764 (1972); P. Belland *et al.*, Appl. Phys. Lett. 21, 32 (1972); K. Büchl *et al.*, in *Laser Interaction and Related Plasma Phenomena,* edited by H. J. Schwarz and H. Hora (Plenum, New York, 1971), p. 503; K. Eidmann and R. Sigel, Max-Planck-Institut für Plasmaphysik, Garching, Report No. IPP IV/46, 1972 (unpublished).

Josephson Effects at High Current Density*

H. A. Notarys, M. L. Yu,† and J. E. Mercereau
California Institute of Technology, Pasadena, California 91109
(Received 26 February 1973)

Effects of large phase gradients on the Josephson relation have been estimated by a modification of the Aslamazov-Larkin theory. The results are applied to explain the Dayem-Wyatt effect.

In some circumstances current density associated with the Josephson phenomena can be quite large. When this occurs, effects due to the spatial variation of the quantum phase may become important. Ordinarily, for insulating tunnel barriers, such effects are negligibly small because of the relatively low current density associated with this type of barrier. However, these effects could become significant in junctions in which the dimension parallel to the current flow approximates the de Broglie wavelength of the current. This Letter outlines an analysis of that situation based on a modification of the Aslamazov-Larkin theory.[1] The results of the analysis are used to describe the large Dayem-Wyatt effect[2] in proximity-effect bridges.[3]

In the proximity-effect bridges, the cross section is typically about 10^{-8} cm^2 and thus, for a given current, the current density is about 10^5 times that in the usual superconducting-normal-superconducting (SNS) tunnel junction.[4] Experimental results on the variation of critical current with temperature for a typical short tin-gold proximity-effect bridge are given as the solid line in Fig. 1. The current is exponential in temperature as predicted[5] for SNS junctions, but varies with temperature somewhat more rapidly than anticipated. Application of rf radiation in general enhances the critical current of these bridges. Figure 1 also shows the maximum enhanced supercurrent for this bridge when irradiated by 2-GHz radiation.

The usual Josephson analysis considers current flow through a junction region separating two superconductors of constant phase, φ_1 and φ_2. Aslamazov and Larkin (AL) showed that at low current density Josephson's result also followed from a model which treated the junction as a two-phase region in which the amplitude of the separate phases was strongly position dependent.

They assumed that in the region of a junction separating two similar superconductors 1 and 2, the wave function was composed of two terms,

$$\Psi = \psi_1 + \psi_2 = \psi_0\{f(x)\exp(i\varphi_1) + [1 - f(x)]\exp(i\varphi_2)\},$$

where $f(x)$ is a function which rapidly goes to unity in superconductor 1 and zero in superconductor 2, and ψ_0 is the undisturbed amplitude far from the junction. Thus the first part of the wave function above is phase coherent with superconductor 1 and the second part is phase coherent with superconductor 2. If current is defined in terms of the usual gradient operator, AL showed that the current resulting from this wave function

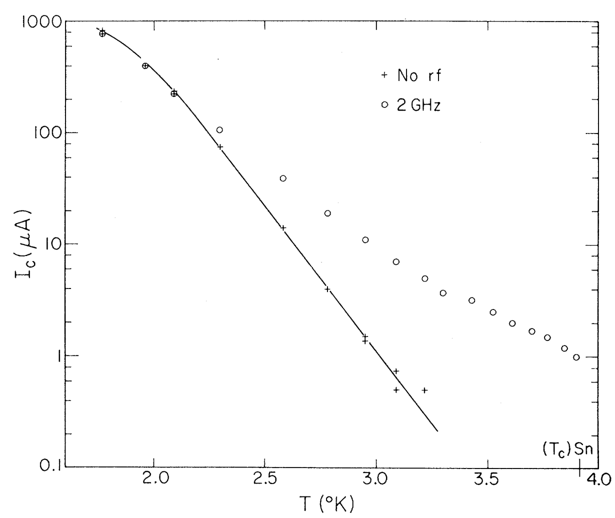

FIG. 1. Temperature dependence of the critical current of a Sn/Au proximity-effect bridge. Crosses, data points without external rf; circles, those for maximum enhancement by a 2-GHz rf field. T_c(Sn) is the transition temperature of the tin film. Bridge dimensions: length, 1 μm; width, 39 μm; thickness, 0.1 μm and resistance, 10 mΩ.

is $j_s \propto \psi_0^2 f \nabla f \sin(\varphi_1 - \varphi_2)$.

We have attempted to include high-current-density effects in this model by allowing the phase to be a function of position as well, $\varphi = \varphi(x)$, and then calculating current in a similar manner to that of AL. The result is

$$j_s \propto \psi_0^2 f \nabla f \sin(\varphi_1 - \varphi_2) + \psi_0^2 f^2 (\nabla\varphi_1 + \nabla\varphi_2)[1 + \cos(\varphi_1 - \varphi_2)]. \quad (1)$$

In a real junction, such as SNS, we interpret ∇f as being primarily determined by the spatial variation of the wave function as it decays into the normal (N) region, while $\nabla\varphi$ is determined primarily by the current source. If $\nabla\varphi_1$ and $\nabla\varphi_2$ are correlated, the second term in Eq. (1) also represents a phase-dependent supercurrent. In this model, phase coupling results through the overlap of the two induced wave functions ψ_1 and ψ_2. Since the superconducting pairs carry the same linear momentum independent of their origin in ψ_1 or ψ_2, a momentum-conserving coupling condition is $\nabla\varphi_1 = \nabla\varphi_2 = \nabla\varphi$, and hence the supercurrent j_s in this phase-coupled condition is

$$j_s \propto \psi_0^2 f \nabla f \sin(\varphi_1 - \varphi_2) + \psi_0^2 f^2 \nabla\varphi [1 + \cos(\varphi_1 - \varphi_2)]. \quad (2)$$

For this coupling scheme the time evolution of the phase difference $\varphi_1 - \varphi_2$ is independent of position and is governed by the potential difference between the two superconductors, $\varphi_1 - \varphi_2 = (2e/\hbar)\int V\,dt + \beta$.

Equation (2) is a strong function of position within the two-phase region. However, neglecting magnetic field effects we assume that the salient features of the phase dependence of the current in a small junction can be expressed by an average taken over the junction region:

$$j_s = j_J \sin(\varphi_1 - \varphi_2) + B\nabla\varphi [1 + \cos(\varphi_1 - \varphi_2)]. \quad (3)$$

In this approximation, f and ∇f are assumed always to be time-independent parameters, and thus J_J and B are temperature-dependent average coefficients for the first and second terms. The phase gradient remains as a variable to be determined by the current or voltage source.

The phase-gradient term is proportional to the reciprocal of the de Broglie wavelength, and thus the second term of Eq. (3) is the two-phase equivalent of the expression for current which would occur in a single-phase system of constant amplitude. With this modification, the current depends both on the sine and the cosine of the phase difference as well as the phase gradient. However, the cosine dependence in this case does not have the same physical origin as the similar cosine dependence in tunnel junctions.[6] In tunnel junctions, current with a cosine dependence on the relative phase occurs only at finite voltage because of quasiparticle interference effects. Here the cosine term exists even at zero voltage and reflects the dependence of the amplitude of the wave function Ψ on the relative phase of the two components ψ_1 and ψ_2.

In this model the current depends on phase both in terms of the usual phase difference $(\varphi_1 - \varphi_2)$, and also as the phase gradient $\nabla\varphi$. Below the critical current the response of the supercurrent to a voltage source can be approximated from Eq. (3) by relating the phase gradient to the electric field. Since the electric field is defined as $E = -\nabla\mu/e$ and $\dot\varphi = -2\mu/\hbar$, the B term in Eq. (3) can be written as

$$j_B \propto \left(\frac{2e}{\hbar}\int E\,dt + \alpha\right)\left[1 + \cos\left(\frac{2e}{\hbar}\int V\,dt + \beta\right)\right].$$

In this case the amplitude of the B term is time dependent directly, as well as through the time dependence of the relative phase. If the applied voltage is $v = v_0 \cos\omega t$ and we assume a linear relationship between E and V, this twofold time dependence results in a complex frequency response of the supercurrent within which is a "rectified" component proportional to $\sin^2\omega t$ of magnitude

$$j_B \propto (2ev_0/\hbar\omega)J_1(2ev_0/\hbar\omega)\sin\beta. \quad (4)$$

This rectified current represents a dc supercurrent, induced from an ac source, whose maximum amplitude (when $\beta = \frac{1}{2}\pi$) is periodic in v_0. It should be noted that such "rectification" would not take place without the phase-gradient term in Eq. (1) and will not occur in the low-current approximation. We suggest that this current may be responsible for the Dayem-Wyatt effect.

In short proximity-effect bridges, the Dayem-Wyatt effect is particularly large[3] and supercurrent can apparently be "created" at temperatures considerably above the "transition temperature" of the bridge by the application of radio-frequency voltages. Figure 1 shows the maximum critical current as a function of temperature for a typical proximity-effect bridge with and without the application of 2-GHz radiation. It can be seen that there is a strong enhancement of critical current over a wide range of temperature. Without radiation the apparent transition temperature was 3.2°K, which was increased to 3.9°K

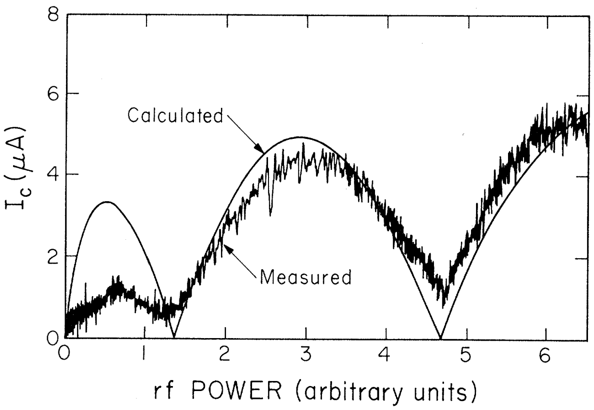

FIG. 2. Enhanced supercurrent as a function of 10-GHz microwave power. These data were taken at 3.6°K where no supercurrent was observed without external radiation. Similar effects persist up to 3.9°K. Smooth solid line, power dependence to be expected from Eq. (4).

by the rf fields.

There is an apparent discontinuity in the enhanced supercurrent at the transition temperature of the tin (~ 3.9°K). Above this temperature T_c(Sn) there is no supercurrent; at T_c(Sn) the current rises abruptly, as shown, to about 1 μA and then increases slowly with decreasing temperature, requiring ~ 0.03°K to increase to 2 μA. Whether this is a real effect or due to film inhomogeneities is still under investigation. For 10-GHz radiation we find that the rf enhancement persists over a wider temperature range than indicated in Fig. 1, down to at least 1.8°K where it still represents a 10% effect. Current noise in the measuring system was < 0.5 μA. We also find that the rf-enhanced current can be modulated periodically by a dc magnetic field.

The rf enhancement of the dc supercurrent in these high-current bridges can be described by assuming that the supercurrent is dominated by the B term in Eq. (3), but that noise voltages act to disrupt the correlation of phase gradients ($\nabla\varphi_1$ and $\nabla\varphi_2$) on a time scale set by the noise bandwidth. Thus superconductivity is suppressed at low frequency, but reappears at frequencies above the noise band. These bridges have been shown[7] to have noise currents equivalent to Johnson noise over a bandwidth of approximately $RI_c\varphi_0^{-1}$, where R is the bridge resistance, I_c is the critical current, and φ_0 is the flux quantum. However, if the frequency of the applied voltage is above the noise cutoff frequency, the "rectifying" indicated in Eq. (4) can still take place and induce a time-average dc supercurrent of maximum amplitude:

$$j_B \propto (2ev_0/\hbar\omega)J_1(2ev_0/\hbar\omega). \tag{5}$$

Figure 2 shows the amplitude of the excess current stimulated by an rf signal at 10 GHz as a function of rf power. These data were taken at about 3.6°K, nearly 0.4°K above the apparent transition temperature of the bridge. Equation (5) has been fitted to the data (solid line in Fig. 2) and agrees well except at low voltage where fluctuations may still predominate. The limitation on the enhancement is apparently set by a critical velocity (or maximum phase gradient), which for Fig. 2 occurs for powers just larger than indicated therein. For the situation illustrated in Fig. 2, the bandwidth $RI_c\varphi_0^{-1}$ is about 10^9 Hz. This noise bandwidth is consistent with our observations that there is no rf-enhanced current at 400 MHz, and a greater enhancement effect at 10 GHz than at 2 GHz. In the temperature range above 3.2°K, we also find that we can induce "steps" in the I-V curve at finite voltage corresponding to $V = n\hbar\omega/2e$ for 2- and 10-GHz radiation. This behavior is also expected from Eq. (3).

Above the critical current, which depends on the maximum gradient (or critical velocity) in this situation, this model also contains many of the features of the "phase slip" process without the analytically undesirable requirement of artificially shifting the phase state at some predetermined instant. By assuming a two-phase quantum state, the effect of phase slip occurs automatically since the amplitude of the wave function is a periodic function of $\varphi_1 - \varphi_2$. Arguments similar to those used in the phase-slip model[8] can also be applied here to account for the excess supercurrent at high voltage which is usually observed in high-current bridges.

*Work supported by the U.S. Office of Naval Research under Contract No. N00014-67-A-0094-0013.

†Work supported by a Schlumberger Foundation Fellowship.

[1]L. G. Aslamazov and A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. 9, 150 (1969) [JETP Lett. 9, 87 (1969)].

[2]A. F. G. Wyatt, V. M. Dmitriev, S. W. Moore, and F. W. Sheard, Phys. Rev. Lett. 16, 1166 (1966).

[3]H. A. Notarys and J. E. Mercereau, Physica (Utrecht) 55, 424 (1971), and to be published.

[4]J. Clarke, Proc. Roy. Soc., Ser. A 308, 447 (1969); J. G. Shepherd, Proc. Roy. Soc., Ser. A 326, 421 (1972).

[5]J. Bardeen and J. L. Johnson, Phys. Rev. B 5, 72

(1972).
[6]B. D. Josephson, Advan. Phys. 14, 419 (1965).
[7]R. K. Kirschman and J. E. Mercereau, Phys. Lett. 35A, 177 (1971).
[8]T. J. Rieger, D. J. Scalapino, and J. E. Mercereau, Phys. Rev. B 6, 1734 (1972).

Random Transfer Integrals and the Electronic Structure of Disordered Alloys*

L. Schwartz and H. Krakauer
Department of Physics, Brandeis University, Waltham, Massachusetts 02154

and

H. Fukuyama
Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 13 November 1972)

Previous theories of off-diagonal disorder in the tight binding model of a binary alloy are found to be incorrect in the dilute limit. We treat this limit exactly, and discuss the extension to arbitrary impurity concentrations.

Recently, authors[1,2] have devoted a great deal of attention to the following model of the binary substitutional alloy A_xB_{1-x}:

$$H=\sum_n |n\rangle \epsilon_n \langle n| + \sum_{n\neq m} |n\rangle h_{nm} \langle m|. \tag{1}$$

The simplifying feature of the model is that the effects of the disorder do not extend over any finite distance. Thus, the local energy levels ϵ_n may take either of the two values ϵ^A or ϵ^B, but the hopping integrals h_{nm} are always periodic. On the basis of a comparison with exact results concerning the limits of the allowed energy spectrum and the values of its leading moments,[2] it has been generally agreed that an excellent mean field description of the model is provided by the coherent potential approximation (CPA). There is, however, another class of exact results that are beyond the scope of this localized perturbation model. These results, the most basic of which is the Friedel sum rule,[3] concern the choice of self-consistent atomic potentials. As Stern[4] has shown, the essential difficulty lies in the assumption of zero-range scattering forces. Accordingly, a more physical description of the alloy must include off-diagonal disorder, i.e., we must allow both h_{nm} and ϵ_n to vary randomly.

The problem of extending the CPA to treat off-diagonal disorder has been discussed by several authors.[5] In addition, their methods have recently been applied to the spin wave spectrum in alloys of Heisenberg magnets,[6] a system for which the diagonal and off-diagonal scattering must certainly be treated on an equal footing. It is important then to realize that *none* of the proposed approximations is correct in the low-density limit. Unfortunately, this is precisely the limit of interest for the Friedel sum rule in metallic alloys, and also for making contact with the exact results of Wolfram and Callaway[7] in the case of insulating magnets. The present paper is limited to a discussion of the electronic problem; the formalism however is directly applicable to magnetic systems.

We begin by considering the Hamiltonian for a single A impurity at the origin of an otherwise perfect B crystal,

$$H=H_B+|0\rangle\delta_0\langle 0| + {\sum_{n\neq 0}}' [|n\rangle\delta_1\langle 0| + |0\rangle \delta_1\langle n|], \tag{2}$$

$$\delta_0=(\epsilon^A-\epsilon^B), \quad \delta_1=(h^{AB}-h^{BB}); \tag{3}$$

the prime indicates that only nearest neighbors are included in the summation. h^{BB} and $h^{AB}=h^{BA}$ describe host-host and impurity-host hopping, respectively. In a momentum representation, the impurity potential $v(\vec{k},\vec{k}')$ may be written

$$v(\vec{k},\vec{k}')=\delta_0+\delta_1[s(\vec{k})+s(\vec{k}')], \tag{4}$$

where $s(\vec{k})={\sum_n}'\exp(i\vec{k}\cdot\vec{R}_n)$ [the host E versus $\vec{k}$ relation is then $E^B(\vec{k})=\epsilon^B+h^{BB}s(\vec{k})$]. For a given (complex) energy z, Eq. (4), together with the unperturbed Green's function $G^{(0)}(\vec{k})=[z-E^B(\vec{k})]^{-1}$, permit an exact solution for the matrix elements of scattering operator $t(\vec{k},\vec{k}')=\langle\vec{k}|[1-vG^{(0)}]^{-1}v$