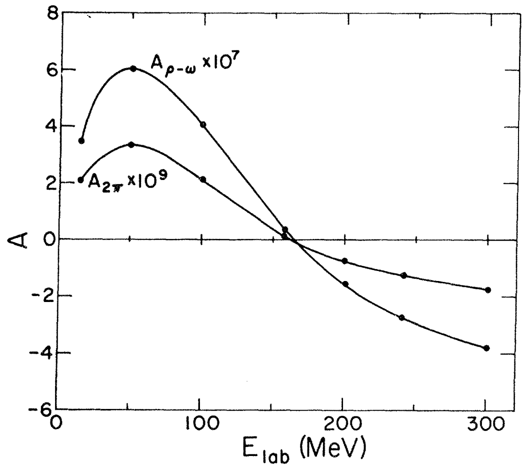

FIG. 2. The asymmetry A as a function of laboratory energy. The points correspond to the calculated energies. The $\rho-\omega$ and 2π contributions are shown separately. The 2π curve shown should be multiplied by a factor of -4.75 ($+4.75$) for p-p (n-n) scattering.

in n-p scattering, which we are presently investigating. Details of this work and extensions will be published elsewhere. Regardless of the validity of the detailed form of $V_{\rm PNC}$, we believe that the order of magnitude and energy dependence of A is correct; an experimental search for the asymmetry is thus highly desirable.

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New Technique for Vertex Graphs

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We introduce a new technique which we use to analytically compute six graphs contributing to the magnetic moment of the electron in sixth order.

We have developed a new technique for evaluating the contribution of certain Feynman graphs to vertex functions. In particular, we have analytically calculated 6 of the 28 different graphs without fermion loops, which contribute to the anomalous moment of the electron in sixth order. The graphs are illustrated in Fig. 1 and our results are indicated in Table I. We used the Feynman gauge throughout.

The idea for the calculation stems from the observation that the form factors are scalar functions of a single four-vector q, and thus do not depend on the orientation of q. Since no angle is intrinsically defined by the problem, it might be possible to do the four-dimensional angular integrals for each Feynman graph analytically. There remain only l "radial" integrals, where l is the number of independent loop momenta.

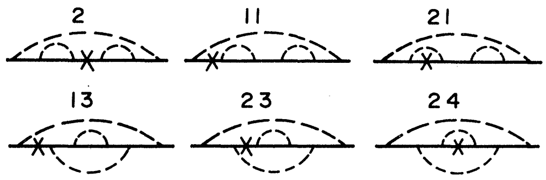

FIG. 1. Six graphs which we calculated, numbered according to the scheme of Ref. 1. Crosses denote the external vertices.

This dimensionality is lower than that obtained by introducing Feynman parameters, and so this technique could prove very useful in evaluating the resulting integrals numerically. As we shall see, it also allows us to evaluate some graphs completely analytically.

We proceed with a brief discussion of the technique. Details will be published later.[2] For clarity, we make certain hypotheses whose justification is presented in the next paragraph.

(i) Suppose we can perform a Wick rotation of the Feynman integrand in momentum space so that the momenta become Euclidean. (ii) Suppose also that we can route the loop momenta so that the momentum of each propagator depends on at most two of the independent momenta (including both loop and external momenta). Then the propagators are of the form[3]

$$1/(\underline{K}^2+m^2) \text{ or } 1/[(\underline{K}-\underline{L})^2+m^2], \tag{1}$$

where m is the mass of the line. Recognizing $(1+z^2-2z\cos\theta)^{-1}$ as the generating function for the spherical harmonics on the sphere in four dimensions, we have

$$[(\underline{K}-\underline{L})^2+m^2]^{-1}=(Z/KL)\sum_{n=0}^{\infty}Z^nC_n(\underline{\hat{K}}\cdot\underline{\hat{L}}), \tag{2}$$

where

$$Z=\{K^2+L^2+m^2-[(K^2+L^2+m^2)^2-4K^2L^2]^{1/2}\}\times(2KL)^{-1}, \tag{3}$$

$\underline{\hat{K}}$ is a unit vector along $\underline{K}$, and C_n is a Gegenbauer polynomial. (iii) By using the addition theorems and orthogonality relations for the C_n, it may be possible to evaluate all the angular integrals and perform the infinite sums over the Z's.[4] We turn now to the justification of remarks (i), (ii), and (iii).

(i) It is legitimate to perform the Wick rotation (recall that we do not introduce Feynman parameters) provided that the external momenta are spacelike. To obtain the magnetic moment, it

TABLE I. Analytic value for the contribution of each graph to $\frac{1}{2}(g-2)$. λ stands for the photon mass. The expressions independent of λ are evaluated and compared with the numerical estimates obtained in Ref. 7.

Graph	Result $\times(\frac{\alpha}{\pi})^3$	Value	Numerical Value Ref. (7)
2	$169/576 - 143\pi^2/432 - \zeta(3)/3$ $+ (11/48 - \pi^2/36)\log\lambda^2 + \log^2\lambda^2/8$	-3.3743	-3.374 ± .003
11	$51/64 - 19\pi^2/96 - \zeta(3)/16$ $+ (11/48 - \pi^2/36)\log\lambda^2 + \log^2\lambda^2/8$	-1.2316	-1.232 ± .002
13	$33/64 + \pi^2/16$ $+ (35/96 - \pi^2/72)\log\lambda^2 + \log^2\lambda^2/16$	1.1325	1.135 ± .005
21	$95/288 + 139\pi^2/432 - 17\zeta(3)/24$ $+ (\pi^2/24)\log\lambda^2 - \log^2\lambda^2/16$	2.6540	2.654 ± .003
23	$-595/864 + 11\pi^2/648 - 7\zeta(3)/36$ $+ (11/48 + \pi^2/72)\log\lambda^2 + \log^2\lambda^2/16$	-0.7549	- .756 ± .003
24	$733/1728 + 59\pi^2/648 + 7\zeta(3)/18$	1.7903	1.789 ± .003
Total (2) + (24) + 2 x [(11) + (13) + (21) + (23)]	$21/8 + \pi^2/6 - 15\zeta(3)/8$ $+ 15\log\lambda^2/8 + \log^2\lambda^2/2$	2.016	2.017 ±.014

is necessary to then continue the electron momentum p to the mass shell. This can be done by deforming the radial integration contours into the complex plane, and setting $p^2 = m^2$ in the integrand.[2]

From now on, we confine our discussion to form factors, or their derivatives, at $q^2 = 0$. This allows us to set $q = 0$ in the propagators, or their derivatives, and the topological structure of the graph is the same as that of the self-energy graph obtained by deleting the external vertex of the vertex graph. We now join the two external electron lines to form a bounded graph. We refer to this graph as the associated self-energy graph (ASEG). See Fig. 2.

(ii) If the ASEG is planar, the loop momenta can be chosen so that no propagator depends on more than two momenta.[5] This is easily seen by associating a circulating loop momentum with each region of the ASEG (the external momentum p is associated with the region bounded by the joined external lines). Then each propagator is associated only with the two loop momenta of the regions it divides.

(iii) If the ASEG, considered as a map, can be colored with only two colors, the angular integrals can be done, and the infinite sum over the Z's collapses to a finite sum.[2]

The graphs we calculated all have two-color (ASEG's). In these graphs, it is always possible to choose the routing of the loop momenta so that all the fermion propagators depend only on a single momentum, while the photon propagators depend on two momenta. But since the photon mass vanishes, the expression for Z in (3) reduces to

$$Z = \min(K/L, L/K).$$

So in these graphs, after performing the angular integrals, the integrand is rational in the radial variables. It is then possible to perform all the integrals in closed form.[6]

The calculations were done using a symbolic algebra program called ASHMEDAI,[7] written by one of us (M.L.). The Dirac algebra was done in the usual fashion. The integrals were performed by a series of substitution commands, in which the integrated forms replaced the integrands.

We feel that these techniques are extremely powerful for such calculations, since they lead to much simpler calculations than the Feynman parameter approach. Work is now continuing on the three-color graphs.

Finally, it should be emphasized that our analytic values fall well within the range obtained numerically for each graph by Levine and Wright.[1] The only other available numerical value for single graphs is that of de Rújula, Lautrup, and Peterman[8] for graph 2. Their result (-3.332 ± 0.011) disagrees with ours.

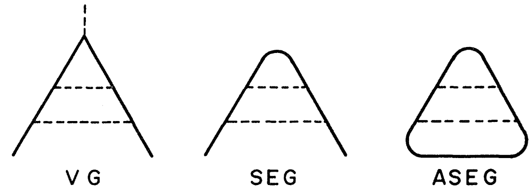

FIG. 2. Examples of a vertex graph (VG), a self-energy graph (SEG) obtained by deleting the external vertex, and what we call the associated self-energy graph (ASEG).

It is not possible to compare our numerical values with those of Kinoshita and Cvitanovic,[9] since they first subtract ultraviolet and infrared counterterms from each graph. These contributions cancel when summed over all graphs, but they change the value of individual graphs by amounts which have not been calculated.

It is a pleasure to acknowledge fruitful conversations with K. Johnson and J. Wright.

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