symmetry for the output-octet trajectories is preserved.

Taking the ratio of Eqs. (12) and (13), we obtain $A_{ab}{}^{\mathrm{R}}/A_{ab}{}^{\mathrm{P}} \simeq \frac{1}{2}\exp[(\alpha_{\mathrm{R}}-1)Y]$, where the coefficient in the right-hand side is independent of a or b. This *universality relation*[15] agrees quite remarkably with the experimental data (with a typical error of 20%) for $A_{pp}{}^{f_0}/A_{pp}{}^{\mathrm{P}} \approx A_{Kp}{}^{f_0}/A_{Kp}{}^{\mathrm{P}} \approx 1.2e^{-0.5Y}$. Other interesting relationships[16] can be derived by generalizing our result to cases with quantum numbers, such as (for ab exotic)

$$\sum_{n=2}^{\infty}\frac{n(\sigma_n{}^{a\bar{b}}-\sigma_n{}^{ab})}{\sigma^{a\bar{b}}-\sigma^{ab}}=\frac{1}{2}\sum_{n=2}^{\infty}\frac{n\sigma_n{}^{ab}}{\sigma^{ab}},\quad \frac{\sigma_n{}^{a\bar{b}}}{\sigma_n{}^{a\bar{b}}-\sigma_n{}^{ab}}=2^{n-1}.$$

Unfortunately, the present data available are not sensitive enough for a meaningful comparison of these relations.

In closing let us stress that this model provides a promising bootstrap framework for a realistic calculation of the Regge parameters appearing in both exclusive and inclusive cross sections. Applications along this line and refinements of the model will be published elsewhere.

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[12]When we consider quantum numbers, it is then easy to see that $|A_n{}^b|_{\mathrm{s.t.}}{}^2$ but not $|A_n{}^b|^2$ should be identified with the Pomeranchukon.

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[16]Notice that these relations and the result $\alpha_{\mathrm{P}}=1$ are all independent of g^2, hence their validity is probably more general than the Chew-Pignotti scheme which admits no correlations among produced particles. One can argue that when the kernel is improved to admit correlations, Eqs. (14) and (15) will retain the same form except with g^2 replaced by the strength of the new kernel.

New Sum Rule for Photoproduction Amplitudes Based on Local Current Commutators*

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A sum rule for isovector Compton scattering, previously derived using dispersive techniques, is rederived using the ++ light-cone commutator. The (nontrivial) limit of the sum rule at $q^2=0$ is given, and is found to be in excellent agreement with experiment. The deep-inelastic limit of the sum rule is discussed in the context of the covariant parton model, where it is shown that the free-field realization is untenable.

Some time ago, several authors[1,2] derived sum rules for the absorptive parts of certain of the invariant amplitudes appearing in a tensor decomposition of the full, spin-dependent, nonforward scattering of vector or axial-vector currents from nucleons. The sum rules were derived using the covariant Fubini approach,[3] with its attendant assumptions concerning unsubtracted dispersion relations for some of the amplitudes. Two of these sum rules are of particular interest, since they can also be derived from a knowledge of the simple ++ light-cone commutator, using the approach of Dicus, Jackiw, and Teplitz.[4,5] One of these, of course, is the Dashen–Gell-Mann–Fubini (DGF) sum rule.[6] Support for the DGF sum rule is grounded in the validity of its q^2 derivative at $q^2=0$, the Cabibbo-Radicati sum rule.[7] Its extension to physical neutrino scattering (the Adler sum rule[8]) in the large-q^2 region has played a seminal role in the development of the scaling hypothesis by Bjorken.[9]

The other sum rule, which is of the form

$$\int_0^\infty [a_1^-(\nu, q^2, q^2, t) - a_1^+(\nu, q^2, q^2, t)]\,d\nu = -F_2^v(t),$$

was examined cursorily (and inconclusively) in Ref. 1 for the case of nonconserved axial-vector currents. In this note, after briefly rederiving the sum rule using light-cone techniques,[4,5,10] I would like to display its form at $q^2=0$ for the case of vector currents, and show that agreement with the data is excellent (to within better than 5%). I will then go on to discuss the deep-inelastic limit of the sum rule in the context of a covariant parton model.

The nonforward commutator function

$$W_{\mu\nu}^{ab}(p_1q_1;p_2q_2) = (2\pi)^{-1}\int d^4x\, e^{iQ\cdot x}\langle p_2|[V_\mu^a(\tfrac{1}{2}x), V_\nu^b(-\tfrac{1}{2}x)]|p_1\rangle \tag{1}$$

[with $Q=\frac{1}{2}(q_1+q_2)$] may be expanded into a complete set of second-rank tensors[1,2] in which the coefficients (the invariant amplitudes) are free of kinematic singularities and zeros. Keeping $q_1^2=q_2^2$ allows time-reversal invariance to reduce the 32 possible invariant amplitudes[1,2] to 20. With the definitions $P=\frac{1}{2}(p_1+p_2)$, $\Delta=p_1-p_2=q_2-q_1$, we write (omitting isotopic indices)

$$\begin{aligned} W_{\mu\nu} = \bar{u}(p_2)\{&(a_1/m^2)P_\mu P_\nu + a_2(P_\mu Q_\nu + P_\nu Q_\mu) + (a_3/2m)(P_\mu\gamma_\nu + P_\nu\gamma_\mu) + a_4 Q_\mu Q_\nu + a_5(Q_\mu\gamma_\nu + Q_\nu\gamma_\mu) + a_6\Delta_\mu\Delta_\nu \\ &+ a_7 g_{\mu\nu} + b_1(P_\mu\Delta_\nu - P_\nu\Delta_\mu) + b_2(Q_\mu\Delta_\nu - Q_\nu\Delta_\mu) + b_3(\Delta_\mu\gamma_\nu - \Delta_\nu\gamma_\mu) + (c_1/m^2)P_\mu Q_\nu \gamma\cdot Q \\ &+ c_2(P_\mu Q_\nu + P_\nu Q_\mu)\gamma\cdot Q + c_3 Q_\mu Q_\nu \gamma\cdot Q + c_4\Delta_\mu\Delta_\mu\gamma\cdot Q + c_5 g_{\mu\nu}\gamma\cdot Q + c_6([\gamma_\nu, \gamma\cdot Q]\Delta_\mu + [\gamma_\mu, \gamma\cdot Q]\Delta_\nu) \\ &+ d_1[\gamma_\mu, \gamma_\nu] + d_2(\gamma_\mu\gamma\cdot Q\gamma_\nu - \gamma_\nu\gamma\cdot Q\gamma_\mu) + d_3(P_\mu\Delta_\nu - P_\nu\Delta_\mu)\gamma\cdot Q \\ &+ d_4([\gamma_\nu, \gamma\cdot Q]Q_\mu - [\gamma_\mu, \gamma\cdot Q]Q_\nu)\}u(p_1), \end{aligned} \tag{2}$$

where the $\{a, b, c, d\}$ are functions of Q^2, $P\cdot Q$, and Δ^2; and m is the nucleon mass. We consider only the conserved nonstrange isovector currents. Then gauge invariance of $W_{\mu\nu}$ provides eight linear relations which reduced to twelve the number of independent scalar amplitudes. For the discussion which follows, only one of these relations is of importance:

$$\nu a_1/m = (\tfrac{1}{4}t - q^2)(a_2 + 2d_4) + \tfrac{1}{2}t(b_1 - 2c_6), \tag{3}$$

where $m\nu = P\cdot Q$. We adopt a conventional normalization: $\langle p'/p\rangle = (p^0/M)(2\pi)^3\,\delta^3(p'-p)$ for baryonic states; $\bar{u}u=1$ for spinors. Now set $q_{1+}=q_{2+}=Q_+=0$, and integrate the $\mu=+$, $\nu=+$ component of Eq. (1) over Q_-:

$$\int_{-\infty}^\infty dQ_-\, W_{++}^{ab} = \int d^2x_\perp\, dx_-\, \exp(-i\vec{Q}_\perp\cdot\vec{x}_\perp)\langle p_2|[V_+^a(\tfrac{1}{2}x), V_+^b(-\tfrac{1}{2}x)]|p_1\rangle_{x_+=0}. \tag{4}$$

From Eqs. (2) and (4) and the knowledge of the ++ light-cone commutator,[4,11] one can obtain

$$\begin{aligned} \int_{-\infty}^\infty d\nu\,\{&(P_+/m)a_1^{ab}(\nu, q^2, t)\bar{u}_2u_1 + [a_3^{ab}(\nu, q^2, t) + \nu c_1^{ab}(\nu, q^2, t)]\bar{u}_2\gamma_+u_1 + (P_+/m)c_1^{ab}\bar{u}_2\vec{\gamma}_\perp\cdot\vec{Q}_\perp u_1\} \\ &= i\epsilon_{abc}\langle p_2|V_+^c(0)|p_1\rangle, \end{aligned} \tag{5}$$

with $\vec{P}_\perp\cdot\vec{Q}_\perp=0$. Now we specialize to the odd-charged combination $W_{\mu\nu}^{(-)} \equiv W_{\mu\nu}^{1+i2,1-i2} - W_{\mu\nu}^{1-i2,1+i2} \equiv W_{\mu\nu}^- - W_{\mu\nu}^+$. With the isovector current written as

$$\langle p_2|V_\mu^c|p_1\rangle = \bar{u}(p_2)\tfrac{1}{2}\tau_c\{[F_1^v(t) + F_2^v(t)]\gamma_\mu - F_2^v(t)P_\mu/m\}u(p_1), \tag{6}$$

the *linear independence* of the quantities $\bar{u}_2 u_1$, $\bar{u}_2\gamma_+ u_1$, and $\bar{u}_2\vec{\gamma}_\perp\cdot\vec{Q}_\perp u_1$ allows us to write the sum rules

$$\int_{-\infty}^{\infty} a_1^{(-)}(\nu, q^2, q^2, t)\,d\nu = -2F_2^{\,v}(t), \tag{7}$$

$$\int_{-\infty}^{\infty}[a_3^{(-)}(\nu, q^2, q^2, t) + \nu c_1^{(-)}(\nu, q^2, q^2, t)]\,d\nu = 2[F_1^{\,v}(t) + F_2^{\,v}(t)], \tag{8}$$

$$\int_{-\infty}^{\infty} c_1^{(-)}(\nu, q^2, q^2, t)\,d\nu = 0, \tag{9}$$

where $a_1^{(-)} = a_1^- - a_1^+$, etc. The odd crossing property of $c_1^{(-)}$ in ν makes the last sum rule trivial, while the even crossing properties of $a_1^{(-)}$, $a_3^{(-)}$, and $\nu c_1^{(-)}$ lead to the final versions

$$\int_0^{\infty}[W_2^-(\nu, q^2, q^2, t) - W_2^+(\nu, q^2, q^2, t)]\,d\nu = F_1^{\,v}(t), \tag{10}$$

$$\int_0^{\infty}[a_1^-(\nu, q^2, q^2, t) - a_1^+(\nu, q^2, q^2, t)]\,d\nu = -F_2^{\,v}(t), \tag{11}$$

where $W_2 = a_1 + a_3 + \nu c_1$ reduces to the usual W_2 at $t=0$. The first is the $t\neq 0$ generalization of the DGF sum rule, while the second is the sum rule derived in Refs. 1 and 2, and the object of our study.

First, we isolate the one-neutron contribution, and rewrite Eq. (11) as

$$-F_1^{\,v}(q^2)F_2^{\,v}(q^2) + \int_{\nu_{\rm th}}^{\infty} a_1^{(-)}(\nu, q^2, q^2, t)\,d\nu = -F_2^{\,v}(t), \tag{12}$$

where $\nu_{\rm th} = m_\pi + (m_\pi^{\,2} - q^2 + \frac{1}{2}t)/2m$, with $q^2 \leqslant 0$; we shall deal always with t very small or zero, so that $\nu_{\rm th} > 0$. For $q^2 = 0$, $t=0$, $\nu\neq 0$, we find from the gauge condition (3) that $a_1(\nu, 0, 0, 0) = 0$, and Eq. (12) reduces to the statement $F_1^{\,v}(0) = 1$.

Let us now set $q^2 = 0$, $t \neq 0$; Eq. (3) shows that $\nu a_1 = \frac{1}{4}tm(a_2 + 2b_1 - 4c_6 + 2d_4)$, and since none of the amplitudes have kinematic singularities, $da_1/dt = \lim(t\to 0)(a_1/t)$. We then take the derivative of Eq. (12), with respect to t, to obtain

$$-\int_{m_\pi}^{\infty} d\nu \lim_{t\to 0}\frac{a_1^{(-)}(\nu, 0, 0, t)}{t} = \frac{dF_2^{\,v}(t)}{dt}\bigg|_{t=0} \cong 11.0\ \mathrm{GeV}^{-2}. \tag{13}$$

One may test this sum rule by relating $[a_1^{(-)}(\nu, 0, 0, t)/t]_{t\to 0}$ to isovector photon-scattering amplitudes. This is most easily done by noting that our a_1 is related to the invariants A_1 and A_2 of Hearn and Leader[12] by

$$a_1(\nu, 0, 0, t) = -(t/4\pi\nu^2)\,\mathrm{Im}(A_1 - A_2). \tag{14}$$

Then one proceeds straightforwardly to write $A_1 - A_2$ in terms of helicity amplitudes,[12] and to use the Wigner-Eckart theorem to do the necessary isospin rotation to isovector photons for the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ intermediate states. Unitarity may then be used to relate the Compton helicity amplitudes to the multipole amplitudes found in analyses of single-pion photoproduction (at least at low energies). This kinematic detail will be presented in a future publication. It will suffice for us at present to write the sum rule in terms of purely resonant contributions, and express these in terms of Walker's amplitudes A_λ, $\lambda = \frac{1}{2}, \frac{3}{2}$, which are defined[13] as the amplitudes for decay of a resonance at rest of spin $S_z = \lambda$ into a photon going up the z axis with helicity $+1$, and a nucleon going down the z axis with helicity $1-\lambda$. The normalization is given in Ref. (13). In terms of these, the sum rule can be reduced to

$$\frac{m}{4\pi\alpha}\sum_{\rm res}\epsilon_r C_r \nu_r^{-2}\left\{A_{1/2}^{\,2} + A_{3/2}^{\,2} - \frac{2M_r}{m}[(J-\tfrac{1}{2})(J+\tfrac{3}{2})]^{1/2} A_{1/2}A_{3/2}\right\} = \frac{d}{dt}F_2^{\,v}(t)\bigg|_{t=0} \simeq 11.0\ \mathrm{GeV}^{-2}, \tag{15}$$

where $C_r = 2\ (-1)$ for $I=\frac{1}{2}$ $(\frac{3}{2})$ resonances, and ϵ_r is the fraction of the photoexcitation which proceeds through the isovector component of the photon. Using Walker's values for the A_λ's,[13] the left-hand side of Eq. (15) reads

$$8.70 + 2.05 + 0.24 + 0.18\ \mathrm{GeV}^{-2} \cong 11.2\ \mathrm{GeV}^{-2}, \tag{16}$$

where the contributions are from the $P_{33}(1236)$, $D_{13}(1512)$, $S_{11}(1550)$, and $F_{15}(1688)$, respectively. The agreement is quite remarkable and implies a very weak contribution from the amplitudes above 1700 MeV. The simplest way in which this may occur is if the s-channel helicity-flip amplitude $\langle 1\ -\frac{1}{2}|\varphi|1\ \frac{1}{2}\rangle$ is small. Then the continuum contribution (in a nonresonant treatment) lacks the analog of the large second term inside the braces in (15), and can be estimated[14] to contribute $\lesssim 0.1$ to the left-hand side of (15).

We can then say that the saturation of the sum rule at $q^2=0$, $t\to 0$ is *at least* as successful as that of the Cabibbo-Radicati[7] sum rule. Suppose then that this encourages us to assume its validity in the deep-inelastic region $q^2\to -\infty$. The neutron pole disappears and, assuming that $\nu a_1 = \mathfrak{A}_1(\nu, q^2)$ scales as $\nu\to\infty$, $q^2\to -\infty$ (see next paragraph), we obtain

$$\int_0^1 (d\omega/\omega)[\mathfrak{A}_1^-(\omega) - \mathfrak{A}_1^+(\omega)]_{t=0} = -F_2^v(0) = -3.70, \tag{17}$$

with $\omega \equiv -q^2/2m\nu$. The structure function $\mathfrak{A}_1(\omega)$ can be expressed either as a Fourier transform of the bilocal scalar form factor $v_2(P\cdot x, 0, 0)$ appearing in the expansion[15]

$$\langle p'|V_\mu(x|0)|p\rangle = \bar{u}(p_2)[v_1(P\cdot x, x^2, t)\gamma_\mu - v_2(P\cdot x, x^2, t)(P_\mu/m) + \text{terms in } \Delta_\mu, x_\mu, P_\mu\gamma\cdot x, \ldots]u(p_1)$$

or, equivalently, as an integral over some of the off-shell spin-flip parton-proton scattering amplitudes.[16-18] As opposed to the case for electroproduction, these amplitudes cannot be heuristically interpreted as parton densities, or squares of wave functions in momentum space. Rather, they resemble overlap integrals of such wave functions for different values of proton helicity. It is easily seen that the amplitude $\mathfrak{A}_1$ and form factor F_2 are zero in the Born approximation. Then, the formula shows that the free-field realization of the parton model is inconsistent with nature, i.e., with $F_2^v \neq 0$.

Finally, it is of interest to compare the present light-cone derivation of the sum rule (11) with the previous derivations[1,2] using the Fubini method.[3] The latter approach necessitated that the amplitudes $A_2^{(-)}$ and $D_1^{(-)}$ [corresponding to the absorptive parts $a_2^{(-)}$ and $d_1^{(-)}$ in Eq. (2)] satisfy unsubtracted dispersion relations in ν for fixed q^2 at $t=0$. The t-channel analysis shows that this is a standard requirement for $A_2^{(-)}$. The case of $D_1^{(-)}$ is more unusual in that it involves the question of a trajectory with $I^G = 1^+$, even signature, and odd parity. Particles corresponding to such a trajectory do not seem to exist. But these quantum numbers may also be reached through a $P\otimes\rho$ cut, which has an effective intercept $\alpha_{\text{eff}}(0)\simeq\frac{1}{2}$; in this case, the correctness of the sum rule may indicate that such a cut does not couple to nucleons and/or charged isovector currents.

In summary, the purpose of this note was to demonstrate convincingly the validity at $q^2=0$, $t\to 0$ of one of the two sum rules derivable from the local ++ light-cone commutators and to stimulate interest in it as a new theoretical constraint on the data. In a future publication, the kinematic details of the reduction of the sum rule to the form (15) will be given, as well as some further discussion on its usefulness in the phenomenology of photoproduction.

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it is $\simeq\frac{1}{2}$. The A_λ's are real by time-reversal invariance.

[14]Vector dominance allows the helicity nonflip term in the sum rule to be written as $(m/4\pi\gamma_\rho^2)\int(d\nu/\nu^2)[\sigma_{\rho^- p}^T(\nu) - \sigma_{\rho^+ p}^T(\nu)]$, where $\sigma_{\rho^\pm p}^T(\nu)$ are total cross sections for the scattering from protons of charged, unpolarized, transverse, zero-mass ρ mesons, and $\gamma_\rho^2/4\pi\simeq 0.53$. A simple assumption is that for energies above, say, $\nu=1$ GeV, $\sigma_{\rho^- p}^T(\nu)-\sigma_{\rho^+ p}^T(\nu)\sim\sigma_{\pi^- p}(\nu)-\sigma_{\pi^+ p}(\nu)\approx 4mb(\nu/\nu_0)^{-1/2}$, $\nu_0=1$ GeV [see, e.g., V. Barger and M. Olsson, Phys. Rev. Lett. 18, 294 (1967)]. In that case the nonflip contribution to the sum rule from the region above the third resonance comes to $\lesssim 0.1\ \mathrm{GeV}^{-2}$.

[15]Use of the light-cone commutators in Ref. 11 allows one, in a standard manner, to demonstrate the scaling of νa_1 by expressing it as a Fourier transform of a bilocal scalar form factor $v_2(P\cdot x, x^2, t)$, $\nu a_1=\int d\alpha\, e^{-i\omega\alpha}(d/d\alpha)\times v_2(\alpha,0,t)$, where $\alpha=P\cdot x$.

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[18]Let $T=\int d^4x\, e^{ik\cdot x}\langle p_2|\psi(x)\bar\psi(0)-\psi(0)\bar\psi(x)|p_1\rangle=\int d^4x\, e^{ik\cdot x}\langle p_2|S(x|0)|p_1\rangle$. Except for isospin matrices, T is the off-shell parton-proton scattering amplitude. The leading contribution to the amplitude $\mathfrak{A}_1$ is obtained from the coefficient of $P_\mu P_\nu \bar u(p_2)u(p_1)$ in $(2\pi)^{-4}i\int d^4k\,\delta((k+q)^2)\epsilon(k_0+q_0)\,\mathrm{Tr}[(k+q)_\mu\gamma_\nu+(k+q)_\nu\gamma_\mu-g_{\mu\nu}(k+q)\cdot\gamma]\,T$, where the trace is taken with respect to parton spinor indices (see Ref. 16). Ignoring terms in Δ_μ, we may expand $T=b_1 k\cdot\gamma\bar u_2 u_1+b_2 k\cdot\gamma\bar u_2 k\cdot\gamma u_1+b_3\gamma_\alpha\bar u_2\gamma^\alpha u_1+\cdots$ and show that, as $\Delta^2\to 0$, $\mathfrak{A}_1(\omega)=\omega^2\int_{-\infty}^{\infty}dk^2\,d(k\cdot p)\theta(-2\omega k\cdot p-k^2-m^2\omega^2)[b_1(k^2,k\cdot p)-m\omega b_2(k^2,k\cdot p)]$.