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Phase-Space Evolution of a Trapped Electron Beam

K. W. Gentle and J. Lohr
University of Texas at Austin, Austin, Texas 78712
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The nonlinear development of the instability excited by a cold, weak electron beam injected into a one-dimensional plasma is examined with a fast electron velocity analyzer. The distribution of the electrons in phase space is observed as the instability grows, traps the beam, and oscillates. The observed distributions mirror those first seen in computer simulations which lead to the trapping hypothesis.

One of the first problems examined by computer simulation of plasmas was that of the nonlinear saturation of the beam-plasma instability.[1] The analytical model by Drummond *et al.*,[2] and the subsequent developments by O'Neil, Winfrey, and Malmberg[3] and Thompson,[4] have led to a simple physical picture of the nonlinear development supported by a careful and complete quantitative theory. The theory for the limit of a cold, weak beam in one dimension is well developed.

The linearly most unstable wave grows rapidly with a phase velocity only slightly less than the beam velocity. Since the spectrum of unstable modes is very narrow, it appears as if only a single wave is growing, and it continues to grow until its amplitude is sufficient to trap the beam. The trapping is most obvious in the electron phase-space distribution as the beam breaks up into sections 1 wavelength long, which bunch and circulate in the potential wells. As the electrons are trapped, they slow down and actually reverse direction in the wave frame. In this state, they have lost the maximum amount of energy, and hence the wave energy is at maximum. As they continue to circulate in the wave potential well, they begin to gain energy again, and the wave amplitude decreases. The wave amplitude continues to oscillate slowly as energy is exchanged between beam electrons and the wave, until other processes broaden the spectrum and spread the electrons out in phase space.

Experimental evidence for this picture has been presented by Gentle and Roberson.[5] Amplitude oscillations of the predicted form and field strength were found. Additional evidence of trapping was reported by van Wakeren and Hopman,[6] and most recently Mizuno and Takana[7] observed time-average electron distribution functions of the form expected for trapped electrons. Although their experiment does not correspond exactly to the natural instability because the wave was externally excited, the physical trapping process for the beam should be the same. The most direct evidence for the trapping process, however, is an actual measurement of the electron distribution in phase space equivalent to the computer outputs. In this Letter, we report such measurements.

The basic apparatus has been previously described in detail.[8] Plasma from a coaxial source diffuses down a magnetic field to form a homogeneous plasma column. The center conductor of the source is hollow to permit injection of an electron beam on the axis. The beam-plasma column is terminated with a fast velocity analyzer which can be moved to sample the electron distribution after varying distances of interaction. The background plasma is quiet and collisionless with densities near $10^9/\text{cm}^3$ and electron temperatures of 20 eV. The magnetic field of a kilogauss is sufficient to render the dynamics one-dimensional.

The velocity analyzer is a conventional gridded type, but with elements spaced at millimeter intervals to minimize transit time. The plasma terminates on the grounded front grid. A second grid is positively biased to repel ions, and the third grid determines the minimum energy that electrons must have to reach the collector plate behind. The data are taken with the technique described by Chang, Nix, and Swain,[9] in which the collector is connected directly to a sampling scope preamplifier, all matched at 50 Ω. The sampling unit is triggered by the high-frequency electric field of the instability as detected with a small probe placed in the plasma near the analyzer. With the sampling unit in manual mode, the vertical output indicates the collector current at a chosen time (phase) relative to the trigger signal. By sweeping the discriminator grid voltage linearly on a time scale of seconds and plotting the derivative of the sampler output as a function of voltage, a plot of the electron distribution at the chosen time may be obtained. A series of such plots at successive times in the wave form gives a complete picture of electron distribution in phase space. Moving the analyzer back allows observation of the distribution at later stages in the evolution of the instability after it begins from noise near the source.

The computer results for phase-space distribution are shown in Fig. 1, taken from Ref. 3. It shows the electron distribution in velocity as a function of phase in the wave. The parameter τ is a dimensionless measure of the stage of development of the instability. In an experiment like the present one in which the instability grows in

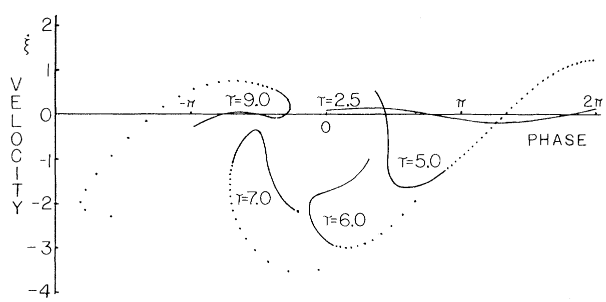

FIG. 1. Computer solution for the electron distribution in phase space. Each line shows the electron velocity as a function of phase (time) in the high-frequency instability. Larger τ's show the distribution at later stages of development of the instability.

space from the point of injection, τ is proportional to the distance along the column at which the phase-space distribution is measured. It is obvious that there is a breaking up of the beam and a clumping of the distribution at lower energy as the instability reaches its maximum at $\tau=6.5$. The return of the beam toward its initial velocity is also clear at $\tau=9$.

Quantitatively, $\tau \cong k_0(\eta/2)^{1/3}$ for this case, where k_0 is the wave number of the most unstable mode and η is the ratio of beam to plasma density, the small expansion parameter of the theory. The dimensionless velocity ξ scales to the real velocity relative to the beam velocity u as $v=(\eta/2)^{1/3}\xi u$.

The equivalent experimental measurements are shown in Fig. 2. The results are scaled to the same dimensionless parameters used in the theory, but the phase reference is arbitrary for each distribution. At small τ, the beam remains continuous with only adiabatic rippling. With the onset of trapping, the distribution breaks up and begins to rotate downward, just as in the simulation. Near $\tau=6$, the distribution is strongly bunched in phase and lower in energy. At later stages, the distribution begins to rotate upward in the well, returning to nearly the original beam velocity, although more smeared out.

Most of these measurements were taken for a 300-V beam of 1 to 2 mA current. The small expansion parameter of the theory had a value $\eta^{1/3} \sim 0.1$. The frequency of the instability was approximately 100 MHz, and the contours were constructed from distributions taken at 1-nsec intervals, giving good resolution of the phase-space distribution. Similar results were obtained over a range of beam energies, beam currents, and plasma densities.

Results for early τ were obtained with the analyzer close to the source, and later τ were seen by moving the analyzer back. A fraction of the wave is reflected by the analyzer, and a small fraction of this reflected wave again reflects at the source end to give some feedback. Therefore, the instability may grow from a level somewhat above the background noise. This feedback is significant only when the analyzer terminates the column near the peak of the wave amplitude. Even then, it affects only the initial level from which the instability develops, which is always much smaller than the maximum wave amplitude. Movement of the analyzer did not affect either the background plasma density or the frequency of the instability. Because the initial level

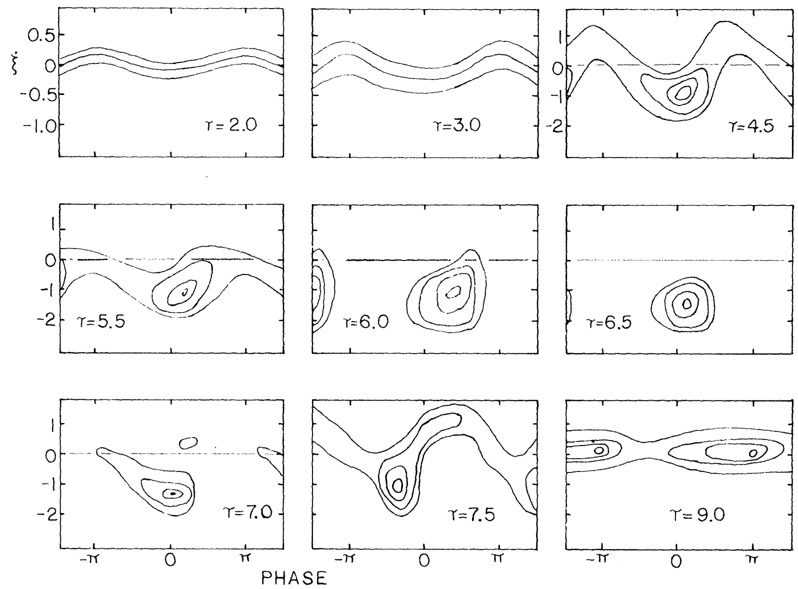

FIG. 2. Experimental contour plots of electron phase-space distribution. For $\tau=2,3$, the central line shows the contour of maximum electron density, with upper and lower curves indicating half-maximum contours. The width is near the resolution limit of the analyzer. At later τ, the inner contour is the maximum density, normalized to 1.0, and contours are plotted at 0.8, 0.5, and 0.2.

changed somewhat with feedback, the τ values were not linearly proportional to the length of the plasma column. The τ values were assigned by observing the time-averaged $f(v)$ and the pattern of $E^2(x)$ up to the analyzer at each position. These could be compared with earlier experiments on the machine which traced the evolution over all τ to determine the appropriate τ values.

The phase-space distributions give the clearest and most direct evidence of trapping as the limiting process in the beam-plasma instability. They show the trapping of the beam and the subsequent rotation vortices associated with the oscillation in wave power.

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