to be constant, we obtain its value as 1.261 ± 0.009.

In this paper we report the results of a measurement of the cosmic-ray muon charge ratio, μ^+/μ^-, over an energy range of 10 GeV to 1.5 TeV. The data were collected with the University of California, San Diego, sea-level cosmic-ray spectrometer,[1] which has a maximum resolvable energy of 3.5 TeV. The spectrometer consists of a solid iron toroidal magnet, six modules of wire spark chambers, and four planes of scintillators. The entire setup can be rotated through 360° about a horizontal axis located at the midpoint, thereby accepting particles with equal solid angle and efficiency at any zenith angle. The wire-chamber information is read out through magnetostrictive delay lines, recorded on magnetic tape together with information from the scintillators, and analyzed off line on a computer.

The importance of the muon charge-ratio measurements lies in their interpretation at high energies, where they reflect the momentum and angular distributions and multiplicities of the parent particles (primarily π's and K's) produced in the primary interactions. There have been a number of theoretical efforts to discriminate among various models of high-energy hadronic interactions using the existing charge-ratio data, but the limited statistical accuracy of these measurements has made it difficult to draw significant conclusions from them.[2-4]

Chou and Yang[2] pointed out that the hypothesis of limiting fragmentation implies that the μ^+/μ^- charge ratio will be larger than unity at high energies. Frazer *et al.*[3] concluded that the steeply falling primary cosmic-ray spectrum causes the projectile fragmentation to be the dominant process in producing cosmic-ray muons. They also showed that the scaling hypothesis predicts a constant value for the charge ratio at high energies and, using accelerator data available at that time, estimated the ratio to be 1.56. Garraffo, Pignotti, and Zgrablich[4] included corrections due to multiple inelastic scattering inside the target nucleus in conjunction with the scaling hypothesis in order to calculate the muon charge ratio, and found a constant value of 1.38. As the theoretical predictions become more accurate, a more statistically significant measurement of the charge ratio becomes meaningful.

We have collected data at eight different zenith angles between 0° and 90° with an angular acceptance of ±4°. The polarity of the magnetic field in the spectrometer was reversed often to measure any systematic effects, collecting approximately half the data at each polarity. Any time variation in the muon detection efficiency of the telescope does not affect the charge ratio measurement. The values of the charge ratios obtained at each angle are presented in Fig. 1. as a function of muon momentum. The dominant features of the charge ratio can be observed by dividing the momentum interval into two regions: (1) 10–50 GeV/c and (2) 50–1500 GeV/c. In the 10–50-GeV/c region the charge ratio shows both an energy and angular dependence; the charge ratio decreases with energy and increases with zenith angle. The zenith angle dependence can be explained as a manifestation of the geomagnetic effect, our spectrometer pointing always to the western sky during the accumulation of the of the present data. As for the 50–1500-GeV/c region, the measured charge ratio is constant over all energies and angles.

In Fig. 2(a) we have combined our data from all

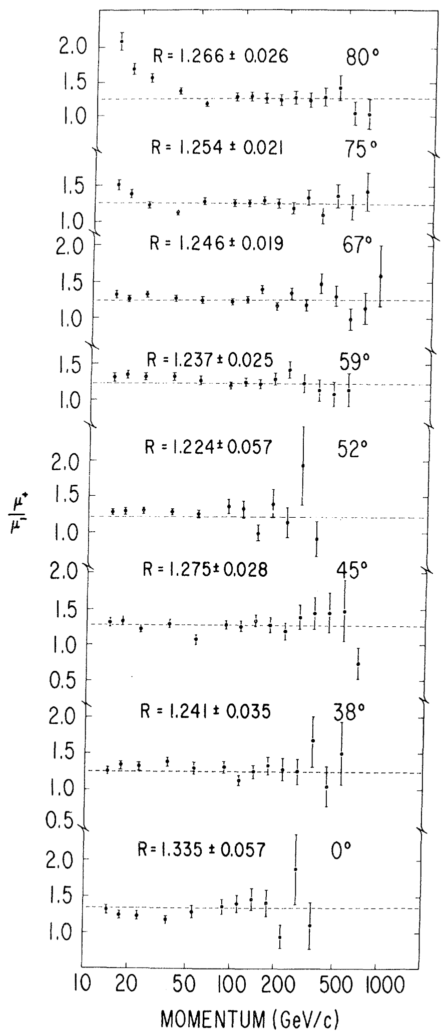

FIG. 1. Measured charge ratios at different zenith angles, plotted as function of muon momentum. R is the charge ratio above 50 GeV/c.

zenith angles and have plotted them against muon momentum at production. [We have reproduced the existing data on charge ratio from a compilation by Fujii *et al.*[5] for comparison in Fig. 2(b).] In the 10–50-GeV/c region we have used data from zenith angles ≤ 60° where the geomagnetic effect on charge ratio is negligible. Our results agree within statistics with the existing sea-level measurements[5-10] [see Fig. 2(b)]. We do not find any support for dips or rises in charge ratio as previously reported.[7,8,10] Referring to the results presented in Fig. 2(a), we notice a slight rise in the value of the charge ratio at energies lower than 50 GeV. We interpret this as due to the nonscaling behavior of the production processes at these energies. For energies greater than 50

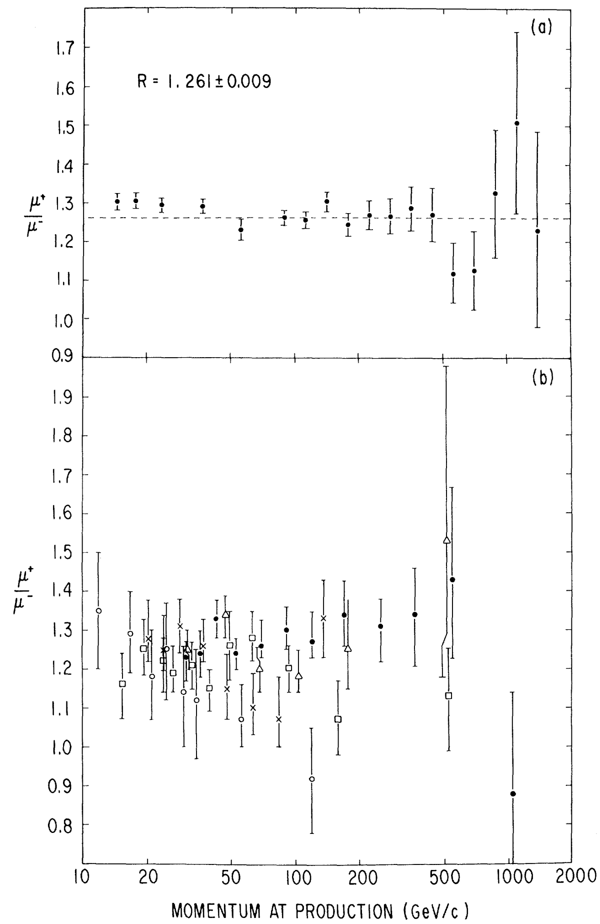

FIG. 2. (a) Combined data from all zenith angles plotted as a function of muon momentum at production. The dotted line is the fit to a constant charge ratio for momentum greater than 50 GeV/c. (b) Charge-ratio data reproduced for comparison, from a 1969 compilation by Fujii *et al.* (Ref. 5).

GeV, however, the charge ratio is essentially constant, consistent with predictions of the scaling hypothesis.[3,4] Assuming a constant charge ratio above 50 GeV, we obtain a value for the μ^+/μ^- charge ratio of 1.261 ± 0.009, in agreement with previous measurements,[5,10] with a χ^2 value of 10.0 for 12 degrees of freedom. The errors are statistical only; we have considered possible sources of systematic error and have concluded that all are negligible. Thus our measured value of the charge ratio is lower than the predictions of earlier calculations[3,4] which were based on imprecise knowledge of the relevant production cross sections. A recent CERN Intersecting Storage Rings measurement[11] of the π^+/π^- production ratio in the relevant X region (~ 0.1) is in fact closer to unity than low-energy accelerator measurements, and a preliminary calculation[12] shows that these data are in agreement with our charge ratio.

It is of interest to note that the results of this experiment exhibit a manifestation of the scaling hypothesis over a very large energy region, from its apparent onset at about 50 GeV/c into the TeV region. The measurement of the cosmic-ray muon charge ratio at these energies corresponds to a test of the scaling hypothesis with primary protons whose energies are, on the average, ten times higher than the muon energies. In our case these correspond to proton energies between 500 GeV and 15 TeV.

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[12]The complete analysis will be reported later together with the overall fit to the observed muon energy spectrum.

ERRATA

COUPLED-CHANNEL BORN-APPROXIMATION CALCULATION FOR THE TRANSITION $Ne^{22}(p, t)Ne^{20}(2^-$, 4.97 MeV). David K. Olsen, Takeshi Udagawa, Taro Tamura, and Ronald E. Brown [Phys. Rev. Lett. 29, 1178 (1972)].

On page 1179, the spin factor in Eq. (1) should be

$$\left(\frac{2I_i+1}{2I_f+1}\right)^{1/2}$$

instead of

$$\left(\frac{2I_f+1}{2I_i+1}\right)^{1/2}.$$

The calculations were done with the correct spin factor.

(^{16}O, ^{12}C) AND (^6Li, d) AS α-TRANSFER REACTIONS. R. M. DeVries [Phys. Rev. Lett. 30, 666 (1973)].

The distorted-wave Born-approximation predictions presented for $^{12}C^*(2^+)$ cross sections were incorrectly normalized and should be 2 times larger than shown. The conclusions remain unchanged and are strengthened by new calculations using experimentally derived spectroscopic factors for $^{16}O = {}^{12}C + \alpha$ (to be published).