Normalization of the N-d Tail of the Trinucleon Wave Function

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The normalization constant C of the N-d tail of the trinucleon wave function is derived from various sources. The measure of consistency between the extracted values, with $C^2(^3\mathrm{He})=3.0$ and $C^2(^3\mathrm{H})=3.4$, is strong support for according C the same status as other trinucleon parameters such as the binding energy, charge radius, and form factor.

The overlap function of the trinucleon and deuteron, defined by

$$u(\vec{r})=\int d^3r_{23}\,\varphi_3(\vec{r},\vec{r}_{23})\varphi_d(\vec{r}_{23}), \tag{1}$$

together with its Fourier transform, appears in diverse branches of few-nucleon physics.[1] Recently, Bolsterli and Hale,[2] upon applying a modified phase-shift analysis to p-^{3}He elastic scattering, and Kim and Tubis,[3] from a simple extrapolation of the exact ^{3}He wave function which they had calculated using the Reid soft-core potential, found

$$u(\vec{r})\xrightarrow[r\to\infty]{} C\varphi(\vec{r}),\quad \varphi(\vec{r})=(\alpha/2\pi)^{1/2}e^{-\alpha r}/r. \tag{2}$$

The asymptotic decay constant α is related to the separation energy in the usual way. Bolsterli and Hale extracted a value for $C^2(^3\mathrm{He})$ of 2.1 from the p-^{3}He data, while Kim and Tubis's value was 2.86. This constant has an important bearing on the study of the trinucleon system. The two extracted values differ sufficiently for us to seek independent confirmation of the correct magnitude from other experimental data. In this paper, $C^2(^3\mathrm{He})$ as well as $C^2(^3\mathrm{H})$ have been extracted from *reliable* sources wherein the overlap function $u(\vec{r})$ appears and must have the correct asymptotic behavior and magnitude. Specifically, I consider two-body (1) photodisintegration and (2) electrodisintegration of ^{3}He, (3) realistic triton wave functions utilized in transfer reactions, and (4) three-body elastic-scattering data.

(1) It is well known[4] that the main contribution to the electric-dipole matrix element for the reaction $^3\mathrm{He}+\gamma\to d+p$ comes from outside the range of nuclear forces where the asymptotic form of the wave function can be used. The expression for the cross section in this approximation is given by

$$d\sigma=(2\pi)^2(e^2/\hbar c)\,|M_{fi}|^2E_\gamma\rho_f, \tag{3}$$

where E_γ is the photon energy and ρ_f is the density of final states. The transition amplitude M_{fi} in Ref. 4 has the form

$$M_{fi}\propto N(1-\alpha_d r_0)^{-1/2}\iint d^3r_{23}\,d^3r\,\varphi_d{}^*(\vec{r}_{23})e^{-i\vec{q}\cdot\vec{r}}\vec{\epsilon}\cdot\vec{r}\,\varphi_d(\vec{r}_{23})\varphi(\vec{r}). \tag{4}$$

N is a renormalization factor, the constant $\alpha_d=0.232\ \mathrm{fm}^{-1}$, and r_0, the nucleon-nucleon triplet effective range, is 1.749 fm. The polarization vector is $\vec{\epsilon}$ while the normalized deuteron function

$$\varphi_d(\vec{r}_{23})=(\alpha_d/2\pi)^{1/2}\exp(-\alpha_d r_{23})/r_{23}. \tag{5}$$

It is easily seen upon carrying out the integration over $\vec{r}_{23}$ that

$$M_{fi}\propto N(1-\alpha_d r_0)^{-1/2}\int d^3r\,e^{-i\vec{q}\cdot\vec{r}}\vec{\epsilon}\cdot\vec{r}\,\varphi(\vec{r}) \tag{6}$$

and that

$$C^2(^3\mathrm{He})=N^2/(1-\alpha_d r_0)=3.0, \tag{7}$$

since Knight, O'Connell, and Prats found N^2 to be equal to 1.82 in order to fit experiment.

(2) The coincidence cross section for two-body electrodisintegration of ^{3}He ($e+{}^3\mathrm{He}\to e'+d+p$) presents a sensitive test of the asymptotic behavior of the ^{3}He wave function. In the analysis of the experimental data, the Fourier transform of the overlap function appears in the expression for the cross section:

$$d^3\sigma/dE\,d\Omega_e\,d\Omega_p\propto|I|^2, \tag{8}$$

where

$$I=\iint d^3r_{23}\,d^3r\,\varphi_d{}^*(\vec{r}_{23})\,e^{i\vec{q}\cdot\vec{r}}\varphi_3(\vec{r},\vec{r}_{23}) = \int d^3r\,e^{i\vec{q}\cdot\vec{r}}u(\vec{r}). \tag{9}$$

The important contributions to the $\vec{r}$ integration come from the intermediate and asymptotic regions of the overlap function for small values of the momentum $\vec{q}$. The absolute magnitude of the peak and neighboring dropoff in the cross section plotted as a function of the proton scattering angle will therefore depend directly on the magnitude of the asymptotical proton function in the d-p system. Since the expression for the cross section is not as heavily weighted in favor of the asymptotic region as in the photodisintegration reaction, a Hulthén form was chosen by the author so that the overlap function would manifest good intermediate behavior as well. Thus

$$u(\vec{r}) = \frac{1}{\beta - \alpha}\left[\frac{\alpha\beta(\alpha+\beta)}{2\pi}\right]^{1/2}\frac{e^{-\alpha r} - e^{-\beta r}}{r}; \qquad (10)$$

therefore,

$$C^2 = \beta(\alpha+\beta)/(\beta-\alpha)^2. \qquad (11)$$

The peak and dropoff in Johansson's data[5] was then fitted. The appropriate value of β gave $C^2(^3\mathrm{He}) = 3.0$.

(3) The overlap function appears in Goldfarb and Parry's analysis[6] of (^{3}H, d) Coulomb stripping reactions. Using the hard-core wave functions of Tang, Schmid, and Herndon[7] parametrized to fit the asymptotic form of the triton wave function correctly, Goldfarb and Parry found that

$$u(\vec{r}) \xrightarrow[r\to\infty]{} N_t e^{-\alpha r}/r, \qquad (12)$$

where

$$N_t = MN_0^{1/2}/3\pi\hbar^2. \qquad (13)$$

N_0, the zero-range normalization for their distorted-wave Born-approximation expression, was 3.32×10^4 MeV^2 fm^3. The extracted value of the asymptotic constant is $C^2(^3\mathrm{H}) = 3.3$.

(4) The ^2S channel for elastic n-d scattering shows unique anomalous features around threshold.[8] From a fit to the straight section of this anomalous effective-range plot with a Hulthén parametrization of $u(\vec{r})$, Kok and Rinat[9] determined that $\beta = 1.18$ fm^{-1}. The forward-dispersion-relations technique of Ericson and Locher[10] likewise gives this same value of β. From this source, a value of $C^2(^3\mathrm{H}) = 3.5$ is found.

In conclusion, there is mutual agreement about the magnitude of C^2 between the various sources considered in this paper. The value given by Kim and Tubis is favored. The agreement should be even better if the experimental separation energy is used in their analysis. This work indicates that C has direct experimental significance and that it should be elevated to the same status as other experimental trinucleon parameters. It is also clear that $C^2(^3\mathrm{He})$ and $C^2(^3\mathrm{H})$ may differ by 10%.

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Evidence for Well-Developed Band Structure in ^{101}Pd

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Three γ-ray cascades have been excited in ^{101}Pd by (heavy-ion, xn) reactions. Excitation functions, γ-ray angular distributions, and γ-γ coincidence measurements were used to show that the γ rays came from three bands built on $\frac{5}{2}^+$, $\frac{7}{2}^+$, and $\frac{11}{2}^-$ intrinsic states.

We have observed three γ-ray cascades descending through high-angular-momentum states in the even-odd nucleus ^{101}Pd. The cascades are produced by states which seem to be members of collective bands built on the $\frac{5}{2}^+$, $\frac{7}{2}^+$, and $\frac{11}{2}^-$ intrinsic states of the nucleus as shown in Fig. 1. This is the first time that three well-developed, apparently collective bands have been seen in a nucleus which is not strongly deformed. The angular-momentum difference between most of the