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[9]This equation can be satisfied only if for all X there exists gauge transformation Ω such that $(\dot{X}^\Omega - X'^\Omega)^2 = 0$ and $X_+^\Omega - (2/\pi)^{1/2} P_+ \tau = 0$. The latter condition can be satisfied because X cannot vanish since $X^2 > 0$.

[10]This follows easily from Eq. (26) (see below). The factor $\prod \mathcal{L}$ which appears in $W(J)$ should be considered as gauge invariant since a more careful definition of the functional integral shows that it should be written as $\prod(\mathcal{L}\, d\sigma\, d\tau)$. We are grateful to B. Zumino for this remark.

[11]We shall discuss the problem of factorization elsewhere.

How to Generate the Pomeranchukon from the Background in a Dual Multiperipheral Model*

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A dual multiperipheral model is proposed in which duality plays an important role alongside multi-Reggeism and unitarity. Assuming that the Regge trajectories are generated through the resonating contributions, I find that the leading Reggeon has to bootstrap itself. Furthermore, a Pomeranchukon with intercept equal to unity is generated naturally through the background contributions, independent of the strength of the coupling constant. The bootstrapping potential and other interesting results of the model are discussed.

It is a difficult yet compelling problem for any model of hadronic production amplitudes to produce a reasonable total cross section. In Regge theory this amounts to understanding how the Pomeranchuk singularity is generated. The conspicuous energy dependence of the various topological cross sections, in contrast with the observed constant total cross sections, suggests that the Pomeranchukon is generated by a nondiffractive mechanism. In this Letter I propose a model in which duality plays an important role alongside multi-Reggeism and unitarity. The motivation is based on the following observations: First, the success of finite-energy sum rules[1] and the two-component scheme of Harari and Freund[2] indicates that the dual nature of hadronic amplitudes deserves serious attention in any dynamical model. Second, a multiperipheral approach[3,4] appears to be an attractive means of imposing unitarity to the dual-resonance amplitudes[5] (DRA's). This is because the DRA's approximate the actual amplitudes (with diffractive components ignored) best in the multiperipheral region, where the multiperipheral Regge model (MRM) is best suited to handle the unitarity sum. Thus it is very appealing to put together these two complementary models. Before going into detail, let us list the key features and assumptions of our model.

(a) Duality is incorporated into the model by using DRA's as input amplitudes[6] in the unitarity equation, and only stable particles are included in the intermediate states.

(b) With DRA's, each amplitude can be unambiguously separated into a resonating and a background component.

(c) Duality forces upon us a consistency condition that the leading Reggeon is generated by the sum of resonating components.

(d) We assume that the Pomeranchukon is generated by the sum of background components with the vacuum quantum number.

(e) We still have the basic ingredient of all MRM, that the amplitudes have factorized multi-Regge behavior with a strong cutoff in transverse momentum. There is, however, no need to assume this separately, since DRA's embody it.

For simplicity, we ignore the internal symmetries. It will become clear later that their inclusion does not affect our main results. Let us consider that there exists only one kind of stable particle, taken as a scalar meson of mass m, and carrying no additively conserved quantum numbers. By assumption (a), the amplitude for the process $a+b\rightarrow 1+2+\cdots+n$ can be written as a sum of $(n+2)$-point DRA's:

$$A_{ab\rightarrow n}=\sum_{\mathcal{P}(12\cdots n)} B_n(-p_a,p_1,p_2,\ldots,p_n,-p_b)+\tfrac{1}{2}\sum_{\mathcal{P}(12\cdots n)}\sum_{j=1}^{n-1} B_n^{(j)}(-p_a,p_1,p_2,\ldots,p_j,-p_b,p_{j+1},\ldots,p_n)$$
$$\equiv A_n^{r}(p_a,p_b;p_1,p_2,\ldots,p_n)+A_n^{b}(p_a,p_b;p_1,p_2,\ldots,p_n), \qquad (1)$$

where B and $B^{(j)}$ are DRA's[7] with cyclic ordering [see Fig. 1(a)], the same as shown by the momentum variables. The symbol $\mathcal{P}(12\cdots n)$ denotes permutations among the n final particles as required by Bose statistics. The factor $\frac{1}{2}$ eliminates the double counting due to $B_n^{(j)}=B_n^{(n-j)}$. Notice that every B ($B^{(j)}$) term has (has no) s-channel resonances since a and b are in adjacent (nonadjacent) order. So we identify the first (second) sum in (1) as the resonating (background) component of $A_{ab\rightarrow n}$, and have denoted it by A_n^{r} (A_n^{b}). The forward unitarity equation is

$$\mathrm{Im}A_{ab}(s,0)=\sum_{n=2}^{\infty}\frac{1}{n!}\int d\Phi_n\,|A_{ab\rightarrow n}|^2 \qquad (2a)$$
$$=\sum_{n=2}^{\infty}\frac{1}{n!}\int d\Phi_n[|A_n^{b}|^2+|A_n^{r}|^2+2\,\mathrm{Re}(A_n^{b}A_n^{r*})], \qquad (2b)$$

where

$$d\Phi_n=\tfrac{1}{2}(2\pi)^{4-3n}\,\delta^4(p_a+p_b-\sum_{i=1}^{n}p_i)\prod_{j=1}^{n}d^4p_j\delta^+(p_j^2-m^2).$$

The phase space $d\Phi_n$ can be separated into $n!$ equivalent regions corresponding to all the different orderings of the longitudinal momenta. Let us restrict ourselves to a particular region Φ_n^{0} specified by

$$p_{1\parallel}>p_{2\parallel}>\cdots>p_{n-1\parallel}>p_{n\parallel}. \qquad (3)$$

[We later replace $(1/n!)\int d\Phi_n$ by $\int d\Phi_n^{0}$.] With the order given by (3), we take s, s_i, η_i, and t_i as independent kinematical variables defined by

$$s=(p_a+p_b)^2,\quad t_i=(\sum_{j=1}^{i}p_j-p_a)^2,$$
$$s_i=s_{i,i+1},\quad s_{ij}=(p_i+p_{i+1}+\cdots+p_j)^2,$$
$$\eta_i=s_{i-1}s_i/s_{i-1,\,i+1}.$$

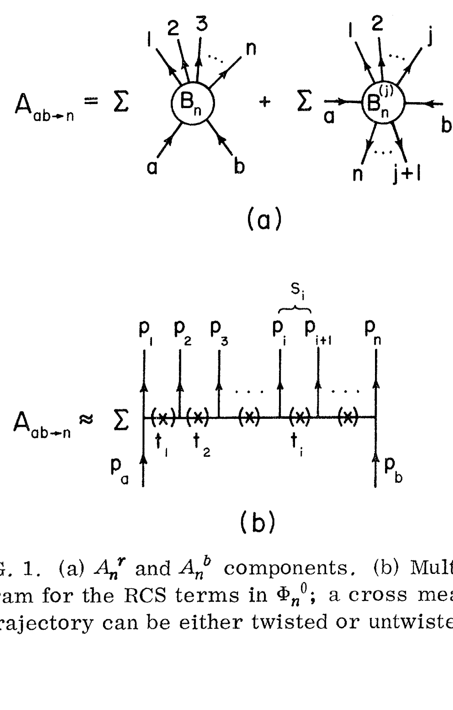

FIG. 1. (a) A_n^{r} and A_n^{b} components. (b) Multi-Regge diagram for the RCS terms in Φ_n^{0}; a cross means that the trajectory can be either twisted or untwisted.

The multiperipheral region (MR) in Φ_n^{0} is specified by letting all s_i become large and all transverse momenta $|\vec{p}_{i\perp}|$ small. We shall say a DRA has right chain structure (RCS) in Φ_n^{0} if it has Regge poles in *every* t_i channel; otherwise it has wrong chain structure (WCS). In the MR, the RCS term has power dependence on all s_i, whereas the WCS term is expected to be exponentially damped in s_j whenever t_j has no pole.[8] Hence, one is quite justified to neglect the WCS terms in favor of the RCS terms in any ordered MR. It is easy to see that each B_n term has RCS only in one region, while each $B_n^{(j)}$ term has RCS in as many as $n!/j!(n-j)!$ regions because of duality. *So in any ordered MR, there are effectively 2^{n-1} RCS terms contributing, all but one belonging to the background component.* The multi-Regge graphs representing the RCS terms in the MR of Φ_n^{0} is shown in Fig. 1(b), where a twisted propagator[9] is denoted by a cross. Taking $n=3$, for example, in the region Φ_3^{0}, the four RCS terms are those having cyclic ordering ($a123b$), ($a12b3$), ($a1b32$), and ($a13b2$). In the double-Regge limit, their sum (in respective order) is

$$A_{ab\to 3} \simeq (\varphi_1\varphi_2 + \varphi_1 + \varphi_2 + \rho)G_a(t_1)s_1^{\alpha_1} \times V(t_1, \eta_1 + i\epsilon, t_2)s_2^{\alpha_2}G_b(t_2), \qquad (4)$$

where G (V) is the single- (double-) Regge vertex function, $\alpha_i \equiv \alpha(t_i)$, $\varphi_i \equiv \exp(-i\pi\alpha_i)$, and $\rho \equiv V(t_1, \eta_1 - i\epsilon, t_2)/V(t_1, \eta_1 + i\epsilon, t_2)$. Notice that all four terms differ only by a phase. Since the vertex function $V_i \equiv V(t_i, \eta_i, t_{i+1})$ has a cut in η_i, factorization in DRA's for general n is rather complicated,[10] but can be formulated in matrix representation.[11] In order not to obscure the physics in the model, we shall ignore the cut of V_i. (The cut can be handled[11] and the results remain essentially the same.) Thus in the MR of Φ_n^0, we can write

$$A_{ab\to n} \simeq \left[\sum_{i=1}^{n-1} (1+\varphi_i)s_i^{\alpha_i}\right]G_a(t_1) \times \prod_{j=1}^{n-2} V_j G_b(t_{n-1}). \qquad (5)$$

Expansion of the product in Eq. (5) gives 2^{n-1} terms; each corresponds to a multi-Regge diagram by associating a twist propagator in t_i whenever φ_i is absent. Clearly only the term with phase

$$\prod_{i=1}^{n-1} \varphi_i$$

is the resonating component; hence we have

$$|A_n^r|^2 = |G_a(t_1)\prod_{i=1}^{n-2}(s_i^{\alpha_i}V_i)s_{n-1}^{\alpha_{n-1}}G_b(t_{n-1})|^2, \qquad (6)$$

$$|A_n^b|^2 = |\prod_{i=1}^{n-1}(1+\varphi_i) - \prod_{i=1}^{n-1}\varphi_i|^2 M_n^2, \qquad (7)$$

$$|A_n^b|_{\text{s.t.}}^2 = (2^{n-1}-1)M_n^2, \qquad (8)$$

$$A_n^{r*}A_n^b = \left[\prod_{i=1}^{n-1}\varphi_i^*(1+\varphi_i) - 1\right]M_n^2, \qquad (9)$$

where M_n^2 stands for the right-hand side of (6), and the subscript s.t. means taking only the squared terms. Let A_{ab}^{P} (A_{ab}^{R}) be the Pomeranchukon (leading Reggeon) exchange term in the forward absorptive part of $A_{ab}(s,t)$, with the asymptotic power $s^{\alpha_{\text{P}}}$ ($s^{\alpha_{\text{R}}}$). By our consistency condition (c) and assumption (d), we equate[12]

$$A_{ab}^{\text{R}} = \sum_{n=2}^{\infty} \int d\Phi_n^0 |A_n^r|^2, \qquad (10)$$

$$A_{ab}^{\text{P}} = \sum_{n=2}^{\infty} \int d\Phi_n^0 |A_n^b|_{\text{s.t.}}^2. \qquad (11)$$

It is straightforward to set up integral equations of the Chew-Goldberger-Low[4] type to evaluate the asymptotic powers of (10) and (11). But it is more transparent physically to use the scheme of the Chew-Pignotti[3] model. Following these authors, let G_a^2, G_b^2, and g^2 be, respectively, the effective values of $|G_a(t_1)|^2$, $|G_b(t_{n-1})|^2$, and $|V(t_i, \eta_i, t_{i+1})|^2$ integrated over the t dependence. Now (10) and (11) are simply given by[13] (let $m = n-2$)

$$A_{ab}^{\text{R}} \simeq e^{(2\alpha-1)Y}\sum_{m=0}^{\infty} G_a^2G_b^2(g^2)^m Y^m/m! = G_a^2G_b^2\exp[(2\alpha - 1 + g^2)Y], \qquad (12)$$

$$A_{ab}^{\text{P}} \simeq 2G_a^2G_b^2\exp[(2\alpha - 1 + 2g^2)Y], \qquad (13)$$

where $\alpha = \alpha(0)$, and Y is defined by $s = m_a^2 + m_b^2 + 2m_am_b\cosh Y$ [$Y \simeq \ln(s/m_am_b)$ for large s]. Identifying the leading power in both sides of (12) and (13), we finally obtain

$$\alpha_{\text{R}} = 2\alpha - 1 + g^2, \quad \alpha_{\text{P}} = 2\alpha - 1 + 2g^2. \qquad (14)$$

Hence

$$\alpha_{\text{P}} = 1 + 2(\alpha_{\text{R}} - \alpha). \qquad (15)$$

As α_{R} is the leading Reggeon, (15) tells us α has to be α_{R} itself; otherwise the Froissart bound is violated. With $\alpha = \alpha_{\text{R}}$, we have $\alpha_{\text{P}} = 1$ and $\alpha = 1 - g^2$. Thus we arrive at the very interesting conclusion that the *leading Reggeon must bootstrap itself through its multiexchanges in the resonating components, and its multiexchanges in the background components generate a Pomeranchukon with* $\alpha_{\text{P}} = 1$. It is remarkable that the bootstrap condition (c) actually demands the strength of the coupling constant to reach its maximum value allowed by unitarity. This offers a very natural and physically appealing explanation to the observed constancy of the total cross sections.

From Eq. (2b) we see that (10) and (11) are only the positive-definite part; the rest are all cross terms. It is difficult to sum them because of their complicated dependence on the phases φ_i. We can, however, give a rough estimate by setting all φ_i's equal to $e^{-i\pi\alpha}$. It is interesting to find that all the cross terms sum up to zero when $\alpha = \frac{1}{2}$.

The best way to include the internal symmetry in the model is perhaps to use the Chan-Paton factors.[14] This is rather involved and will not be discussed here. We merely mention that the twisted ladders occur only in the t-channel SU(3) singlet state, while the untwisted ladders have projection in both the singlet and octet states, hence the same kind of mechanism will generate the Pomeranchukon as a higher trajectory. The symmetry for the output-octet trajectories is preserved.

Taking the ratio of Eqs. (12) and (13), we obtain $A_{ab}{}^{\mathrm{R}}/A_{ab}{}^{\mathrm{P}} \simeq \frac{1}{2}\exp[(\alpha_{\mathrm{R}}-1)Y]$, where the coefficient in the right-hand side is independent of a or b. This *universality relation*[15] agrees quite remarkably with the experimental data (with a typical error of 20%) for $A_{pp}{}^{f_0}/A_{pp}{}^{\mathrm{P}} \approx A_{Kp}{}^{f_0}/A_{Kp}{}^{\mathrm{P}} \approx 1.2e^{-0.5Y}$. Other interesting relationships[16] can be derived by generalizing our result to cases with quantum numbers, such as (for ab exotic)

$$\sum_{n=2}^{\infty}\frac{n(\sigma_n{}^{a\bar{b}}-\sigma_n{}^{ab})}{\sigma^{a\bar{b}}-\sigma^{ab}}=\frac{1}{2}\sum_{n=2}^{\infty}\frac{n\sigma_n{}^{ab}}{\sigma^{ab}},\quad \frac{\sigma_n{}^{a\bar{b}}}{\sigma_n{}^{a\bar{b}}-\sigma_n{}^{ab}}=2^{n-1}.$$

Unfortunately, the present data available are not sensitive enough for a meaningful comparison of these relations.

In closing let us stress that this model provides a promising bootstrap framework for a realistic calculation of the Regge parameters appearing in both exclusive and inclusive cross sections. Applications along this line and refinements of the model will be published elsewhere.

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[12]When we consider quantum numbers, it is then easy to see that $|A_n{}^b|_{\mathrm{s.t.}}{}^2$ but not $|A_n{}^b|^2$ should be identified with the Pomeranchukon.

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[16]Notice that these relations and the result $\alpha_{\mathrm{P}}=1$ are all independent of g^2, hence their validity is probably more general than the Chew-Pignotti scheme which admits no correlations among produced particles. One can argue that when the kernel is improved to admit correlations, Eqs. (14) and (15) will retain the same form except with g^2 replaced by the strength of the new kernel.

New Sum Rule for Photoproduction Amplitudes Based on Local Current Commutators*

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A sum rule for isovector Compton scattering, previously derived using dispersive techniques, is rederived using the ++ light-cone commutator. The (nontrivial) limit of the sum rule at $q^2=0$ is given, and is found to be in excellent agreement with experiment. The deep-inelastic limit of the sum rule is discussed in the context of the covariant parton model, where it is shown that the free-field realization is untenable.

Some time ago, several authors[1,2] derived sum rules for the absorptive parts of certain of the invariant amplitudes appearing in a tensor decomposition of the full, spin-dependent, nonforward scattering of vector or axial-vector currents from nucleons. The sum rules were derived using the co-