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[8]A more subtle criticism of the analysis performed in Ref. 1 is that even the sharpest lines observed there are $\approx 20\times$ the natural linewidth, and that the line shapes are very complex, resulting from a combination of unresolved hyperfine spectra in source and absorber. Under such conditions, it is virtually impossible to make precise measurements of f.

Narrow-Frequency Tunable γ-Ray Nuclear Resonances Induced by Laser Radiation

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A method is proposed for obtaining very narrow-frequency tunable γ-ray resonances of emission and absorption in nuclear transitions. The method is based on frequency modulation of the nuclear transition in molecules vibrationally excited with a coherent laser wave and having the same tunable projection of the translational velocity on the direction of the wave propagation. Laser-induced nuclear resonances have widths 10^{-2}–10^{-4} times the Doppler width of nuclear transitions and a tuning range of about 10^{-5}.

The present Letter proposes a method for obtaining very narrow-frequency tunable nuclear resonances in gas, which extends the methods for obtaining narrow resonances by means of laser absorption saturation[1,2] for a quite new frequency region. The method proposed is based on the frequency modulation of the nuclear transition in molecules vibrationally excited with the coherent light wave and having the same tunable projection of the translational velocity on the direction of the wave propagation. The method enables one to obtain resonances in both absorption and emission of nuclei having widths 10^{-2}–10^{-4} times the Doppler width of nuclear transitions, and to tune their frequency in $\Delta\omega/\omega_\gamma \simeq 10^{-5}$, considerably exceeding the tuning range of Mössbauer resonances.[3] The Letter also considers the principal experiments which might be carried out with these narrow resonances.

Let us consider molecules in a low-pressure gas with an inhomogeneous broadening of the vibrational-rotational absorption line ω_0, belonging to a certain ith vibration of the molecule. With a coherent traveling light wave of frequency ω and wave vector $\vec{k}_0 = \vec{n}\omega/c$, one can excite molecules with a quite definite projection of velocity $\vec{v}$ on the wave direction: $\vec{k}_0 \cdot \vec{v} = \omega - \omega_0 = \Omega$. The molecule density excited with a light wave on the $v'' = 1$ level is found from the expression

$$N_1(\vec{v}) = \frac{q}{2} N_0(\vec{v}) \frac{x}{1+x}, \quad x = \frac{G\Gamma_0^2}{(\Omega - \vec{k}_0\vec{v})^2 + \Gamma_0^2}, \qquad (1)$$

where $N_0(\vec{v})$ is the Maxwellian distribution of the molecule population on the lower vibrational level, $v' = 0$, when field is absent; Γ_0 is the homogeneous half-width of the optical transition determined by the collisions of molecules and (under a very low pressure) by the finite time of molecule transit through the light beam; G is the saturation parameter of the absorption transition determined by the light wave intensity; $q = g_0 Z_{\mathrm{rot}}^{-1} \times \exp(-E_0/kT)$ is the relative fraction of molecules occupying the rotational sublevel of the lower vibrational level of transitions determined by the g_0 level degeneracy and statistical weight Z_{rot} of the rotational states; E_0 is the rotational energy of the transition's lower level.

From the classical point of view, nuclei in vibrationally excited molecules vibrate with the frequency ω_0, which can be viewed as the average frequency of the motion from one turning point to another in the potential curve of the molecule.[4] First let us consider the simplest case to excite the lower rotational state with $J = 0$ of the vibrational level $v'' = 1$. In this case in a classical approach the time-dependent amplitude of the Sth atom can be represented in the form

$$\vec{r}_{iS}(t) = \vec{a}_{iS}^{\,v} \cos\omega_0 t, \qquad (2)$$

where $\vec{a}_{iS}^{\,v}$ is the average oscillation amplitude of the Sth atom in the excitation of the ith normal vibration of the molecule, depending upon molecular force constants and orientation of the molecule with respect to the polarization vector of the light wave. The vibration amplitudes of atoms vary from 0.1 to 10^{-3} Å.[4,5] The most es-

sential thing is that the nuclear displacement $\vec{a}_{iS}$ is of the same order of magnitude or even more than the wavelength λ_γ of γ quanta with energy from 10 keV to 1 MeV. The vibration of the nuclear emitter with this amplitude should cause a considerable frequency modulation of γ radiation.

The transition frequency of nuclei in the molecule vibrationally excited by a light wave, for collinear observation, is equal to

$$\omega_{\mathrm{nucl}}^{(0)} = \omega_\gamma \pm \Delta_\gamma + \vec{k}_\gamma \cdot \vec{v} = \omega_\gamma \pm \Delta_\gamma + (\omega - \omega_0)\omega_\gamma/\omega_0, \qquad (3)$$

where ω_γ is the transition frequency of the recoilless nucleus; Δ_γ is the recoil energy, signs + and − denote absorption and emission, respectively; $\vec{k}_\gamma$ is the wave vector of the γ quantum. Because of the modulation of the nuclear position with frequency ω_0 and the average amplitude $\vec{a}_{iS}{}^v$, there occurs a splitting of the nuclear frequency $\omega_{\mathrm{nucl}}^{(0)}$ into a number of spectral components at frequencies $\omega_{\mathrm{nucl}}^{(m)} = \omega_{\mathrm{nucl}}^{(0)} \pm m\omega_0$ ($m = 0, 1, 2, \ldots$). A relative component intensity at frequency $\omega_{\mathrm{nucl}}^{(m)}$ is determined from the expression

$$I_m = \left\langle y_m{}^2\left(\frac{\vec{a}_{iS}{}^v \vec{n}}{\lambda_\gamma}\right)\right\rangle \simeq J_m{}^2\left(\frac{\tilde{a}_{iS}{}^v}{\lambda_\gamma}\right), \qquad (4)$$

where J_m is the Bessel function of the mth order, and the averaging $\langle \cdots \rangle$ is carried out over all orientations of molecules; $\tilde{a}_{iS}{}^v$ is the average value of the displacement projection of the Sth atom on the direction of the light wave exciting the ith normal vibration of the molecule.

Under a collinear observation of the nuclear line ($\vec{k}_\gamma = \vec{k}_0\omega_\gamma/\omega_0$) only a narrow spectral interval will be modulated by a light wave at frequency $\omega_{\mathrm{nucl}}^{(0)}$ with width

$$\Gamma_{\mathrm{nucl}} = \Gamma_0(1+G)^{1/2}\omega_\gamma/\omega_0 \qquad (5)$$

from the whole Doppler-broadened line of the nuclear transition. As a result the shape of the inhomogeneously broadened nuclear line will be noticeably distorted by a light wave. At frequency $\omega_{\mathrm{nucl}}^{(0)}$ determined by expression (3) a dip is produced with the half-width over the half-height Γ_{nucl} and the depth $\beta_0 = 1 - \frac{1}{2}qI_0G/(1+G)$; and at frequency $\omega_{\mathrm{nucl}}^{(m)} = \omega_{\mathrm{nucl}}^{(0)} \pm m\omega_0$ the peaks are formed with the same width and the relative value $\beta_m = \frac{1}{2}qI_mG/(1+G)$. The shape of the spectral line of the nuclear absorption (emission) for observation in the direction of the light wave saturating the vibrational-rotational transition of the molecule is depicted in Fig. 1.

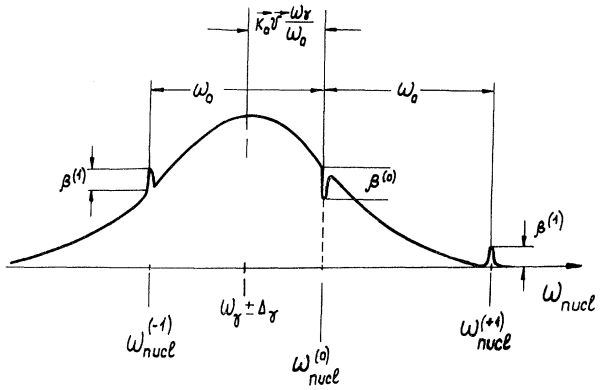

FIG. 1. Shape of the spectral line of nuclear absorption (emission) for observation in the direction of the light wave saturating the vibrational-rotational transition of the molecule.

Because a relatively small part q of molecules is in resonance with the coherent light wave (q is a factor which determines the population of one rotational level and usually lies within the interval from 10^{-1} for simple molecules to 10^{-3} for many-atom molecules), narrow γ resonances have a relative amplitude in the interval 0.1–10%.

The principal feature of this effect is an automatic changeover of narrow γ-resonance frequency under a light-wave frequency scanning along the Doppler contour of the optical transition. A relative region of continuous tuning of the dip at frequency $\omega_{\mathrm{nucl}}^{(0)}$ is determined by the Doppler width of transitions. However, the tuning region of all the resonances $\Delta\omega_{\mathrm{tun}}$ exceeds the Doppler width:

$$\frac{\Delta\omega_{\mathrm{tun}}}{\omega_\gamma} = \frac{\Delta\omega_{\mathrm{Dopp}}{}^\gamma}{\omega_\gamma} + \bar{m}\frac{\omega_0}{\omega_\gamma}. \qquad (6)$$

The ω_0 can be changed by selecting the vibrational frequencies of the molecule in which the nucleus is incorporated, and the number m of the resonance can be raised by increasing the amplitude of the $\tilde{a}_{iS}{}^v$ nucleus vibrations. The maximum number $\bar{m}$ of peaks is approximately found by the ratio $\tilde{a}_{iS}{}^v/\lambda_\gamma$, and in principle $\bar{m} \gg 1$ is possible. In any case, the tuning region of the narrow resonances overlaps the interval $\Delta\omega_{\mathrm{tun}}/\omega_\gamma = 10^{-5}$ in the vicinity of the nuclear transition.

In the case of the excitation of the vibrational rotational state with $J \neq 0$, it is necessary to take into account the fact that the light wave causes both the vibrational movement of the nucleus according to law (2) and an additional oscillator

movement with the frequency of the molecular rotation. In the classical limit the nuclear displacement can be represented as

$$\vec{r}_{iS}(t) = \vec{a}_{iS}{}^{v} \cos\omega_v t + \vec{a}_{iSJ}{}^{r} \cos\omega_r t, \qquad (7)$$

where $\vec{a}_{iSJ}{}^{r}$ is the average amplitude of the oscillations of the Sth atom in the $\vec{k}_0$ direction due to the molecular rotation excited on the level with angular momentum J. In this case the light-wave frequency is $\omega = \omega_v + \omega_r$. The peculiarity is that the amplitude of the nuclear oscillations due to the rotation can be much higher than that of the nuclear oscillations due to the vibrations. The frequency modulation of the line due to the nuclear emitter motion according to law (7) results in the splitting of each component at frequency $\omega_{\mathrm{nucl}}{}^{(0)} \pm m\omega_r$ into many close components at frequencies $\pm n\omega_r$.

Since in the typical case $\tilde{a}^r \gg \lambda\!\!\!^{-}_{\gamma}$, then the modulation by the molecular rotation broadens the nuclear resonance at frequency $\omega_{\mathrm{nucl}}{}^{(m)}$ by the relative value

$$\delta = \frac{\bar{n}\omega_r}{\omega_\gamma} \simeq \frac{\tilde{a}_{iSJ}{}^{r}}{\lambda\!\!\!^{-}_{\gamma}} \frac{\omega_r}{\omega_0}, \qquad (8)$$

where $\bar{n}$ is the number of the rotational component for which the intensity maximum is attained, i.e., the maximum value of $J_n^2(\tilde{a}_{iSJ}{}^{r}/\lambda\!\!\!^{-}_{\gamma})$ as a function of n. If we take $\omega_r = 2\pi c\beta(2J+1)$ then approximately the estimate $\delta \simeq \tilde{a}_{iSJ}{}^{r} 4\pi\beta J$ is valid, where β is the rotational constant. For example, at $\tilde{a}^r \simeq 1$ Å, $\beta = 0.5$ cm^{-1}, the value of the rotational broadening of the narrow nuclear resonance is $\delta \simeq 6\times10^{-8}J$. This value is much smaller than the Doppler broadening, but it exceeds the ultimate width of the resonance found with (5). Therefore, to eliminate this effect one should work either at the transition with $J=0$, or with the nucleus placed on the rotation axis ($\tilde{a}^r = 0$) of the molecule, or with heavy molecules for which $\beta \ll 1$ cm^{-1}.

The classical treatment of the modulation effect and the frequency tuning of γ-radiation resonances by the laser radiation considered above should describe correctly the qualitative aspect of the effect. The strict quantitative theory is naturally quantum mechanical. In the quantum theory the general wave function of the nucleus in the excited molecule will have oscillatory terms responsible for the additional components near the $\omega_{\mathrm{nucl}}{}^{(0)}$ frequency. As a result there arise the combination frequencies $\omega_{\mathrm{nucl}}{}^{(m,n)}$ different from the central frequency $\omega_{\mathrm{nucl}}{}^{(0)}$ by the frequencies of vibrational and rotational transitions. The results of the quantum-mechanical treatment of the effect considered will be presented later.

Narrow absorption resonances on the nuclear transition can be detected with a Mössbauer source of γ radiation using the same nuclei as in a molecular gas. In this case by tuning the dip frequency $\omega_{\mathrm{nucl}}{}^{(0)}$ on the recoilless emission line of source frequency ω_γ one can register the absorption minimum. Similarly by tuning the frequency $\omega_{\mathrm{nucl}}{}^{(-1)}$ on the frequency ω_γ one can detect a narrow absorption peak. Since the resonant absorption coefficient for nuclear transitions in a low-pressure gas (0.01–0.1 Torr) is very small, then it is possible to observe narrow resonances in absorption through the change of the intensity of γ fluorescence of the gas. It is still more convenient to observe narrow nuclear resonances in the emission of the molecular gas with γ-active nuclei.

The method proposed enables us to fulfill several basic experiments. For example, by tuning the narrow resonance of γ radiation $\omega_{\mathrm{nucl}}{}^{(+1)} = \omega_\gamma - \Delta_\gamma + \vec{k}_0\cdot\vec{v}\omega_\gamma/\omega_0 + \omega_0$ in the vicinity of the line of the nuclear absorption ω_γ in a condensed target, one can accurately measure the phonon spectrum over the wing shape in the nuclear absorption line.[6] It is natural that in these experiments one can measure the recoil energies of nuclei with an accuracy inaccessible at present. Since the value of $\vec{k}_0\cdot\vec{v}$ can be measured with a very high accuracy $\delta\Omega$ by measuring the detuning Ω of the frequency of the light wave with respect to the Doppler contour center ω_0, then the resonance frequency of γ radiation can change over with accuracy $\delta\omega_0/\omega_\gamma = \delta\Omega/\omega_0$. This value can reach 10^{-11}–10^{-13}. It enables absolute measurements of γ-quantum energy with accuracy ω_γ/ω_0 (10^{-11}–10^{-13}) which, in principle, can attain 10^{-8}. With this method one can connect the energy scales of optical and γ ranges with an accuracy of 10^{-8}. It is natural that the method suggested allows one to carry out γ spectroscopy of nuclei in the vicinity of nuclear transitions 10^{-5} with resolution 10^{-9}, inaccessible for present methods. Notice that this method is also suitable for transitions where the Mössbauer effect does not arise (for instance, for the γ-quantum energy much more than 100 keV).

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Two-Stream Instability Heating of Plasmas by Relativistic Electron Beams*

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We discuss the two-stream interaction of relativistic electron beams with a plasma in the nonlinear regime supported by computer simulation experiments.

For relativistic electron beams with current I in excess of the critical Alfvén current I_A, i.e., $I/I_A = \nu/\gamma \gg 1$, the dominant mechanism by which the beam transfers its energy to the plasma is expected to be that due to the turbulent decay of the return current proposed by Lovelace and Sudan.[1] Here we use conventional notation: $\nu = Nr_e$, where N is the number of beam electrons per unit length and r_e is the classical electron radius; γ is the beam energy in rest mass units. On the other hand, for beams with $\nu/\gamma \sim 1$ the competing process of the electrostatic two-stream interaction becomes important. The present Letter is devoted to a discussion of the two-stream interaction, supported by computer simulation studies for a homogeneous beam-plasma system.

The gross magnetohydrodynamic stability of the beam can be assured on the time scale of the two-stream instability provided the beam propagates along a guide magnetic field such that the beam kinetic-energy density is less than the magnetic energy density.[2] In addition, the beam velocity distribution is assumed to satisfy $\Delta v_\parallel/c \ll \gamma_0^{-1}(n_b/2n_e)^{1/3}$, where n_b and n_e are the respective beam and plasma particle densities, $\Delta v_\parallel$ is the velocity spread in the direction of the average beam velocity $\vec{v}_0$, and $\gamma_0 = (1 - v_0^2/c^2)^{-1/2}$. Furthermore, we shall assume a one-dimensional spectrum for the unstable waves. There are four situations where the last assumption is valid: (i) beam propagation along a sufficiently strong magnetic field; (ii) $\Delta v_\perp/c \gtrsim (n_b/2\gamma_0 n_e)^{1/3}$, which reduces the growth rates of modes with finite[3,4] $k_\perp$ below $\gamma_0^{-1}(n_b/2n_e)^{1/3}$, where $\Delta v_\perp$ is the velocity spread perpendicular to $\vec{v}_0$; (iii) the plasma density profile has a minimum at the beam axis, and waves with finite $k_\perp$ will be reflected towards the direction of beam propagation; (iv) the beam has been modulated at a wavelength for maximum growth of the two-stream instability.

Under the above restrictions, the dispersion relation for electrostatic waves is

$$1 - \omega_e^2/\omega^2 - \omega_b^2/(\omega - kv_0)^2 = 0, \qquad (1)$$

where $\omega_e^2 = 4\pi n_e e^2/m$ and $\omega_b^2 = 4\pi n_b e^2/\gamma_0^3 m$. After N e-folds in amplitude, the width of the unstable spectrum predicted by (1) is $\delta k/k_0 \sim \gamma_0^{-1}(n_b/2n_e)^{1/3} \times N^{-1/2}$, where $k_0 \approx \omega_e/v_0$ is the wave number corresponding to the most unstable wave. It is clear that the spectrum can be very narrow when a high-energy beam interacts with a cold plasma. In this case, the spectrum may be approximated by a single wave with wave number k_0, the so-called single-wave model.

There are two main questions to be resolved: (i) the maximum intensity to which the unstable waves grow in the two-stream instability phase of the interaction, and (ii) the rate of energy transfer to the plasma after wave saturation. To characterize the strength of the interaction, we define the parameter $S \equiv \beta_0^2 \gamma_0 (n_b/2n_e)^{1/3}$. Several investigators have examined the regime $S \ll 1$. From a quasilinear analysis of a one-dimensional spectrum Fainberg, Shapiro, and Shevchenko[3] obtain for the saturation amplitude

$$W \equiv \textstyle\sum_k |E_k|^2/8\pi n_b \gamma_0 mc^2 = 0.158S,$$

while Kovtun and Rukhadze[5] obtain $W = 0.198S$ using the single-wave model for the interaction.