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Magnetic Braiding in a Toroidal Plasma*

Thomas H. Stix
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
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Minute deviations from symmetry in the plasma current flow pattern are sufficient to break up magnetic surfaces and braid the lines of force in a toroidal plasma under magnetic confinement. Magnetic braiding can be expected to enhance current penetration, electron heat transport, and particle loss rates; sudden onset of braiding may be accompanied by voltage spikes.

The magnetic topology in a plasma under toroidal confinement is frequently described as a system of nested magnetic surfaces. Their existence is easily demonstrated when the magnetic field possesses toroidal symmetry; an asymptotic calculation indicates their presence under a much more general set of conditions.[1] On the other hand, it is well known that lack of symmetry in the magnet windings can lead to the appearance of magnetic islands and meandering lines of force.[2,3] In this paper, we examine the breakup of magnetic surfaces by lack of toroidal symmetry in the plasma current flow pattern. It will be found that minute variation of the current density in the vicinity of the rational surfaces is sufficient to braid the magnetic lines of force and may be expected to lead to reduction of the skin effect together with enhancement of the radial transport of heat and particles. Moreover, sudden onsets of braiding—even localized in the plasma—would produce quick changes in the plasma inductance and could therefore cause voltage spikes to appear in the external circuits.

Let us start by finding what might be considered the natural modes for small-amplitude perturbations to the parallel current flow in a toroidal plasma with magnetic shear. We adopt the usual toroidal coordinate system with major radius R, minor radius r, poloidal angle θ, and toroidal angle φ. The zero-order current density $\vec{j}$ is independent of φ and lies in the magnetic surfaces which are given approximately by $r = \text{const}$. Let us now consider first-order divergence-free perturbations $j_1(r, \theta, \varphi)$ to the zero-order parallel current $j_0(r)$. On the rational surfaces, the zero-order magnetic field lines close after an integral number (n) of toroidal circumnavigations of the torus, during which an integral number (m) of poloidal circumnavigations will also have occurred. Exactly on these rational surfaces it is possible that j_1 may vary as $\cos[p(m\theta + n\varphi)]$, where p is an integer. It will be assumed that a thin layer of current with such a spatial variation does, in fact, flow on the m, n rational surface, and will be shown, in the following calculation, that a chain of magnetic islands appears in the vicinity. For simplicity, we use slab geometry to describe the immediate region around the m, n surface: $\hat{z}$ is the direction of $\vec{B}_0$, $y = r - r_{mn}$, and the spatial variation of the current flow at $y = 0$ is described by $\int j_1\, dy = \alpha + \beta\cos(kx)$. The shear of the zero-order magnetic field in the vicinity of the m, n surface is characterized by a shear length L, so that the magnetostatic vector potential in the slab approximation is

$$A_z = \frac{By^2}{2L} - \frac{2\pi\alpha|y|}{c} + \frac{2\pi\beta}{kc}\cos(kx)\exp(-|ky|). \tag{1}$$

Solution of the equation for y is simplified by approximating $|ky| \ll 1$ and also $\pi\alpha^2 kL \ll \beta Bc$. Then defining $a \equiv A_z kc/2\pi\beta$, the magnetic surfaces are described for various a values by

$$|y| \approx \mu\left(\frac{a - \cos(kx)}{2}\right)^{1/2}, \quad \mu \equiv \left(\frac{8\pi\beta L}{kcB}\right)^{1/2}, \tag{2}$$

and island structure appears for values of $|y| < \mu \times \sin(kx/2)$. We can now give finite width to the thin layer of perturbed current: Little further change occurs in the magnetic surfaces as the perturbed current flows along the new lines of force and distributes itself into the interior of each island. The variation $\langle j_1 \rangle = -(\beta/\mu)\sin(kx/2)$

will produce an equivalent sheet current showing the spatial dependence assumed above with $\alpha = -\beta$.

Returning now to the toroidal configuration, we replace kx by $m\theta + n\varphi$. B now varies as $[1+(r/R)\times\cos\theta]^{-1}$ which, for large m, only causes a slow adiabatic variation of the island thickness μ. [In fact, some resonant coupling exists, but its coefficient will be of order $(r/R)^m$.] It is helpful to express the maximum value of $\langle j_1\rangle$ as a fraction of the local zero-order parallel current, $\max\langle j_1\rangle = \epsilon j_0$, and to express the shear length in terms of the change of B_θ, $L^{-1} = (r/B)d(B_\theta/r)/dr$. Then using $q \equiv rB/RB_\theta$, we can write

$$4\pi j_0 L/Bc = 1 + 2L/qR \equiv \xi,$$

$$\mu = 2\epsilon\xi/k, \quad \beta = 2\epsilon^2 j_0\xi/k.$$

Consider ξ to be of order unity. k in these formulas actually stems from the y dependence of A_z; in the magnetostatic case it is $[(m/r)^2+(n/R)^2]^{1/2} \approx m/r$, but for time-varying fields the penetration wave number would be more appropriate.

One next asks how these "natural modes" might be excited. They may be the straightforward result of an instability—the plasma is, for instance, unstable with respect to current filamentation due to local heating with its associated increase in conductivity. However, this instability grows only on the Ohmic heating time scale and may be stabilized by classical or anomalous cross-B electron heat transport. High-m-number resistive modes[4] are additional candidates, as are any inhomogeneities in the plasma creation or recycling process. Another point of view, however, is to consider that the plasma state is one of mild turbulence accompanied by a slow equipartition of poloidal magnetic energy away from the inductive energy associated with the zero-order plasma current, W_0. With this in mind, we make a rough comparison of the magnetic energies W_{mn}, $(m,n) \neq (0,0)$:

$$W_{mn} \approx (\epsilon^4\xi^2/m^3)W_0,$$

in which $\epsilon = \epsilon(m,n)$. Proceeding in this vein, let us hypothesize that the perturbation energy is evenly divided among the m, n modes up to a maximum value of $n = N$. We further make use of the fact that $q = q(r)$ takes on the value n/m on the rational surfaces. The total number of perturbation modes is then

$$M \approx (N^2/2)[q(a) - q(0)][q(a)q(0)]^{-1}.$$

For the case $q(0)=1$, $q(a) \gg 1$, we would have $M \approx N^2/2$. Finally, if the *total* perturbation energy is equal to fW_0, we find the relative island full width is

$$2\mu(m,n)/r \approx 4(f\xi^2/mM)^{1/4} \approx 4(f/mN^2)^{1/4}.$$

Now, by eliminating duplication, the number of distinct rational surfaces is of the order of $2M/3$. Then using $M \approx N^2/2$ as cited above, we find that overlap between islands on adjacent rational surfaces will occur approximately for $f \gtrsim N^{-5}$. For example, if we choose $N = 10$, island overlap occurs when only 0.001% of the in-plasma poloidal magnetic energy is distributed among the perturbation modes!

The special significance of island overlap, pointed out in Ref. 3, is the occurrence of a Brownian motion for flux lines. It is interesting to see physically how this may come about. Let us consider the effect on the surfaces described by Eqs. (1) and (2) due to the presence of a current perturbation on an adjacent rational surface with $k' = k + \Delta k$. We solve (1) locally, i.e., for a fixed value of z. Superposition of the contributions from the two current sheets to A_z produces a term similar to the last term on the right in (1), but in which the amplitude of β exhibits a beat phenomenon with the spatial wave number Δk. μ and a in Eq. (2) will then display the same beat phenomenon, and one can make a qualitative sketch, as in Fig. 1, of the local poloidal magnetic field pattern. The first effect of mode coupling—one which occurs just as a result of super-

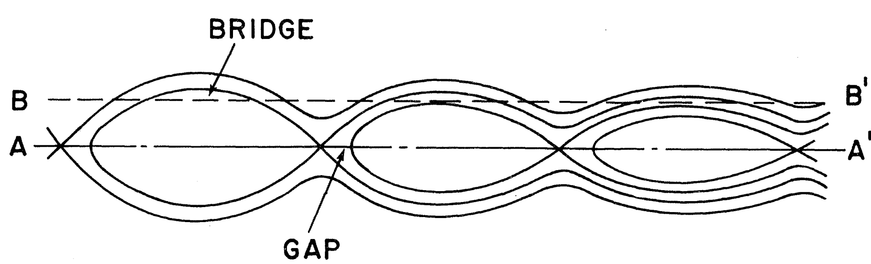

FIG. 1. Sketch of local poloidal magnetic field relative to rational surface AA'. Islands of adjacent magnetic archipelago (not shown) extend down to BB'.

position and does not require island overlap—is the production of gaps between islands in the original archipelago. (Our natural modes are not normal modes. The magnetic islands inside of which the perturbation j_1 flows are affected by superposition. From the point of view of orthogonality in this sense, it would be better to choose j_1 in the form of δ functions periodically spaced on the rational surfaces. However, the mode energy is then divergent and, in addition, the intervening magnetic structure would still be affected by superposition.) The second effect of mode coupling, which does require island overlap, is the creation of local bridges between islands of adjacent archipelagos. Moreover, it must be borne in mind that the bridging and gapping will shift, relative to the original archipelago, as one goes to different z values; it is by this means that the magnetic surfaces are destroyed and the magnetic lines of force become braided.

Having established the existence of braiding, we note immediately that it will try to destroy itself: Time evolution of the braiding will tend to destroy the local coherency of the current filaments (except for those sufficiently akin to δ functions). Tending to maintain filamentation, on the other hand, will be its own magnetic energy as well as instabilities, inhomogeneities, and equipartition. The determination of the actual level of braiding is therefore best left to the laboratory, but meanwhile it is worthwhile to consider the probable consequences of moderate to strong braiding.

First, the ordinary skin effect for current penetration will be reduced, and complete braiding would, like ideal Litz wire, bring the plasma inductance to its d.c. value. If one may, in fact, depend on the occurrence of braiding in the early stages of Ohmic heating, the engineering design of large stellarators and tokamaks will be much simplified.

Second, radial electron heat transport will be enhanced. To move one radial step length Δ, electrons would have to move along B a distance at least of order $\pi q(r)R$. Thus the electron heat-loss time will be of order $\langle (a/\Delta)^2 \pi q R (2kT_e/m_e)^{-1/2}\rangle$. Now taking into account the wide range of velocities for the electrons which comprise the hypothesized current filaments, a *minimum* value for Δ would be the same as the minimum plausible thickness for the filaments, namely of the order of an electron banana thickness.

A flat electron temperature profile would be expected to appear in any plasma region which exhibits strong local braiding. In this regard, the $q\sim 1$ core of a tokamak plasma must be especially susceptible to braiding as a result of both incipient hydromagnetic instability and low shear [note, in Eq. (2), that island thickness varies as the square root of the shear length].

The same diffusive process would also be expected to enhance the loss of fast ions, produced for example in neutral injection heating or as fusion α particles, and of runaway electrons. The large Larmor orbits of the ions will average out the very fine-scale braiding, and the appropriate ion step length Δ_i must therefore correspond in some sense to just those Fourier components of the braiding with radial wave numbers smaller than the reciprocal of the ion Larmor radius.

Third, ambipolar particle loss will be enhanced. With the fast-moving electrons performing the necessary exploration of the labyrinth, thermal ions may be pulled out along the most direct escape routes at velocities up to the acoustic velocity. Thus the confinement time by this loss mechanism alone could be as short as $\langle (a/\Delta_i)\pi q R (2kT_e/m_i)^{-1/2}\rangle$.

Fourth, onset of localized braiding may cause voltage spikes. Let us consider the sudden occurrence of strong braiding in some portion of a toroidal plasma. The new current distribution in the braided region will be uniform and if, in this region, the original current density had decreased with radius, the braiding will have produced a sudden decrease of inductance. To conserve magnetic energy, the plasma current must increase. Penetration of the electric fields to the plasma surface—this process itself facilitated, perhaps, by braiding—would in this example produce a negative voltage spike in the external circuits.

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