Research under Contract No. N00014-67-A-0077-0010, Technical Report No. 30, and by the National Science Foundation through Grant No. GP 27355. Additional support was received from the National Science Foundation (Grant No. GH 33637) through the Materials Science Center, Cornell University, Report No. 1918.

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[5]The measurements in Refs. 3 reveal only the relative splitting of the A transition.

[6]One should not overlook the possibility of degeneracy, given the likelihood that the v_L vary considerably with pressure.

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[10]For similar reasons we are unprepared to offer a specific model for the B transition, since this requires testing the stability of the best A_3 phase well below the normal-superfluid transition temperature. Some specific possibilities have been mentioned by P. W. Anderson and C. M. Varma (to be published).

[11]This formula and Eq. (3) have been somewhat simplified by using $\omega \gg T_c$.

Nonlinear Mode Competition in Beam-Plasma Instability*

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(Received 28 September 1972)

In a cold beam-plasma system unstable for $f > f_{ce}$, two waves are launched, one at f_0, with large growth rate, and a small test wave at f_T, with small growth rate. The wave at f_0 saturates because of beam trapping, independent of the test-wave amplitude. After the beam is trapped, the test wave ceases growing and exhibits amplitude oscillations in phase with large wave. This is consistent with a linear interaction of the test wave with the modified electron beam.

Recent work on cold beam-plasma instabilities shows the trapping of beam particles in the wave potentials to be the dominant nonlinear saturation process. Gentle and Roberson[1] observed a narrow wave spectrum at the onset of saturation which showed the amplitude oscillations characteristic of beam trapping in a single wave. More recently, Mizuna and Tanaka,[2] Bollinger *et al.*,[3] and Gentle and Roberson[4] observed changes in the beam distribution function which are in good qualitative agreement with theoretical work, particularly recent one-dimensional computer calculations for beam trapping by a single wave.[5-9]

Of interest here is the subsequent nonlinear development, which is not as clearly established. Onishchenko *et al.*[5] and O'Neil, Winfrey, and Malmberg[6] discuss the later nonlinear picture and state that eventually a broad wave spectrum will develop governed by quasilinear theory. There are several possible mechanisms to generate the broad wave spectrum, the most straightforward one being the continued growth of those frequency components which are small at the point of saturation of the principal wave.

In the present experiment we examined the behavior of a small-amplitude test wave (frequency f_T) in the presence of a large-amplitude wave (frequency f_0) in a cold beam-plasma system, $f_0, f_T > f_{ce}$. Briefly, we find that the test wave does not continue to grow after saturation, but remains small. Thus, the small components of the spectrum do not lead to a transition to quasilinear behavior in a straightforward way.

The experiments were done in a machine described elsewhere[10]; the system parameters are as follows: axial background field $B_0 = 180$ G ($f_{ce} = 0.5$ GHz), plasma frequency $f_p = 0.6$ GHz, plasma temperature ~3 eV, beam energy 350 eV, beam density $n_B \cong 2\times 10^{-3} n_p$, beam diameter 6 mm, plasma diameter 5 cm, and interaction region ~1 m (50 cm used in the experiment has $\Delta n_p / n_p < 5\%$). The waves are launched from a transmitter near the electron gun and detected by a movable axial probe and tuned receiver. In this experiment the beam density is kept small enough so that amplified noise does not grow to observable size within the machine. Under experimental conditions perturbations within a frequency band from 600 to 760 MHz are spatially amplified. Two waves are launched, E_0 at $f_0 = 710$ MHz with linear growth rate $\gamma_0 = 1$ dB/cm (maximum growth rate in the band) and E_T at $f_T = 670$ MHz with $\gamma_T = 0.5$ dB/cm.

The z dependence of the amplitude of the two waves is shown in Fig. 1 for several input powers. Curve A shows the principal wave E_0. This remains constant for all test waves shown. There is evidence for trapping after saturation although the amplitude oscillations are superimposed on a decay which is not completely understood at present. Curves B, C, and D show the axial dependence of the test wave E_T in the presence of E_0 for three different incident powers. The position of saturation and the subsequent amplitude oscillations do not change and all of these match closely the corresponding features of the principal wave. Curve E shows the z dependence of E_T when E_0 is absent. Here the saturation amplitude is much larger and occurs much later. This leads us to the conclusion that the saturation of the test wave in B, C, and D is completely determined by the nonlinear behavior of the principal wave.

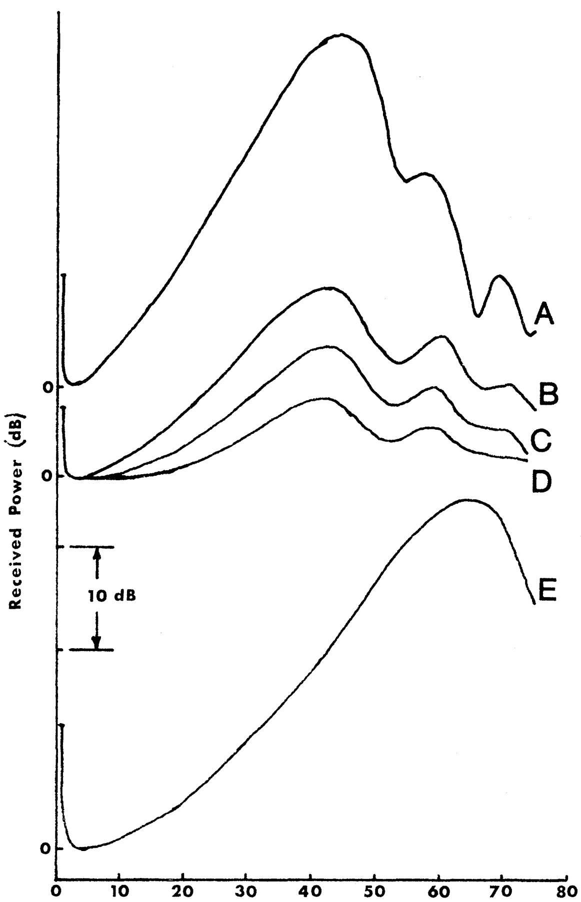

FIG. 1. Spatial dependence of wave amplitudes. Curve A, principal wave at $f_0 = 710$ MHz. Curves B, C, and D, test wave at $f_T = 670$ MHz at relative input powers 0, −5, and −10 dB, respectively. Curve E, repeat of curve B with principal wave absent.

The experiment suggests that when a single wave grows to large amplitude any other wave with small amplitude is inhibited from further linear growth. Presumably E_0 saturates as a result of single-wave beam trapping since it is unaffected by E_T. Since the test wave has small amplitude everywhere, it is reasonable to consider its behavior as linear. Using these two assumptions we present a picture which we believe qualitatively accounts for the competition process.

The two waves do not interact directly, but through a wave-particle-wave process. Upon saturation the principal wave traps the beam and the electrons oscillate in the wave potentials. E_T then interacts with the modified distribution function and the growth characteristic is locally determined by the beam at that point.

Just at saturation the electrons have undergone one half-cycle of oscillation. E_0 begins to damp and the beam is nearly monoenergetic at a velocity smaller than the wave velocity. E_T also damps since the linear interaction of a beam with a wave faster than the beam results in damping.

After the electrons have undergone one cycle, the beam is again monoenergetic, but now with its original velocity. Here the linear beam–test-wave interaction results in growth. In this way E_T follows E_0 through the changing beam distribution.

These arguments rely on the wave velocities being approximately equal, but smaller than the unperturbed beam velocity. The phase velocities v_0 and v_T were obtained in the linear region by means of interferometric wavelength measurements, with the result that $v_0 = 0.97v_B$ and $v_T = 0.98v_B$ (±1%), where v_B is the unperturbed beam velocity. Although the accuracy is poor, the wave with larger growth rate is consistently found to have the smaller phase velocity. One might thus expect that E_T should damp before E_0, since the average beam velocity falls below v_T first. This tendency is observed in the experiments in Fig. 1. Curve A is maximum at $z = 45$ cm while the corresponding maxima of B, C, and D are near $z = 42$ cm, approximately two wavelengths earlier. However, detailed analysis of effects such as these must be deferred until a quantitative theory is developed.

The competition or mixing between waves, depending on relative amplitude, has been found to be an important aspect of the nonlinear behavior of two waves. We believe that these processes may be important in the nonlinear development of many waves, for example, the suppression of "noise" in a beam-plasma system by strong beam modulation.[11] Further work, both theoretical and experimental, is obviously needed in this area.

Concerning the general decay after saturation, there are two effects not included in present single wave trapping theory. The wave above the cyclotron frequency used here has an appreciable transverse electric field and is subject to cyclotron damping, either of which could be responsible for loss of energy from the wave. The test wave has proved useful in estimating the trapped particle bounce frequency, since the test wave amplitude oscillations are less obscured by the decay. If the test wave interaction is truly linear as proposed, it may be generally useful as a beam diagnostic.

*Work supported by the National Science Foundation and the U. S. Air Force Office of Scientific Research.

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Nonlinear Stabilization of $\vec{E} \times \vec{B}$ Electron Drift Instability with $T_i \sim T_e$

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The resonant nonlinear interactions of two waves with the gyromotion of the electrons stabilize the high-frequency modes of the $\vec{E}\times\vec{B}$ electron drift instability, but further destabilize the low-frequency modes. However, the total wave energy is stabilized by this mechanism, whereas wave-wave scattering processes transfer energy from the unstable region of the spectrum toward the nonlinearly damped one. The predicted turbulence level at $K = 1/\lambda_D$ is in good agreement with experimental results.

Electron Bernstein waves are destabilized through their interaction with resonant ions if their direction of propagation in the electron frame is parallel to the current flow and if they have positive energy in this frame.[1-3] This only occurs when the wave frequency is sufficiently close to a harmonic of the electron cyclotron frequency. Because the growth rates are large compared with the ion cyclotron frequency, the ions can be assumed to have straight-line trajectories.

The main properties of the $\vec{E}\times\vec{B}$ electron drift instability in plasmas with $T_i \sim T_e$ are as follows:

(1) The real part of the linear dispersion relation has a discrete set of roots (branches). The frequency of the modes with maximum linear growth rate is given by

$$\omega_k(n) = n\Omega + \Delta\omega_k(n) \quad (n = \pm 1, \pm 2, \ldots), \tag{1a}$$

$$\Delta\omega_k(n) \cong (2\pi)^{-1/2} \frac{n\Omega^2}{|k| c_e} (1 + k^2\lambda_D{}^2)^{-1} \ll \Omega, \tag{1b}$$

where Ω, c_e, and λ_D are respectively the electron gyrofrequency, thermal velocity, and Debye length. We assume throughout that $|k| c_e/\Omega \gg 1$.

(2) The linear growth rate is maximum when