Coil-Excited Low-Frequency Waves in a Single-Ended Q Machine

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Two modes were excited by applying oscillating currents to a small coil surrounding a plasma column in a single-ended Q machine. One of them had the same dispersion relation as grid-excited waves, beyond about half a wavelength from the exciting position. The other was an electrostatic ion cyclotron wave propagating along a magnetized plasma column.

Propagation and damping of ion acoustic waves were first measured by Hatta and Sato[1] and Little.[2] The former excited the waves by applying oscillating potentials to a grid immersed in a plasma. Ever since grid excitation was used in the experiment on Landau damping of ion acoustic waves in a Q-machine plasma,[3] many experiments have been made on related problems by using the same method in Q machines.[4] It was, however, pointed out that in such a plasma there should be a significant contribution from free-streaming particles to spatial evolutions of excited perturbations.[5] Recently, Estabrook and Alexeff emphasized nonexistence of ion acoustic waves and Landau damping driven by a *grid* in an *ideal* Q machine, according to their computer simulation.[6] In this Letter, two quite different methods of excitation —grid excitation and coil excitation—are reported in order to clarify the dependence of wave properties on the exciting method in a single-ended Q machine. For the case of coil excitation, perturbations are produced by applying oscillating currents to a small coil surrounding a plasma column.[2] Thus, there is no electric field parallel to the propagation, in contrast to the case for grid excitation.

A Cs plasma (about 4 cm in diameter and 120 cm long), produced by surface ionization on a tungsten plate at 1500–2000°K, is confined by a uniform magnetic field B_0, 1.2–2.5 kG, and is terminated at a 5-cm-diam target biased negatively.[7] The plasma density N is $(1-5)\times10^8$ cm^{-3} and the electron temperature T_e is 2100–3000°K. The ion temperature T_i, 1200–2000°K, and the plasma flow speed, comparable to the ion thermal speed, are estimated from wave propagation and damping[7] and from an electrostatic energy analyzer.[8] The collision mean free paths of charged particles are longer than the plasma length. 25 turns of 2-mm-diam Cu wire are wound around the plasma column to make an exciting coil of 4 cm i.d. and 0.8 cm in width, which is set at 40 cm from the hot plate in the chamber [see Fig. 1(a)]. The magnetic perturbation $\vec{B}$, produced by the coil, are 20–100 G peak to peak. A grid 4 cm in diameter made of 0.1-mm-diam

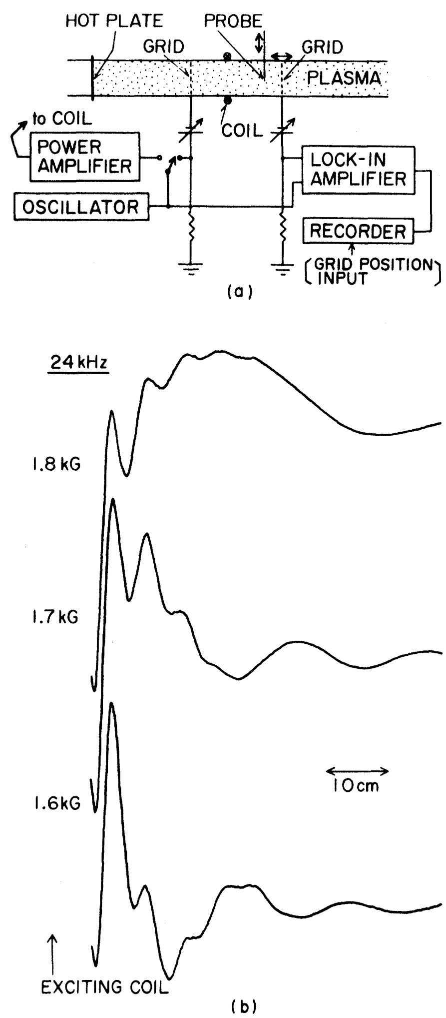

FIG. 1. (a) Schematic diagram of the experimental apparatus. (b) Typical wave patterns of coil-excited waves with $\omega > \omega_{ci}$, demonstrating their dependence on magnetic field B_0 at $\omega/2\pi = 24$ kHz.

Mo wires spaced 2 mm apart (Debye length $\ll$ mesh size $\lesssim$ ion Larmor radius) is set at 10.8 cm upward from the coil. Thus, all particles flow down across the coil after passing through the grid. The grid bias, slightly negative with respect to the plasma ($-0.5 \sim -1.0$ V), is kept at the same value for both methods of excitation. Exciting potentials of frequency $\omega/2\pi$, 5–100 kHz, are in the range 0.01–1 V peak to peak, where the phenomena do not depend on them.[9] The signals are received by a similar grid or a probe, biased at -10 V to pick up ion currents. They are fed into the interferometer system, together with the reference signals, which gives the wave patterns along the plasma column.[7,9]

Measurements are made on the signals propagating in the direction of the plasma flow. As already reported elsewhere,[7] the phase velocity ω/k_r and damping factor k_i of grid-excited waves depend on position up to about half a wavelength from the exciter. Beyond there, however, they are independent of position. Almost the same wave patterns are obtained for the case of coil excitation, as long as $\omega < \omega_{ci}$, the ion cyclotron frequency. Since $\vec{B}$ is not restricted just under the coil, it is impossible to compare coil-excited waves with grid-excited ones in the vicinity of the exciters. Figure 2 shows the values of k_r and k_i versus ω, measured beyond half a wavelength or so from the coil. The results agree with those for grid-excited waves, which are also plotted in this figure.

On the other hand, if ω is larger than a frequency ω_0 slightly above ω_{ci}, the wave patterns are different from those of grid-excited waves. The patterns consist of two modes having different wavelengths [see Fig. 1(b)]. One of them, which has a short wavelength, is the mode mentioned above. Its wavelength is independent of B_0. The wavelength of the other increases with increasing B_0. As shown in Fig. 3(a), there is a cutoff frequency ω_0 slightly above ω_{ci}. As ω is increased, ω/k_r decreases and approaches a constant value. It is also found in Fig. 3(b) that the damping rate k_i/k_r decreases gradually as ω is decreased toward ω_0.

The properties of the fast mode agree with the

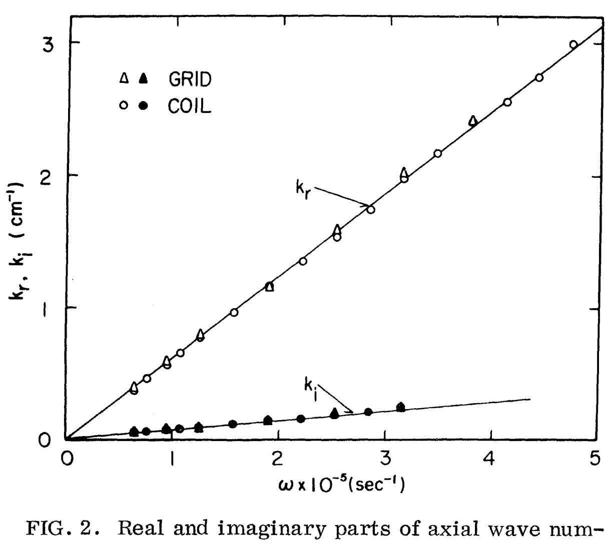

FIG. 2. Real and imaginary parts of axial wave numbers, k_r and k_i, as functions of angular frequency ω, measured for coil-excited waves (the slow mode), together with those of grid-excited waves. The best-fit lines give $\omega/k_r = 1.51\times10^5$ cm/sec and $k_i/k_r = 0.113$. Detailed measurements of grid-excited waves were reported in Ref. 7.

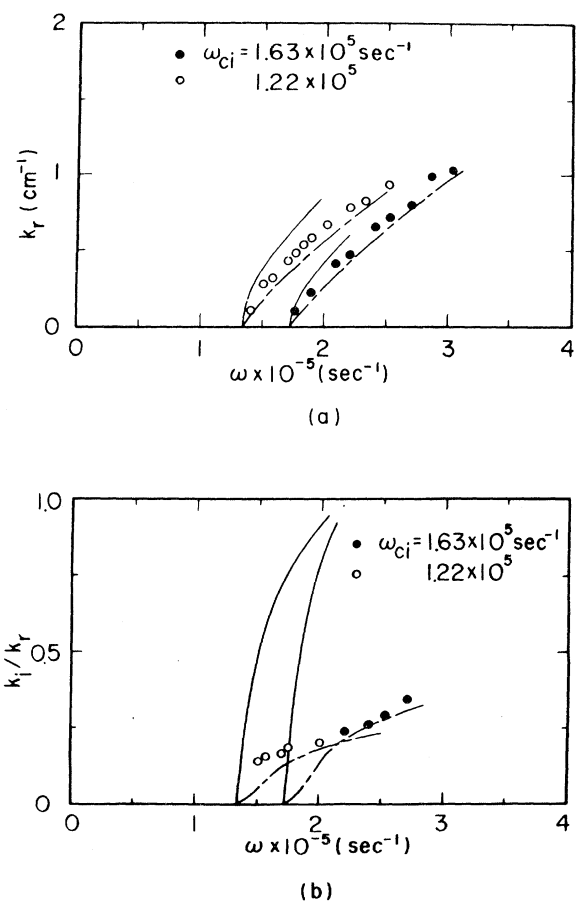

FIG. 3. Examples of measured dispersion curves of coil-excited waves (the fast mode). (a) Real part of axial wave number k_r and (b) damping rate k_i/k_r as a function of angular frequency ω at $\omega_{ci} = 1.22\times10^5$ and 1.63×10^5 sec^{-1}. Theoretical curves are shown by solid lines for $v_0 = 0$ and by dot-dashed lines for $v_0 = 2a_i$, which corresponds to the experiment.

characteristic behavior of electrostatic ion cyclotron waves propagating along a plasma column in an axial magnetic field. A general dispersion relation for the electrostatic waves, under the assumption of Maxwellian velocity distributions, is given by[10]

$$k_{\parallel}^2+k_{\perp}^2+\sum_{e,i}\sum_{n=-\infty}^{\infty}\frac{m\omega_p^2\exp(\lambda)I_n(\lambda)}{\kappa T}[1+\alpha_0 Z(\alpha_n)]\simeq 0, \tag{1}$$

where $\lambda=k_{\perp}^2\kappa T/m\omega_c^2$, $\alpha_n=(\omega+n\omega_c)/k_{\parallel}a$, $\omega_p^2=4\pi Ne^2/m$, $\omega_c=|eB/mc|$, $a^2=2\kappa T/m$, $I_n(\lambda)$ is the nth-order modified Bessel function, $Z(\varphi)$ is the plasma dispersion function,[11] $\sum_{e,i}$ expresses the sum of the quantities for electrons and ions, and $k_{\parallel}$ and $k_{\perp}$ are the components of the wave number parallel and perpendicular to the magnetic field, respectively. In our case, $k_{\parallel}^2, k_{\perp}^2 \ll k_D^2$ $(=4\pi Ne^2/\kappa T_e)$, $\lambda_i \ll 1$, and ω is so small that electrons maintain a Boltzmann equilibrium distribution. Thus, Eq. (1) is approximated by

$$2\frac{T_i}{T_e}\simeq(1-\lambda_i)Z'(\alpha_{i,0})+\lambda_i\left[\frac{2\omega_{ci}^2}{\omega^2-\omega_{ci}^2}+\frac{\alpha_{i,0}}{2}\left(\frac{Z'(\alpha_{i,1})}{\alpha_{i,1}}+\frac{Z'(\alpha_{i,-1})}{\alpha_{i,-1}}\right)\right]. \tag{2}$$

By assuming $1\ll|\alpha_{i,0}|$ $(<|\alpha_{i,1}|)$ so that $Z'(\alpha_{i,0,1}) \simeq \alpha_{i,0,1}^{-2}$, and by neglecting terms small for $2\theta^{-1} = 2T_i/T_e \simeq 1$ and $\beta=\omega_{ci}/\omega \lesssim 1$, Eq. (2) becomes

$$4[1-\beta^2(1+\theta\lambda_i)]/\theta\lambda_i(1+\beta)\simeq Z'(\varphi), \tag{3}$$

where $\alpha_{i,-1}$ is replaced by $\varphi=\varphi_r-j\varphi_i$. The term on the left-hand side of Eq. (3) is determined experimentally, and so this equation can be easily solved for φ (see Ref. 11). The phase velocity ω/k_r and the damping rate k_i/k_r $(k_{\parallel}=k_r+jk_i)$ are obtained from

$$\frac{\omega}{k_r a_i}=\frac{\varphi_r}{1-\beta}\left[1+\left(\frac{\varphi_i}{\varphi_r}\right)^2\right],\quad \frac{k_i}{k_r}=\frac{\varphi_i}{\varphi_r}. \tag{4}$$

The predicted dispersion relation is shown by solid lines in Fig. 3, where the measured values $T_e=2620°$K and $T_i=1360°$K are used. Since the mode (0, 1) is confirmed by the amplitude variations of the plasma cross section, $k_{\perp}$ is calculated from the relation $k_{\perp}=2.40/$[plasma radius $(=1.9)$]. In our experiment, however, there is a plasma flow of velocity v_0 $(\simeq 2a_i)$. Replacing ω by $\omega-k_{\parallel}v_0$ in Eqs. (1) and (2) and φ_r by φ_r+v_0/a_i in Eq. (4), we can derive the dispersion relation for Maxwellian distributions shifted by v_0, which is shown by dot-dashed lines for $v_0=2a_i$ in Fig. 3.[12] In this figure, the measured values are found between the solid curves and the dot-dashed curves, close to the latter. These curves are the dispersion relations for electrostatic ion cyclotron waves with $\omega>\omega_{ci}$, where $|\alpha_{i,0,1}|\gg 1$ is satisfied even for $\theta\simeq 1$. The slight difference between the experimental data and the predicted curves for $v_0=2a_i$ might be mainly attributed to the assumption of the shifted-Maxwellian ion velocity distribution.[8]

It is well known that, under some conditions, a grid also excites a so-called "pseudowave," which is a burst of ions accelerated by an electric field applied parallel to the propagation.[13] For coil excitation, however, the induced electric field is azimuthal and so this kind of pseudowave should not be excited. In the experiment, the grid-excited waves have the same dispersion relation as the slow mode excited by the coil, in spite of quite different exciting mechanisms.[14] Thus, it is concluded that the pseudowaves do not contribute to the spatial evolution of grid-excited waves for our conditions. The mode (0, 0)[15] ("principal mode," $k_{\perp}=0$) is confirmed for grid-excited waves by their almost constant amplitudes of the plasma cross section. It is natural that the radially varying modes are also excited by the coil, because the perturbations are produced by the radial compression of the plasma. Electrostatic ion cyclotron waves propagating along the magnetic field were excited previously with the same method in a weakly ionized plasma having $\theta\gg 1$.[16] They are ion acoustic waves, the mode of which is (0, 1), propagating along a plasma column in an axial magnetic field. In our case, two modes are excited and the collisionless damping due to the finite value of T_i is clearly observed. Finally, there is no significant contribution of free-streaming particles to the fast mode, because the property of this mode can be explained only by the plasma collective motion.

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[14]We must also pay attention to the work done on nonlinear plasma waves in single-ended Q machines, in which grid-excited waves have the same dispersion relation as low-frequency waves (predicted to be ion acoustic waves) produced by quite different exciting processes, i.e., wave-wave coupling [D. Phelps, N. Rynn, and G. Van Hoven, Phys. Rev. Lett. 26, 691 (1971)] and the decay instability [R. N. Franklin, S. M. Hamberger, G. Lampis, and G. L. Smith, Phys. Rev. Lett. 27, 1119 (1971)].

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Correlation Range of Fluctuations of Short-Range Order in the Isotropic Phase of a Liquid Crystal*

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We have measured the correlation range for fluctuations of the orientational order parameter in the isotropic phase of the liquid crystal *p*-methoxy benzylidene, *p-n* butyl aniline (MBBA). The correlation range was found to diverge as $(T - T_c^*)^{-1/2}$, in agreement with the predictions of de Gennes. This result is in contradiction with the behavior reported by Chu, Bak, and Lin.

The nematic-isotropic phase transition has been the subject of considerable recent research.[1-4] Although a first-order phase transition, it displays such a striking variety of pretransitional phenomena that it might be said to be "almost" second order. In the isotropic phase the light-scattering power[1] (Rayleigh ratio), the magnetic[1] and flow[2] induced birefringence, and the relaxation time[3] for the fluctuations in short-range order all increase rapidly as the phase transition is approached. These observations are all satisfactorily accounted for[4] by a phenomenological (Landau type) model proposed by de Gennes.[5] There remains one prediction of this model which has yet to be confirmed—the temperature dependence of the correlation length for short-range order in the isotropic phase. In this Letter we report experimental measurements of this correlation length which agree with the predictions of the de Gennes model.

An appropriate order parameter for a nematic liquid crystal is a symmetric traceless second-rank tensor proportional to the anisotropy in the dielectric-constant tensor.[5] One may write this as $Q_{\alpha\beta} = (3/2\Delta\epsilon)(\epsilon_{\alpha\beta} - \bar{\epsilon}\delta_{\alpha\beta})$, where $\Delta\epsilon$ is the maximum anisotropy for a perfectly ordered liquid crystal. In the de Gennes model the free energy is assumed to be an analytic function of temperature and order parameter. In the vicinity of the phase transition the free-energy density has the form $(\partial_\alpha \equiv \partial/\partial x_\alpha)$

$$\varphi = \varphi_0 + \tfrac{1}{2}AQ_{\alpha\beta}Q_{\beta\alpha} - \tfrac{1}{3}BQ_{\alpha\beta}Q_{\beta\gamma}Q_{\gamma\alpha} + \tfrac{1}{4}C(Q_{\alpha\beta}Q_{\beta\alpha})^2 + \tfrac{1}{2}L_1\,\partial_\alpha Q_{\beta\gamma}\,\partial_\alpha Q_{\beta\gamma} + \tfrac{1}{2}L_2\,\partial_\alpha Q_{\alpha\gamma}\,\partial_\beta Q_{\beta\gamma} + \cdots. \tag{1}$$