# Ghost-Free String Picture of Veneziano Model*

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The recently proposed Lagrangian of a free string is quantized by means of path integrals. It is shown how Veneziano amplitudes are obtained in this picture by coupling the string with external sources.

Recently, Virasoro conditions[1] have been derived by studying the propagation of a free string in physical space-time.[2] For a free point particle the action is proportional to the length of the trajectory. In the case of a string, its successive positions for different times generate a two-dimensional surface $\Sigma$ in space-time, and it is natural to choose the action as proportional to the area of this surface. If one denotes

$$\dot{X}=\partial X/\partial\tau, \quad X'=\partial X/\partial\sigma, \tag{1}$$

the corresponding Lagrangian reads[2]

$$\mathcal{L}=-((\dot{X}\cdot X')^2-\dot{X}^2X'^2)^{1/2}. \tag{2}$$

The crucial remark is that the area of the surface does not depend on its parametrization. Indeed, it is easy to see that the action

$$I(X)=\int_\Sigma d\sigma\, d\tau\, \mathcal{L}(X(\sigma,\tau)) \tag{3}$$

is invariant under the general transformations

$$I(X)=I(X^\Omega), \tag{4}$$

where $\Omega$ is defined through

$$\sigma\to\sigma^\Omega(\sigma,\tau), \quad \tau\to\tau^\Omega(\sigma,\tau), \tag{5}$$

$$X_\mu(\sigma,\tau)\to X_\mu{}^\Omega(\sigma,\tau)=X_\mu(\sigma^{\Omega^{-1}}(\sigma,\tau),\tau^{\Omega^{-1}}(\sigma,\tau)), \tag{6}$$

$\sigma^\Omega(\sigma,\tau)$ and $\tau^\Omega(\sigma,\tau)$ being arbitrary functions of $\sigma$ and $\tau$. Therefore the theory has a built-in gauge invariance. This has been shown to be the origin of the Virasoro gauge conditions.

Let us introduce the notation

$$X_\pm=(1/\sqrt{2})(X_0\mp X_1), \quad \tilde{X}_l=X_l, \quad l=2,3,\ldots,d-1, \tag{7}$$

where $d$ is the number of dimensions of space-time. Goldstone *et al.*[4] have shown that because of the gauge invariance one can quantize the free string by considering only $\tilde{X}$ as an independent dynamical variable:

$$[\tilde{X}_l(\sigma,\tau),\dot{\tilde{X}}_m(\sigma',\tau)]=i\delta_{lm}\delta(\sigma-\sigma'), \quad \ddot{\tilde{X}}-\tilde{X}''=0, \tag{8}$$

provided that $d=26$ and that the intercept of the leading trajectory is 1. The gauge conditions of Ref. 4 are

$$X_+=(2/\pi)^{1/2}P_+\tau \tag{9}$$

and

$$\dot{X}^2+X'^2=\dot{X}\cdot X'=0, \tag{10}$$

where $P_\mu$ is the total momentum of the string, defined by

$$P_\mu=(2\pi)^{-1/2}\int_0^\pi d\sigma\,\partial\mathcal{L}/\partial\dot{X}_\mu. \tag{11}$$

It follows from (9) and (10) that $X_-$ is an $\tilde{X}$-dependent operator which is expressed as a bilinear function of $\tilde{X}$. Thus the basis of the Hilbert space can be taken as eigenvalues of $\tilde{X}(\sigma)$ and $P_+$ for on-shell states.

Up to now, however, only *free* strings have been considered. It is the purpose of this paper to show that, in the same way as in the analog model,[5,6] the dual amplitudes are obtained by coupling the string with external sources. We shall formulate the amplitudes in terms of Feynman path integrals.

In order to see the form of the path integral, let us consider first the following matrix element of the propagator of the string in the Hamiltonian formalism:

$$\langle\tilde{X}_f|\exp[-i(R+p^2)t]|\tilde{X}_i\rangle\exp[i\tilde{p}\cdot(\tilde{q}_i-\tilde{q}_f)], \tag{12}$$

where $R$ is related to the Hamiltonian $H$ of the string in the gauge of (9) and (10):

$$H=\frac{1}{2}\int_0^\pi d\sigma(\dot{\tilde{X}}^2+\tilde{X}'^2)=R+\tilde{p}^2+\langle 0|H|0\rangle . \tag{13}$$

$\tilde{q}$ is the center-of-mass position of the string, defined by

$$\tilde{q}=(2/\pi)^{1/2}\int_0^\pi d\sigma\,\tilde{X}(\sigma). \tag{14}$$

Using the relation between the operator and functional expression of the propagators we can write (12) as

$$\int\cdots\int_{X_i}^{X_f}\mathfrak{D}X(\sigma,\tau)\exp[i\int_0^\pi d\sigma\int_0^t d\tau\tilde{\mathfrak{L}}+2ip_+p_-t+i\tilde{p}\cdot(\tilde{q}_i-\tilde{q}_f)], \tag{15}$$

where $\tilde{\mathfrak{L}}$ is given by

$$\tilde{\mathfrak{L}}=\frac{1}{2}(\dot{X}^2-X'^2). \tag{16}$$

Let us define $D$ by

$$D^{-1}=\int\cdots\int\mathfrak{D}X_-\prod_{\sigma,\tau}\delta(\tfrac{1}{2}[\dot{X}(\sigma,\tau)-X'(\sigma,\tau)]^2); \tag{17}$$

then we obtain

$$\int\cdots\int\mathfrak{D}\tilde{X}\mathfrak{D}X_+\mathfrak{D}X_-D\prod_{\sigma,\tau}\delta(X_+(\sigma,\tau)-(2/\pi)^{1/2}p_+\tau)\delta(\tfrac{1}{2}(\dot{X}-X')^2)\exp[i\iint_\Sigma d\sigma\,d\tau\mathfrak{L}+ip\cdot(q_i-q_f)], \tag{18}$$

where $\mathfrak{L}$ is given by (2), which is linearized as a result of the condition $(\dot{X}-X')^2=0$ as

$$\mathfrak{L}=\frac{1}{2}(\dot{X}^2-X'^2). \tag{19}$$

The expression (18) is similar to the functional integral of gauge-invariant theory,[8] except that the factor $D$ is not exactly the same as the Faddeev-Popov determinant $\Delta$, which is defined by[9]

$$\Delta(X)\int d\Omega\prod_{\sigma,\tau}\delta(X_+^\Omega-(2/\pi)^{1/2}p_+\tau)\,\delta(\tfrac{1}{2}(\dot{X}^\Omega-X^{\Omega\prime})^2)=1 \tag{20}$$

as usual. It is not difficult to verify[10] that

$$D=\Delta(X)[\prod_{\sigma,\tau}\mathfrak{L}(X(\sigma,\tau))]^{-1}, \tag{21}$$

if $(\dot{X}-X')^2=0$ and $X_+=(2/\pi)^{1/2}P_+\tau$. Thus, in the functional integral there appears the factor $[\prod\mathfrak{L}]^{-1}$ as compared to the usual Faddeev-Popov expression.

Generalizing the expression (18) we define the generating functional of Green's functions by

$$W_t(J)=\int\cdots\int\mathfrak{D}^{(4)}X\Delta_f(X)\prod_{\sigma,\tau}[\mathfrak{L}^{-1}(X(\sigma,\tau))\delta(X_+(\sigma,\tau)-f(\sigma,\tau))\delta(\tfrac{1}{2}(\dot{X}-X')^2)]\exp[i\iint_\Sigma d\sigma\,d\tau(\mathfrak{L}+J_\mu X_\mu)], \tag{22}$$

where $f$ is a function of $\sigma,\tau$ and it fixes the gauge; $\Delta_f(X)$ is the Faddeev-Popov determinant defined by an expression similar to (20); and $J_\mu$ is the external source function. The integrations over $X_+$ and $X_-$ are carried out as follows: First the integration of $X_+$ is trivial because of the δ-functional

$$\prod_{\sigma,\tau}\delta(X_+-f).$$

Then we choose $f$ is such a way that $\mathfrak{L}+J\cdot X$ becomes independent of $X_-$. For this purpose we introduce the Green's function $N(y,y')$ which satisfies

$$(\partial^2/\partial\sigma^2-\partial^2/\partial\tau^2)N(y,y')=-i\,\delta^{(2)}(y-y'), \tag{23}$$

where by $y$ we denote a point of coordinates $(\sigma,\tau)$, with the boundary condition that the normal derivative of $N$ on the boundary is constant. The factor in the exponent becomes independent of $X_-$ if we choose

$$f(\sigma,\tau)=-i\int dy'N(y,y')J_+(y'). \tag{24}$$

Formula (22) then takes the form

$$W_t(J)=\exp[-\int dy\,dy'\,J_-(y)N(y,y')J_+(y')]\int\cdots\int \mathfrak{D}\tilde{X}\exp[i\int_\Sigma d\sigma\,d\tau(\tilde{\mathcal{L}}+\tilde{J}\cdot\tilde{X})]$$
$$\times\int\cdots\int \mathfrak{D}X_-\prod_{\sigma,\tau}[\mathcal{L}^{-1}\delta(\tfrac{1}{2}(\dot{X}-X')^2)]\Delta_f(X). \tag{25}$$

We now show that the last integral of (25) is 1. We remark that, in the functional integral, $\Delta_f(X)$ is multiplied by $\delta(X_+-f(\sigma,\tau))\delta(\frac{1}{2}(\dot{X}-X')^2)$.[12] We need, therefore, to determine $\Delta_f(X)$ only when $X_+=f(\sigma,\tau)$ and $(\dot{X}-X')^2=0$, in which case the integral which defines $\Delta_f$ has a contribution only from $\Omega=1$ and is equal to the corresponding Jacobian. Thus one gets

$$\Delta_f(X)\prod_{\sigma,\tau}\mathcal{L}^{-1}=\prod_{\sigma,\tau}(\dot{f}(\sigma,\tau)-f'(\sigma,\tau))\det(\partial_\sigma-\partial_\tau), \tag{26}$$

where $\det(\partial_\sigma-\partial_\tau)$ is to be taken in the functional sense. This formula allows us to integrate trivially over $X_-$ and one gets

$$\int\cdots\int \mathfrak{D}X_-[\prod_{\sigma,\tau}\mathcal{L}^{-1}]\Delta_f(X)\prod_{\sigma,\tau}\delta(\tfrac{1}{2}(\dot{X}-X')^2)=1. \tag{27}$$

The form we obtain is similar to the generating functional of Green's functions of quantum electrodynamics in the Coulomb gauge where the exponential of the Coulomb interaction energy also appears in front of the functional integral.

Finally, the integration over $\tilde{X}$ is performed as usual[6] and one obtains

$$W_t(J)=\exp[\tfrac{1}{2}\int dy\int dy'\,J_\mu(y)N(y,y')J_\mu(y')]\int\cdots\int \mathfrak{D}\tilde{X}\exp(i\int_\Sigma d\sigma\,d\tau\,\mathcal{L}). \tag{28}$$

This formula is similar to the result of the analog model. Differences are that the functional integral is now only over the transverse components and the Green's function $N$ is the one for the hyperbolic differential Eq. (23) instead of Laplace's.

Following the idea of Refs. 6 and 7, we express the integrand of dual amplitudes as quantum-mechanical transition matrix elements of strings, which can be computed from Eq. (28) by choosing the appropriate $J$. We prove now that Venziano amplitude can be derived from the string model. For this purpose we follow the same procedure as in Ref. 5 and compute

$$V(k_i\cdots k_N)=\lim_{\epsilon\to 0}\int\cdots\int \mathfrak{D}J\,W_t(J)\prod_{i=1}^{N}\int\frac{d\tau_i}{E(k_i^2)}\theta(\tau_{i+1}-\tau_i)\prod_{\sigma,\tau}\delta^{(4)}(J(\sigma,\tau)-\sum_{i=1}^{N}k_i\rho_i(\sigma,\tau)), \tag{29}$$

where $k_i$'s are the external momenta, $E(k_i^2)$ are the self-energy factors,[5] and

$$\lim_{\epsilon\to 0}\rho_i(\sigma,\tau)=\sqrt{2\pi}\,\delta(\sigma)\delta(\tau-\tau_i). \tag{30}$$

In this way we obtain the same form of Veneziano amplitude as in Ref. 6 except that we have now the Green's function (23) instead of Neumann's function. However, if we choose external momenta such that no singularities appear in the integrand, we can make a Wick rotation on $\tau_k$; i.e., $\tau_k\to -i\tau_k$, so that we obtain the usual expression, q.e.d.

Our transverse gauge is not factorizable since the gauge conditions depend on the external source function. However, it is possible to change the gauge to a factorizable one[11] defined by the condition $(\dot{X}\pm X')^2=0$.

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[3]Our Lorentz metric is $A_\mu B_\mu = A\cdot B = \vec{A}\cdot\vec{B} - A_0 B_0$. Since $\sigma$ and $\tau$ are considered as space and time variables, respectively, one has $\dot{X}^2 < 0$, $X'^2 > 0$.

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[9]This equation can be satisfied only if for all $X$ there exists gauge transformation $\Omega$ such that $(\dot{X}^\Omega - X'^\Omega)^2 = 0$ and $X_+^\Omega - (2/\pi)^{1/2} P_+ \tau = 0$. The latter condition can be satisfied because $X$ cannot vanish since $X^2 > 0$.

[10]This follows easily from Eq. (26) (see below). The factor $\prod \mathcal{L}$ which appears in $W(J)$ should be considered as gauge invariant since a more careful definition of the functional integral shows that it should be written as $\prod(\mathcal{L}\, d\sigma\, d\tau)$. We are grateful to B. Zumino for this remark.

[11]We shall discuss the problem of factorization elsewhere.

# How to Generate the Pomeranchukon from the Background in a Dual Multiperipheral Model*

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A dual multiperipheral model is proposed in which duality plays an important role alongside multi-Reggeism and unitarity. Assuming that the Regge trajectories are generated through the resonating contributions, I find that the leading Reggeon has to bootstrap itself. Furthermore, a Pomeranchukon with intercept equal to unity is generated naturally through the background contributions, independent of the strength of the coupling constant. The bootstrapping potential and other interesting results of the model are discussed.

It is a difficult yet compelling problem for any model of hadronic production amplitudes to produce a reasonable total cross section. In Regge theory this amounts to understanding how the Pomeranchuk singularity is generated. The conspicuous energy dependence of the various topological cross sections, in contrast with the observed constant total cross sections, suggests that the Pomeranchukon is generated by a nondiffractive mechanism. In this Letter I propose a model in which duality plays an important role alongside multi-Reggeism and unitarity. The motivation is based on the following observations: First, the success of finite-energy sum rules[1] and the two-component scheme of Harari and Freund[2] indicates that the dual nature of hadronic amplitudes deserves serious attention in any dynamical model. Second, a multiperipheral approach[3,4] appears to be an attractive means of imposing unitarity to the dual-resonance amplitudes[5] (DRA's). This is because the DRA's approximate the actual amplitudes (with diffractive components ignored) best in the multiperipheral region, where the multiperipheral Regge model (MRM) is best suited to handle the unitarity sum. Thus it is very appealing to put together these two complementary models. Before going into detail, let us list the key features and assumptions of our model.

(a) Duality is incorporated into the model by using DRA's as input amplitudes[6] in the unitarity equation, and only stable particles are included in the intermediate states.

(b) With DRA's, each amplitude can be unambiguously separated into a resonating and a background component.

(c) Duality forces upon us a consistency condition that the leading Reggeon is generated by the sum of resonating components.

(d) We assume that the Pomeranchukon is generated by the sum of background components with the vacuum quantum number.

(e) We still have the basic ingredient of all MRM, that the amplitudes have factorized multi-Regge behavior with a strong cutoff in transverse momentum. There is, however, no need to assume this separately, since DRA's embody it.

For simplicity, we ignore the internal symmetries. It will become clear later that their inclusion does not affect our main results. Let