PHYSICAL REVIEW LETTERS

VOLUME 30 — 7 MAY 1973 — NUMBER 19

Fourth-Order Vacuum Polarization Correction to the Positronium Hyperfine Structure

David A. Owen*
Centre de Physique Théorique, Centre National de la Recherche Scientifique, Marseille, France
(Received 22 January 1973)

It is shown that the fourth-order vacuum polarization correction to the annihilation diagram contributes $\Delta\nu = -\frac{1}{4}\alpha^4 R_\infty \ln\alpha^{-1}$ to the positronium ground-state hyperfine structure. This leads to a reduction of 11.3 MHz in the previous theoretical prediction which greatly improves the agreement between theory and experiment.

It had been previously believed that the lowest-order contribution to the positronium hyperfine splitting from interaction kernels, $I_K(x, x')$, containing three-photon diagrams would be $m_e\alpha^6$.[1] Generally, it was thought that a factor of α^3 came from the wave-function normalization and an additional factor of α for each photon.

I shall show for a particular diagram, namely, the fourth-order vacuum polarization annihilation diagram (see Fig. 1), that the above argument is not true since this diagram contributes to order $\alpha^6 \ln\alpha$. This is due to the occurrence of a threshold singularity at $4m_e^2$ for the photon propagator.

To be more explicit, we have that the energy shift for the annihilation process is[2]

$$\Delta E_A = -4\pi\alpha\,|\varphi(0)|^2\langle S^2\rangle \bar{D}_F(K^2), \tag{1}$$

where K is the total c.m. positronium energy and

$$\bar{D}_F(q^2) = -\left[\frac{1}{q^2} + \int_0^\infty \frac{dt}{t}\,\frac{\Pi(t)}{(q^2-t)}\right], \tag{2}$$

with

$$\Pi(k^2) = -\frac{(2\pi)^3}{3k^2}\sum_n \delta^4(p_n - k)\langle 0|j_\mu(0)|n\rangle \times\langle n|j^\mu(0)|0\rangle. \tag{3}$$

The $\alpha^6 \ln\alpha^{-1}$ contribution to ΔE_A is readily found for the fourth-order vacuum polarization diagram by using the expression for $\Pi^{(4)}(t)$ calculated by Källén and Sabry.[3] We find that

$$\Delta E_A^{(4)}(\alpha^6 \ln\alpha^{-1}) = -4\pi\alpha\,|\varphi(0)|^2\langle S^2\rangle \times\Pi^{(4)}(4m_e^2)\int_{4m_e^2}^\infty dt\,\frac{1}{t(t-K^2)}, \tag{4}$$

where

$$\Pi^{(4)}(4m_e^2) = (\alpha/\pi)^2\{-\tfrac{1}{2}[4\Phi(-1) + \tfrac{1}{2}\pi^2]\} = \tfrac{1}{4}\alpha^2, \tag{5}$$

with $\Phi(t)$ being the Spence function. Thus we have[4]

$$\Delta E_A^{(4)}(\alpha^6 \ln\alpha^{-1}) = -\frac{\pi\alpha^3}{2m_e^2}|\varphi(0)|^2\langle S^2\rangle \ln\alpha^{-1}, \tag{6}$$

which leads to a frequency shift of

$$\Delta\nu = -\tfrac{1}{4}\alpha^4 R_\infty \ln\alpha^{-1} = -11.3\text{ MHz}, \tag{7}$$

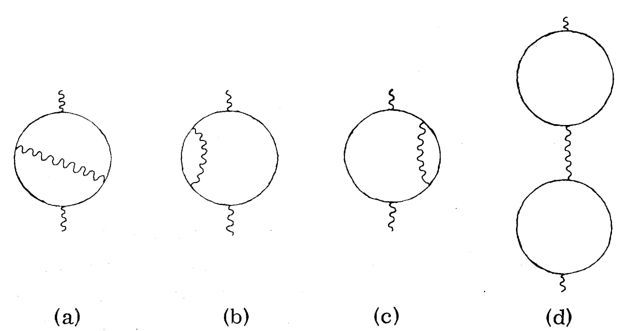

FIG. 1. Fourth-order vacuum polarization diagrams. It is found that (d) does not contribute to order $\alpha^6 \ln\alpha^{-1}$.

where we have taken $\alpha^{-1} = 137.036\,08$.

Combining this result with previous calculations[1,5] we obtain

$$\nu_{\text{theor}} = 2.034\,04 \times 10^5 \text{ MHz} \tag{8}$$

for the ground-state positronium hyperfine interval, which is to be compared with the experimental value[6]

$$\nu_{\text{expt}} = 2.033\,96(5) \times 10^5 \text{ MHz}.$$

One observes the theoretical result is now slightly more than 1 standard deviation above the experimental value. Considering the estimate for uncalculated diagrams in Ref. 5, the agreement between theory and experiment seems quite reasonable.[7]

I wish to thank Professor A. Peterman for checking this calculation.

*Chercheur Associé au Centre National de la Recherche Scientifique.

[1] R. Karplus and A. Klein, Phys. Rev. 87, 848 (1952).

[2] D. A. Owen and W. W. Repko, Phys. Rev. A 5, 1570 (1972). The present paper uses the Bjorken and Drell metric and in this respect differs from this reference.

[3] G. Källén and A. Sabry, Kgl. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 29, No. 17 (1955); B. E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1968).

[4] After the completion of this work, I learned that the same result had been obtained, independently, by R. Barbieri, P. Christillin, and E. Remiddi.

[5] T. Fulton, D. A. Owen, and W. W. Repko, Phys. Rev. A 4, 1802 (1971).

[6] E. R. Carlson, V. W. Hughes, M. L. Lewis, and I. Lindgren, Phys. Rev. Lett. 29, 1069 (1972).

[7] This is, of course, assuming that none of the other three-photon diagrams contribute to order $\alpha^6 \ln\alpha^{-1}$. Although arguments exist which suggest that this is the case, no definitive statement concerning their contribution can be made until there is further investigation. Furthermore, it would be useful to calculate all relevant diagrams to order α^6 as this could bring the agreement between theory and experiment even closer. Estimates to this order have recently been discussed by T. Fulton (to be published) showing that this is quite possible.

High-Resolution Study of Fluorine Metastable X-Ray Emitters*

Patrick Richard, R. L. Kauffman, F. F. Hopkins, C. W. Woods, and K. A. Jamison
Kansas State University, Manhattan, Kansas 66506
(Received 20 February 1973)

A crystal spectrometer is used to study foil-excited fluorine beam x rays. The transitions from 2^1P and 2^3P to 1^1S_0 He-like fluorine states and the transitions from ${}^2P^\circ(1)$ and ${}^4P^\circ(1)$ to $1^2S_{1/2}$ Li-like fluorine states are observed. Lifetimes of 0.537 ± 0.02 and 2.000 ± 0.025 nsec are measured and identified with the 2^3P_1 and either the $2^4P_{3/2}{}^\circ(1)$ or $2^4P_{1/2}{}^\circ(1)$ decays, respectively.

In this paper we report measurements of F beam x rays with sufficient resolution to observe separately singlet and triplet He-like states as well as doublet and quartet Li-like states. The F x rays are measured after passing through a thin C foil and the intensities recorded as a function of time of flight. Lifetimes of two metastable states are determined from these measurements. Allowed transitions are also seen far downstream from the C foil which implies the existence of long-lived cascades.

The first lifetime determination of x-ray–emitting metastable states was performed by Sellin, Donnally, and Fan[1] who, with the use of a proportional counter, measured the lifetimes of the 1^1S_0-2^3P_1 intercombination transitions in N^{+5} and O^{+6} obtained from a tandem Van de Graaff accelerator. Sellin *et al.*[2] later repeated the oxygen measurement with a crystal spectrometer and within the statistics of the experiment observed only the triplet intercombination line downstream from the foil. In the first experiment the exponential decay curves contained a long-lived component whereas in the later experiment no such component was observed. Schmieder and Marrus,[3-5] with the use of a Si(Li) detector, have measured the 1^1S_0-2^3S_1 $M1$, 1^1S_0-2^3P_2 $M2$, and the 1^1S_0-2^1S_0 double $E1$ lifetimes of He-like Ar^{+16} as well as the $1^2S_{1/2}$-$2^2S_{1/2}$ double $E1$ lifetime of He-like Ar^{+17} obtained from the heavy-ion linear accelerator at the University of California, Berkeley. Cocke, Curnutte, and Macdonald[6] have also recently measured x-ray metastable states of Cl with the use of a Si(Li) detector.