Gravitational-Wave Observations as a Tool for Testing Relativistic Gravity*

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Gravitational-wave observations can be powerful tools in the testing of relativistic theories of gravity. Future experiments should be designed to search for *six* different types of polarization, and for anomalies in the propagation speed of the waves: $|c_{\text{grav waves}} - c_{\text{em waves}}| \gtrsim 10^{-7} c_{\text{em waves}}$. This Letter outlines the nature and implications of such measurements.

Several viable gravitation theories now exist that differ radically when describing strong gravitational fields, but that can be made to be identical to each other and to general relativity in the "post-Newtonian limit." During the next twenty years, one will probably not be able to distinguish these theories from general relativity or from each other by means of "solar-system experiments" (gravitational redshift, perihelion shift, light deflection, time delay, gyroscope precession, lunar-laser ranging, gravimetry, Earth rotation, ...). However, gravitational-wave experiments offer hope: These theories differ in their predictions of (i) propagation speed and (ii) polarization properties of gravitational waves.

(i) Some of the competing theories[1-4] predict the same propagation speed for gravitational waves (c_g) as for light (c_{em}). But others[5-7] predict a difference that, in weak gravitational fields, is typically

$$(c_g - c_{em})/c \sim (1/c^2) \times |\text{Newtonian potential}|$$

$\sim 10^{-7}$, for waves traveling in our region of the Galaxy or in the field of the Virgo cluster. An experimental limit of $\lesssim 10^{-8}$ would disprove most such theories and would stringently constrain future theory building. Perhaps the most promising way to obtain such a limit is by comparing arrival times for gravitational waves and for light that come from the onset of a supernova, or from some other discrete event. If current experimental efforts continue unabated, by 1980 one may detect gravitational-wave bursts from supernovae in the Virgo cluster (~three supernovae per year, 11 Mpc from Earth). Then a limit of

$$|c_g - c_{em}|/c \lesssim 10^{-9} \times (\text{time-lag precision})/(1 \text{ week})$$

will be possible.

(ii) All of the currently viable theories fall into a class called "metric theories of gravity."[8,9] Recently, we have completed an analysis of the polarization properties of the most general weak, plane, null wave permitted by any metric theory. In general, the wave involves the metric field $g_{\mu\nu}$ and also auxiliary gravitational fields, such as the scalar field ϕ in Dicke-Brans-Jordan[2] theory. We include all these contributions by basing our analysis on the resultant Riemann tensor, the only directly measurable field. Our analysis also applies to waves that are approximately, rather than exactly, null.[7,10] Details will be published elsewhere.[11]

Our main result is that the Riemann tensor of the most general wave is composed of *six* modes of polarization, which are expressible in terms of the six "electric" components R_{i0j0} (i, j spatial) that govern driving forces in a detector.[12] Consequently, *currently feasible detectors can obtain all measurable information contained in the most general wave permitted by any metric theory of gravity.* It is important that future experiments be designed to measure all six "electric" components.

The amplitudes of the six polarization modes are related to the "electric" components R_{i0j0} in the following manner: Use coordinates $txyz$; let the wave propagate in the $+z$ direction. The six amplitudes are, in the notation of Newman and Penrose,[13] two real functions $\Psi_2(u)$, $\Phi_{22}(u)$ and the real and imaginary parts of two complex functions $\Psi_3(u)$, $\Psi_4(u)$, where $u \equiv t - z/c$ is the "retarded time." Then

$$\Psi_2 = -\tfrac{1}{6}R_{z0z0},$$

$$\Psi_3 = \tfrac{1}{2}(-R_{x0z0} + iR_{y0z0}),$$

$$\Psi_4 = R_{y0y0} - R_{x0x0} + 2iR_{x0y0},$$

$$\Phi_{22} = -(R_{x0x0} + R_{y0y0}).$$

Figure 1 shows the action of each mode on a sphere of test bodies. Ψ_4 and Φ_{22} are purely transverse, Ψ_2 is purely longitudinal, and Ψ_3 is mixed. General relativity permits only the two Ψ_4 modes.

The entire Riemann tensor of any observed wave can be reconstructed from these amplitudes.

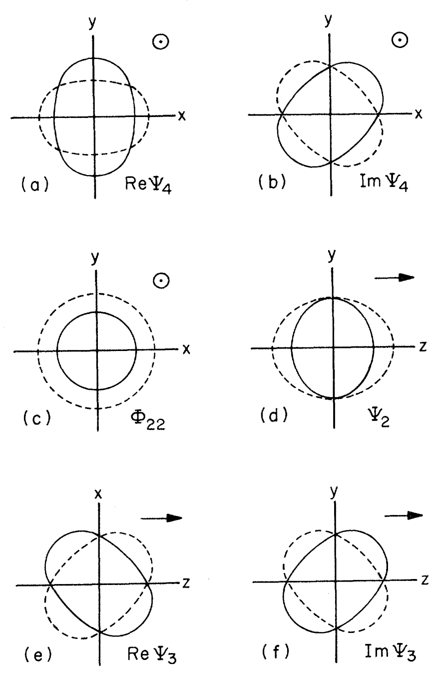

FIG. 1. The six polarization modes of a weak, plane, null gravitational wave permitted in the generic metric theory of gravity. Shown is the displacement that each mode induces on a sphere of test particles. The wave propagates in the $+z$ direction (arrow at upper right) and has time dependence $\cos(\omega t)$. Solid line, snapshot at $\omega t = 0$; the broken line, one at $\omega t = \pi$. There is no displacement perpendicular to the plane of the figure.

Comparison with waves permitted by various metric theories of gravity then allows one to rule out some theories. To facilitate this comparison, we have set up a classification scheme for waves based on the properties of the six amplitudes under certain Lorentz transformations. We choose[14] a restricted set of "standard observers" such that (a) each observer sees the wave traveling in the $+z$ direction, and (b) each observer sees the same Doppler shift, e.g., each measures the same frequency for a monochromatic wave. These standard observers are related by the subgroup of Lorentz transformations that leaves the wave vector $\vec{k}$, $\vec{k} \equiv \nabla u$, invariant ("little group"). The six amplitudes $\{\Psi_2, \Psi_3, \Psi_4, \Phi_{22}\}$ are generally observer dependent. However, there are certain "invariant" statements about them that are true for all standard observers if they are true for one. These statements characterize *invariant classes of waves*:

Class II_6: $\Psi_2 \neq 0$. All standard observers measure the same nonzero amplitude in the Ψ_2 mode. (But the presence or absence of all other modes is observer dependent.)

Class III_5: $\Psi_2 \equiv 0 \neq \Psi_3$. All standard observers measure the absence of Ψ_2 and the presence of Ψ_3. (But the presence or absence of Ψ_4 and Φ_{22} is observer dependent.)

Class N_3: $\Psi_2 \equiv 0 \equiv \Psi_3$, $\Psi_4 \neq 0 \neq \Phi_{22}$. Presence or absence of all modes is independent of observer.

Class N_2: $\Psi_2 \equiv 0 \equiv \Psi_3$, $\Psi_4 \neq 0 \equiv \Phi_{22}$. Independent of observer.

Class O_1: $\Psi_2 \equiv 0 \equiv \Psi_3$, $\Psi_4 \equiv 0 \neq \Phi_{22}$. Independent of observer. Class II_6 is the most general; as one demands that successive amplitudes vanish identically, one descends to less and less general classes. The class of the most general permitted wave in some currently viable metric theories is, for general relativity,[1] N_2; Dicke-Brans-Jordan,[2] N_3; Will-Nordtvedt,[3] III_5; Hellings-Nordtvedt,[4] N_3; Ni's new theory,[5] II_6; and Lightman-Lee,[6] II_6. All these but Dicke-Brans-Jordan theory can be adjusted to have the same post-Newtonian limit as general relativity, for certain choices of possible cosmological models and arbitrary theory parameters.

We see that measuring the polarization of gravitational waves provides a sharp experimental test of theories of gravity. The class of the "correct" theory is at least as general as that of any observed wave. The observation of a wave more general than N_2 would contradict general relativity but would be consistent with other viable theories.[2-6] Weber[15] has initiated such experiments by searching for the Φ_{22} mode, with negative results.

To test theories, an experimenter must classify the waves that he detects. If he knows the direction of a wave *a priori* (e.g., from a particular supernova), he can directly extract the amplitude of each mode from his data and determine the class. If he does not know the direction, he *cannot* extract the amplitudes or determine the direction without applying some further assumption to his data (e.g., that the wave is no more general than N_3 and is therefore purely transverse). But he can always place limitations on what the class may be (e.g., if driving forces in his detector do not remain in one plane, the wave must be more general than N_3, i.e., II_6 or III_5).

We now sketch the arguments that lead to these results. Consider a weak, plane, null wave described by a linearized Riemann tensor $R_{\alpha\beta\gamma\delta}(u)$, with $\nabla u \cdot \nabla u = 0$. Work in an approximately constant quasiorthonormal null tetrad[13] $(\vec{k}, \vec{l}, \vec{m}, \vec{m}^*)$, where $\vec{k} = \nabla u$. The Bianchi identities imply that there are six functionally independent real components of the Riemann tensor; take them to be $\{\Psi_2, \Psi_3, \Psi_4, \Phi_{22}\}$, as above. (The other components are $\Phi_{21} = \Psi_3$, $-2\Lambda = \frac{2}{3}\Phi_{11} = \Psi_3$, $\Phi_{00} = \Phi_{01} = \Phi_{02} = \Psi_0 = \Psi_1 = 0$.) Consider the "little group"[16] $E(2)$ of Lorentz transformations of the tetrad which fix $\vec{k}$: $\vec{k}' = \vec{k}$, $\vec{m}' = e^{i\varphi}(\vec{m} + \alpha\vec{k})$, $\vec{l}' = \vec{l} + \alpha^*\vec{m} + \alpha\vec{m}^* + \alpha\alpha^*\vec{k}$, where α is complex and φ is a real phase. The action of $E(2)$ on the amplitudes $\{\Psi_2, \Psi_3, \Psi_4, \Phi_{22}\}$ is

$$\begin{aligned} &\Psi_2' = \Psi_2, \quad \Psi_3' = e^{-i\varphi}(\Psi_3 + 3\alpha^*\Psi_2), \\ &\Psi_4' = e^{-2i\varphi}(\Psi_4 + 4\alpha^*\Psi_3 + 6\alpha^{*2}\Psi_2), \\ &\Phi_{22}' = \Phi_{22} + 2\alpha\Psi_3 + 2\alpha^*\Psi_3^* + 6\alpha^*\alpha\Psi_2. \end{aligned} \tag{1}$$

The invariant classes of waves that are defined above correspond precisely to the different representations of $E(2)$ that can arise through Eqs. (1).

The helicity (spin) decomposition of a wave is $E(2)$ invariant *only* for classes N_3, N_2, and O_1. Theories in classes N_3, N_2, and O_1 provide a unitary representation of $E(2)$ which is a direct sum of one-dimensional massless-particle representations,[16-18] containing at most spins $0, \pm 2$. Theories in classes II_6 and III_5 provide a reducible representation of $E(2)$ which is not completely reducible and is therefore nonunitary[18]; it is likely that such theories cannot be quantized. No other representation of $E(2)$ (such as one with "continuous spin"[18]) can occur.

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[7]In some metric theories, e.g., Refs. 5 and 6, gravitational waves follow null geodesics of a background metric $\eta_{\mu\nu}$, while photons and neutrinos follow null geodesics of a slightly differing physical metric $g_{\mu\nu}$; therefore c_g differs slightly from c_{em}. A small graviton mass also leads to a slight speed difference.

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[10]For *any* c_g there are six polarization modes, measurable through the "electric" components. Our invariant classes break down if c_g differs greatly from c_{em}, however.

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[14]Requirement (a) ensures that the various transverse and longitudinal modes are defined relative to wave direction. Requirement (b) is inessential; without it one is led to a larger little group but exactly the same invariant classes.

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