tributions are separately divergent.

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[10] A number of papers have recently appeared dealing with hadronic mass differences in the context of specific gauge models. Whether or not such calculations yield finite answers in lowest order does not really matter since these approaches, lacking the support of a general principle, offer no guarantee of yielding finite results in higher orders.

[11] More complicated cancelation mechanisms are at work in classes of theories more general than the one considered in the present Letter. These mechanisms have been exhibited elsewhere: R. N. Mohapatra and P. Vinciarelli, to be published.

Speculations on the Breakdown of Scaling at 10^{-15} cm*

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Hints from existing data suggest that deviations from scaling behavior should appear at momenta $|q^2| \gtrsim 20$ GeV^2. We interpret the deviations as due to structure of the hadronic constituents at 10^{-15} cm and make experimental predictions.

The observed scaling behavior for deep inelastic electron scattering suggests that hadrons may be composed of pointlike spin-$\frac{1}{2}$ constituents (partons) from which the virtual photon scatters incoherently.[1] In configuration space, one says that the virtual photon is probing the leading light-cone singularity of the current commutator.[2] The picture seems almost too good to be true, in that the onset of this presumably asymptotic phenomenon occurs for surprisingly small values of the mass ($Q^2 \equiv -q^2$) and laboratory energy (ν) of the virtual photon. Nonetheless, it is tempting to proclaim that we have glimpsed the elementary, structureless building blocks from which hadrons are constructed: that nothing remains between us and the light cone.

Here we wish to propose a less exuberant perspective on the meaning of scaling: that it represents the probing of just another layer of matter,[3] *and that hints of the structures to be discovered in the next layer are already in evidence.* Our view is old-fashioned,[4] in that it anticipates the repetition of a story which has occurred in other areas of physics.

Phenomena very similar to the scaling in electron-nucleon scattering have been observed previously in the scattering of electrons from atoms and from complex nuclei.[5] For virtual photons with $Q^2 \lesssim (300\text{ MeV})^2$ individual nucleons in the nucleus scatter coherently and the resonant level structures of the nucleus are displayed. But already at $Q^2 \cong (500\text{ MeV})^2$ the coherent excitations have essentially disappeared and the cross section is dominated by incoherent scattering from individual nucleons and by the quasielastic peak, which occurs at $Q^2 \cong 2M\nu$, with M the proton mass. This is in fact similar to the scaling seen in the electron-nucleon case, except that in the nuclear case the would-be scaling is violated by the production of pions and by the nucleon form factors which vary with Q^2. However, we wish here to concentrate on the essential similarity, which is that in both cases the virtual photon scatters incoherently from the constituents of the target. There are two salient features of the nuclear example which we wish to stress:

(a) The onset of incoherence takes place for Q^2 less than the square of the mass of the constituent, and this is perfectly understandable since the nucleus is a weakly bound system. Incoherence sets in when $Q^2 \gg 1/L^2$, where L is the internucleon spacing, $L \sim 1$ fm.

(b) The quanta (pions, ρ mesons) which bind the nucleons to form the nucleus also give the nucleon structure (form factors) which causes the simple scaling behavior to be violated (and in the nuclear case, it happens that it is violated before it can begin, since by the time Q^2 is large enough for the individual nucleons to scatter incoherently, the electromagnetic current is already probing within their structure clouds).

Unimaginatively, we suggest that a similar picture applies to the electron-nucleon case. This suggests a natural explanation of the early onset of incoherence, as in (a) above. And, as in (b), we expect that the quanta (gluons) which bind the constituents also give them sturcture. However, unlike the nuclear analogy, scaling does occur in this case, so we have evidently not yet seen the form factor of the constituent nor have the gluons been produced. These facts are accounted for by asserting that the gluon is very heavy. The mass of the gluon defines a scale of new physics, where simple scaling fails and we begin to study the structure of the constituents of the nucleon. We would still expect the constituents to have such structure even if relativistic effects are important in the nucleon.

What present hints are there for the "scale" at which this "new physics" appears? The proton's magnetic form factor $G_M(Q^2)$ falls with large Q^2 more rapidly[6] than the dipole shape $G_M{}^D(Q^2) = (1 + Q^2/0.71\ \text{GeV}^2)^{-2}$. This point has been emphasized by Massam and Zichichi[7] for a number of years. The exact nature of this falloff and the quantitative behavior of G_M for large Q^2 cannot be specified accurately or uniquely because of the limited data in the observable region $Q^2 \lesssim 25$ GeV^2. However, we extract one conclusion from the analyses: A fit to $G_M(Q^2)$ over the entire experimental range requires a large mass parameter $M_G \sim 10$ GeV to be introduced. A simple modification of the dipole formula which fits the data is

$$G_M(Q^2) \cong (1 + Q^2/0.71\ \text{GeV}^2)^{-2}(1 - Q^2/M_G{}^2). \quad (1)$$

It is possible (see below) that the second factor is an approximation to the familiar resonant form $(1 + Q^2/M_G{}^2)^{-1} \approx 1 - Q^2/M_G{}^2$ for $Q^2/M_G{}^2 \ll 1$. The more detailed analysis of Massam and Zichichi[7] based on vector dominance theory with ω, ρ, φ mixing gives $M_G = 7.7 \pm 1.1$ GeV. The appearance of a large mass $M_G \sim 10$ GeV suggests the possibility of a new large mass scale, or short distance scale, $1/M_G \sim 2 \times 10^{-15}$ cm, on which qualitatively new phenomena will occur.

In bound-state models of the proton the dipole behavior of the form factor G_M emerges naturally if the constituents are pointlike.[8] The large mass scale appearing in the correction in (1) is then associated with a small but finite size, $\sim 1/M_G$, for the constituents themselves. Using a solvable relativistic bound-state model we have verified[9] that it is possible to factor $G_M(Q^2)$ into two terms as in (1). That is, we have shown that one can write, in the kinematic region $M^2 \ll Q^2 \ll M_G{}^2$,

$$G_M(Q^2) \cong G_M{}^D(Q^2) F_C(Q^2), \quad (2)$$

where $G_M{}^D(Q^2) \sim 1/Q^4$ asymptotically is the dipole form for pointlike constituents, and, for $Q^2/M_G{}^2 \ll 1$,

$$F_C(Q^2) \cong (1 - Q^2/M_G{}^2) \quad (3)$$

is the structure function of the individual constituents, due to their gluon clouds.[10] Notice that this factorization (2) is precisely what happens in the nuclear analogy. The details of this analysis will be presented elsewhere in a fuller description of this work.[11]

The second hint as to the scale for "new physics" comes from deep inelastic electron-photon scattering data. The experimental separation of the structure functions W_1 and W_2 has actually been performed for just a small number of data[12] points in the interval $1.5 < Q^2 < 10$ GeV^2. If we wish to use these points to test scaling, we are restricted too small values of $\omega \equiv 2P \cdot q/Q^2 \approx 2$ and we find that $\nu W_2(\omega, Q^2)$ seems to decrease as Q^2 increases from 1.5 GeV^2 to 10 GeV^2. If we attribute this effect to a finite constituent size, the correction multiplies the usual scaling function

$$\nu W_2(\omega, Q^2) \cong \mathfrak{F}_2(\omega)(1 - 2Q^2/M_G{}^2), \quad (4)$$

and a crude analysis of the data yields $M_G{}^2 \sim 100$

± 50 GeV^2, as in (1). We have also verified the factorization property of (4) in a relativistic bound-state model.[9]

The trend in the data for $\omega \cong 2$ which we have interpreted by Eq. (4) has been given an alternative explanation by Bloom and Gilman,[13] who account for it by proposing that the scaling variable is $\omega' \equiv \omega + M^2/Q^2$. Since $d\mathfrak{F}_2(\omega)/d\omega > 0$ at $\omega \cong 2$, their proposal also accounts qualitatively for the observed decrease of $\nu W_2(\omega \approx 2, Q^2)$ as Q^2 increases. To decide between their interpretation and ours, it will be sufficient to have accurate $W_1 - \nu W_2$ separated data for $\omega > 4$ (where $dF_2/d\omega \sim 0$, so that according to Bloom and Gilman the effect should disappear) and/or for larger Q^2 values (where according to Bloom and Gilman the effect diminishes, while according to our hypothesis it becomes more pronounced).

It is then our conjecture that the deviations from dipole behavior and scaling are measurements of the form factor of the nucleon's constituents. If our interpretation of the already available data is correct, then two experimental consequences follow *immediately*. First, as accurate data for the individual structure functions W_1 and νW_2 become available for deep inelastic lepton-nucleon scattering at larger Q^2 values, $Q^2 \approx 20$ GeV^2, deviations from scaling should become quite apparent according to (4).

Second, the total cross section for electron-positron annihilation to hadrons (in the one-photon approximation) should have the form

$$\sigma(e^+ + e^- \rightarrow \gamma \rightarrow \text{hadrons}) \propto q^{-2}(1 + 2q^2/M_G^2) \qquad (5)$$

in the kinematic region $M^2 \ll q^2 \ll M_G^2$. Equation (5) shows the most striking consequence of our speculations: The form factor of the constituent has a positive slope and therefore, to leading order for small $0 < q^2/M_G^2 \ll 1$, *the total annihilation cross section should increase relative to its pointlike behavior* $\propto 1/q^2$. $|F_C(q^2)|^2$ appears in (5) as a modification of pointlike behavior because the emerging pair of constituents produced by the electromagnetic current interact via massive gluon exchange. This interaction has a very short time scale $\tau_I \sim 1/M_G \ll 1/\sqrt{q^2}$. Therefore the produced constituents, which as a result of their final-state interactions become the observed final hadrons, do not behave as if free and pointlike. The presence of a q^2-independent component in (5) should be tested soon by colliding-beam experiments now in progress or being prepared.

The behavior of the constituent form factor at $q^2 \sim M_G^2$ is, strictly speaking, outside the framework of the hypothesis which we have so far advanced. Stated most conservatively, our speculations for $M^2 \ll |q^2| \ll M_G^2$ require only that the nucleon constituents have an electromagnetic structure on the scale of $M_G^{-1} \sim 0.02$ fm, and there need not even be a particle with mass M_G corresponding to the gluon. However, following Yukawa, we do expect that some such particle should exist, and if we suppose that this particle is a vector meson with the quantum numbers of the photon [or, alternatively, a unitary singlet coupled to the photon via SU(3) breaking] then we might actually see it as a resonance in $F_C(q^2)$ at $q^2 \sim M_G^2$. On the other hand, if there is no gluon, or if there is a gluon which does not couple to the photon (e.g., a scalar gluon) then the behavior of $F_C(q^2)$ for $q^2 \sim M_G^2$ is a dynamical question about which we cannot even begin to speculate.

In a completely analogous way the scaling law predicted[14] for the process $p + p \rightarrow \mu^+ + \mu^- +$ hadrons on the basis of a pointlike constituent model is modified by the identical factor as in (5):

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{3q^4}\mathfrak{F}\left(\frac{q^2}{s}\right)\left(1 + 2\frac{q^2}{M_G^2}\right), \qquad (6)$$

where q^2 is the square of the dimuon mass, s is the square of the total collision energy, and the ratio q^2/s is finite.

These ideas of constituent structure and massive gluons have further experimental implications to which we now turn. The deep inelastic neutrino cross sections now being explored at the National Accelerator Laboratory should exhibit corrections to scaling due to constituent size, in analogy to the deep inelastic electron scattering. Moreover, there is an important new feature that will be probed in these experiments: the behavior of the cross section and structure functions above the conjectured threshold for gluon production. Since energy transfers ν as large as several hundred GeV can now be achieved in the lab, hadronic final states of masses $M_f^2 \simeq 2M\nu(1 - Q^2/2M\nu) \gtrsim (20\text{ GeV})^2$ will be produced. Therefore, we can cross the threshold for producing real gluons of mass $M_G \sim 10$ GeV, if indeed they do exist. The salient qualitative feature is the observation of a nonscaling bump when we cross the production threshold. Furthermore, this threshold for the production of the postulated fluons will reveal whether they are indeed heavy or, instead, weakly coupled light objects as described in Ref. 10.

Massive gluons, if they exist, will also be produced in purely hadronic interactions. The natural suggestion is to associate the recently observed rise[15] in total hadron-hadron cross sections at the CERN intersecting storage rings in the energy interval $s \sim (30\ \mathrm{GeV})^2-(55\ \mathrm{GeV})^2$ with the opening of new inelastic gluon production channels. Similarly the long tail[16] in the large transverse momentum distributions for the inclusive particle distribution, which rises sharply above the extrapolated slope $\exp(-bp_\perp)$ for $p_\perp > 2.5$ GeV, suggests the possibility of production and eventual decay of a massive particle. Beyond these crude observations a more detailed hadron dynamics in the framework of the parton model (and with lighter gluon masses, ~1–2 GeV) is given in the recent paper by Casher, Nussinov, and Susskind,[17] whose preprint arrived after we completed the work we are here reporting.

Finally we comment that our interpretation of constituent structures and deviations from scaling implies modifications of the usual view of the light-cone singularities of current products. Independent of the ultimate behavior of such products for $x_\mu x^\mu \to 0$, it is clear that in the presently probed region, $1/Q^2 \approx x_\mu x^\mu \gg 1/M_G{}^2$, we are not yet asymptotic. It is natural then to find non-scale-invariant corrections to the light-cone algebra in this region.[18] As an example of the simplest behavior, consider the modification

$$[J^\mu(x), J^\nu(0)] \cong_{M_G{}^{-2} \ll x^2 \ll M^{-2}} \left(1 - 2\frac{\Box}{M_G{}^2}\right)\left\{\left(\partial_\rho \frac{\epsilon(x^0)\delta(x^2)}{2\pi}\right)[\bar{\psi}(x)\gamma^\mu\gamma^\rho\gamma^\nu\psi(0) - \bar{\psi}(0)\gamma^\nu\gamma^\rho\gamma^\mu\psi(x)]\right\}, \tag{7}$$

where the factor in curly brackets is the usual light-cone commutator,[19] with ψ the triplet of quark fields and Q the quark charge matrix. Evaluating (7) between nucleon states and taking its Fourier transform, we recover the broken scaling behavior of (4). We shall discuss elsewhere[11] the consequences of our hypothesis for current algebra near the light cone and at equal times.

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